ENGINEERING TRIPOS PART IIB, 2005, MODULE 4C8. SOLUTIONS.

Q1

(a) Assumptions used to derive standard "bicycle" model:

- Small angles
- Linear creep, neglect tyre realigning moments
- 2 tyres on each axle have same slip angle
- Neglect motion of sprung mass on suspension

(b) With F applied forward of G, $N = Fx$ and $Y = F$.

In the steady state, $\dot{v} = \dot{\Omega} = 0$, so the eq's become

$$\begin{bmatrix} C/u & Cs/u + mu \\ Cs/u & Cq^2/u \end{bmatrix} \begin{Bmatrix} v_{ss} \\ \Omega_{ss} \end{Bmatrix} = \begin{Bmatrix} F \\ xF \end{Bmatrix} \quad \text{— (1)}$$

Where $C = C_f + C_r$, $s = \dfrac{aC_f - bC_r}{C_f + C_r}$, $q^2 = \dfrac{a^2 C_f + b^2 C_r}{C_f + C_r}$ — (2)

Inverting (1) gives

$$\begin{Bmatrix} v_{ss} \\ \Omega_{ss} \end{Bmatrix} = \frac{F \begin{bmatrix} Cq^2/u & -\left(\frac{Cs}{u} + mu\right) \\ -Cs/u & C/u \end{bmatrix} \begin{Bmatrix} 1 \\ x \end{Bmatrix}}{(C/u)(Cq^2/u) - (Cs/u)(Cs/u + mu)} \quad \text{— (3)}$$

Using (2), denominator is $1/u^2 [C_f C_r l^2 - Csmu^2]$, $(l = a + b)$

So (3) $\rightarrow$

$$\frac{v_{ss}}{u} = \beta_{ss} = \left[\frac{Cq^2 - x(Cs + mu^2)}{C_f C_r l^2 - Csmu^2}\right] F$$

&

$$\frac{\Omega_{ss}}{u} = \left[\frac{C(x - s)}{C_f C_r l^2 - Csmu^2}\right] F \quad \text{— (4)}$$

(c) If $x = s$ then $\Omega_{ss} = 0$ for any vehicle. This point on the vehicle $[x = (aC_f - bC_r)/(C_f + C_r)]$ is known as the Neutral Steer Point (NS).

1 (cont) The Static margin is the distance that the NS point lies behind G, normalised by the wheelbase, l, ie $SM = -s/l$. The vehicle effectively rotates about NS. Understeer and oversteer refer to the motion of the vehicle when F is applied at G, ie $x = 0$.
In this case, from (4):

If $\underline{s=0}$, $\underline{SM=0}$ $\quad \frac{\Omega_{ss}}{u} = 0$ and $\beta_{ss} = \frac{cq^2}{C_f C_r l^2}$ = Const all speeds

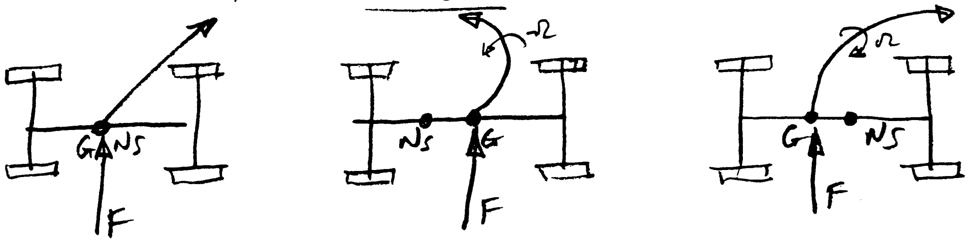
This is neutral steer $\Rightarrow$ NS coincides with G.

If $\underline{s<0}$, $\underline{SM>0}$ $\quad$ then $\frac{\Omega_{ss}}{u} > 0$ & $\beta > 0$ all speeds

This is understeer $\Rightarrow$ NS aft of G

If $\underline{s>0}$, $\underline{SM<0}$ $\quad$ then for $u < \sqrt{\frac{C_f C_r l^2}{c s m}}$, $\frac{\Omega_{ss}}{u} < 0$ & $\beta_{ss} > 0$

This is oversteer $\Rightarrow$ NS forward of G



Neutral steer ($SM=0$) $\qquad$ Understeer ($SM>0$) $\qquad$ Oversteer ($SM<0$)

(d) If δ is applied and held, in the steady state $\Omega = u/R$, $\dot{\Omega} = \dot{v} = 0$, Eq's of motion $\Rightarrow$

$$\begin{bmatrix} c & cs + mu^2 \\ cs & cq^2 \end{bmatrix} \begin{Bmatrix} \beta \\ 1/R \end{Bmatrix} = \begin{Bmatrix} C_f\delta \\ aC_f\delta \end{Bmatrix} \qquad — (5)$$

Solving (5) for R gives $\frac{1/R}{\delta} = \frac{cC_f(a-s)}{C_f C_r l^2 - csmu^2} = \frac{l C_f C_r}{C_f C_r l^2 - csmu^2}$

1(cont) So $\delta = \frac{l}{R}\left(1 - \frac{C_s m u^2}{l^2 C_f C_r}\right)$ —— (6)

Differentiate (6) to find speed sensitivity:

$$\frac{d\delta}{du} = -\frac{2l}{R}\left(\frac{C_s m}{l^2 C_f C_r}\right) \quad \text{—— (7)}$$

Neutral steer $(S=0) \rightarrow \delta = l/R$ & $d\delta/du = 0 \quad \forall u$

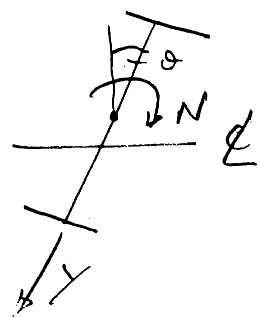
Understeer $(S<0) \rightarrow d\delta/du > 0$, all speeds so more steer angle is needed for higher speeds

Oversteer $(S>0) \rightarrow d\delta/du < 0$, all speeds. Vehicle becomes unstable with $\delta = 0$ when $u = \sqrt{\frac{C_f C_r l^2}{C_s m}}$

Q2(a) Force and moment acting on rigid wheelset see lecture notes (derivation required). In general case, net yawing moment is

$$N = 2dC\left(\frac{\varepsilon y}{r} - \frac{d\dot{\theta}}{u} - d/R\right)$$

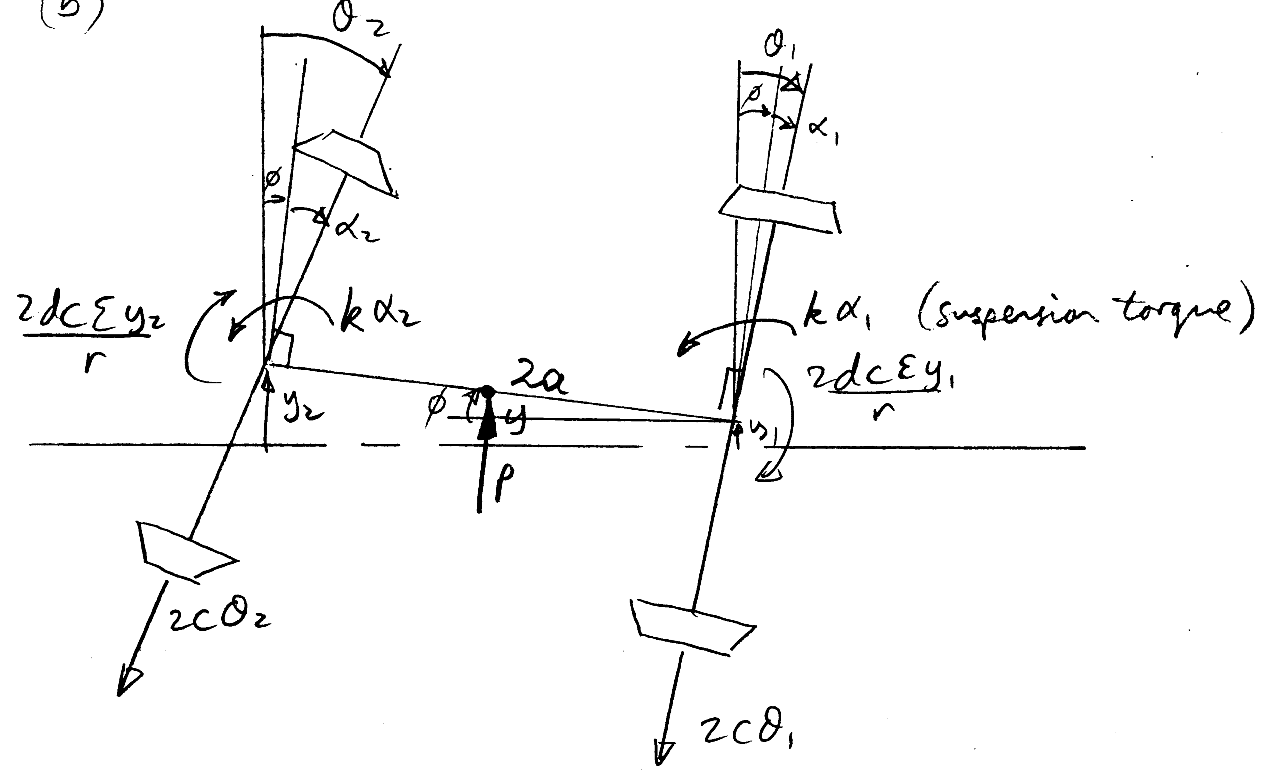
$$\& \quad Y = 2C(\theta + \dot{y}/u)$$



So for straight line motion $R \to \infty$
and in the steady state $\dot{\theta} = \dot{y} = 0$

$$N_1 = \frac{2dC\varepsilon y_1}{r} \quad \& \quad Y_1 = 2C\theta_1$$

(b)



$$\left.\begin{aligned} \phi &= \frac{y_2 - y_1}{2a} & \alpha_2 &= \theta_2 - \phi = \theta_2 - \left(\frac{y_2 - y_1}{2a}\right) \\ y &= \frac{y_1 + y_2}{2} & \alpha_1 &= \theta_1 - \left(\frac{y_2 - y_1}{2a}\right) \end{aligned}\right\} \quad (1)$$

Moment equilibrium for each axle:

$$\frac{2dC\varepsilon y_1}{r} = k\left[\theta_1 + \left(\frac{y_1 - y_2}{2a}\right)\right] \quad \text{———} \quad (2)$$

$$\frac{2dC\varepsilon y_2}{r} = k\left[\theta_2 + \left(\frac{y_1 - y_2}{2a}\right)\right] \quad \text{———} \quad (3)$$

(2) cont

Force equilibrium on whole vehicle

$$2c(\theta_1 + \theta_2) = P \qquad \text{—— (4)}$$

2 Moments about frame centre

$$\frac{2dc\varepsilon}{r}(y_1 + y_2) + 2ac(\theta_1 - \theta_2) = 0 \qquad \text{—— (5)}$$

4 variables $\theta_1, \theta_2, y_1, y_2$ & 4 equations (2)-(5)

(c) If $k \to \infty$, (1) $\Rightarrow$ $\theta_1 + \frac{y_1 - y_2}{2a} = \frac{2dc\varepsilon y_1}{rk} \to 0$

Similarly (2) $\Rightarrow$ $\theta_2 + \frac{y_1 - y_2}{2a} = 0$

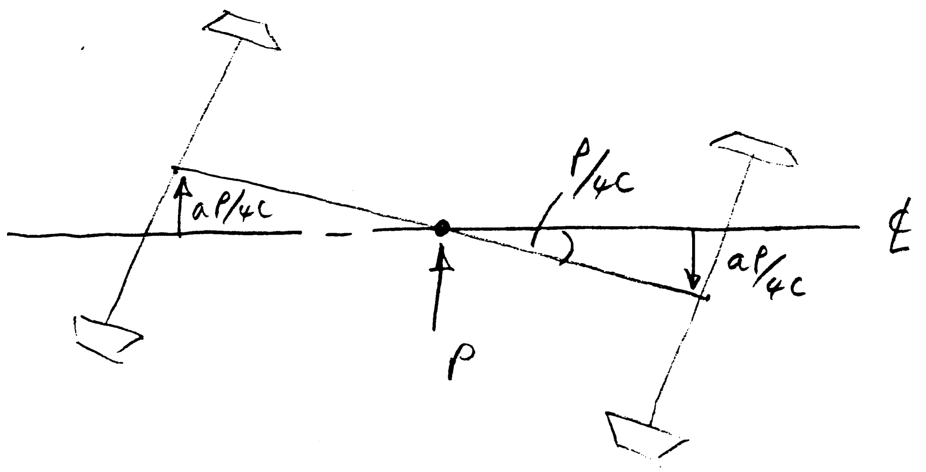
So $\theta_1 = \theta_2 = \frac{y_2 - y_1}{2a} = \phi$ (from (1))

$2y =$

Hence (4) $\Rightarrow$ $\phi = P/4c$

and (5) $\Rightarrow$ $y_1 = -y_2 = a\phi = \frac{aP}{4c}$

$\therefore$ lateral tracking error at the centre of the bogie is zero ie $y = \frac{y_1 + y_2}{2} = 0$



Q~~1~~2 answer

(a)
Sprung mass acceleration – relates to the discomfort felt by the occupants of the vehicle.

Suspension working space – working space is important because of body styling requirements, ground clearance, or articulation limits on the suspension linkage and driveshafts. Working space is usually a constraint rather than something to be minimised. The conflict diagram shows only the working space centred on the static suspension deflection. Braking and cornering manoeuvres can require additional working space, particularly if the suspension is soft.

Dynamic tyre force – fluctuation in the normal force between the tyre and the road reduces the cornering and braking forces that can be generated. In heavy vehicles, the dynamic tyre force can contribute to road damage.

(b)
Note that damping contours are not shown in the conflict diagram for clarity. Damping increases when moving along the stiffness contours from right to left.

The minimum dynamic tyre force occurs at B, with k=20kN/m. However minimum acceleration is achieved with less damping, at point D. In moving from B to D the working space increases, but this is probably still within the acceptable range for a passenger car. Points between B and D represent optimum designs for this value of suspension stiffness; moving the damping outside of this region causes both the acceleration and the tyre force to increase.

The acceleration can be reduced further by decreasing the stiffness to say 10kN/m, point C, with no significant increase in tyre force or working space. However, the low value of stiffness may cause problems with large suspension deflection under the action of cornering or braking manoeuvres. Further reduction in acceleration can be achieved by a further reduction in damping, to point E, but with increases in tyre force and working space.

Point A is more appropriate for a racing car. The stiff suspension, 80kN/m, limits the deflection due to braking and cornering manoeuvres, which compensates for the dynamic tyre force not being the minimum available (point B). Note that at point A one value of damping minimises tyre force and acceleration simultaneously.

An active suspension can provide an infinite static stiffness and thus allow the conflict between acceleration and tyre force to be resolved without concern for suspension deflection due to braking and cornering manoeuvres.

(c)

(i) Use Newton's 2nd law on the two masses to derive the equations of motion.

$$m_s \ddot{z}_s = k(z_u - z_s) + c(\dot{z}_u - \dot{z}_s)$$

$$m_u \ddot{z}_u = k(z_s - z_u) + c(\dot{z}_s - \dot{z}_u) + k_t(z_r - z_u)$$

(ii) Take Laplace transforms and replace s with $j\omega$ to derive the transfer function between z_r and z_s. Then convert to a transfer function H between $\dot{z}_r$ and $\ddot{z}_s$ by multiplying by $j\omega$.

$$H(j\omega) = \frac{\ddot{z}_s(j\omega)}{\dot{z}_r(j\omega)} = j\omega \frac{z_s(j\omega)}{z_r(j\omega)} = \frac{(j\omega)^2 ck_t + (j\omega)kk_t}{D(j\omega)}$$

where

$$D(j\omega) = (j\omega)^4 m_s m_u + (j\omega)^3 (m_s + m_u)c + (j\omega)^2 (m_s(k + k_t) + km_u) + (j\omega)ck_t + kk_t$$

(iii) Evaluate the mean square response of $\ddot{z}_s$ to a white noise velocity input $\dot{z}_r$ of spectral density S_0 using:

$$E\left[\ddot{z}_s^2\right] = \int_{\omega=-\infty}^{\omega=\infty} S_{\ddot{z}_s}(\omega)d\omega = \int_{\omega=-\infty}^{\omega=\infty} |H(\omega)|^2 S_0(\omega)d\omega$$

A closed-form solution can be obtained by using the expression

$$\int_{-\infty}^{\infty} |H(j\omega)|^2 \mathrm{d}\omega = \frac{-\pi\left(A_0 A_1 A_4 B_2^2 + A_0 A_3 A_4 B_1^2\right)}{A_0 A_4 \left(A_0 A_3^2 + A_1^2 A_4 - A_1 A_2 A_3\right)}$$

where the As and Bs are the coefficients of the transfer function H.
The resulting expression for the mean square acceleration is:

$$\mathrm{E}\left[\ddot{z}_s^2\right] = \pi S_0 \left(\frac{(m_u + m_s)k^2 + k_t c^2}{m_s^2 c} \right)$$

Q~~2~~4 answer

(a)
The bounce and roll inputs arising from parallel tracks on a randomly rough road are uncorrelated. In the vehicle model the bounce response depends only on the bounce input, whilst the lateral and roll responses depend only on the roll input. (The matrix equation of motion can be partitioned.) Hence the bounce and roll accelerations of the sprung mass $\ddot{z}_s$ and $\ddot{\theta}_s$ are uncorrelated. The acceleration $\ddot{z}_p$ is related to the accelerations at the centre of the sprung mass by $\ddot{z}_p(t) = \ddot{z}_s(t) + p\ddot{\theta}_s(t)$. Because $\ddot{z}_s$ and $\ddot{\theta}_s$ are uncorrelated the mean square spectral density of $\ddot{z}_p$ is given by $S_{\ddot{z}_p}(\omega) = S_{\ddot{z}_s}(\omega) + p^2 S_{\ddot{\theta}_s}(\omega)$.

(b)
V=30m/s, f=0Hz to 5Hz: The wavelengths (V/f) are very long, and so the inputs from the left and right tracks are highly correlated and in phase. The consequence is negligible roll response $\ddot{\theta}_s$, and the acceleration $\ddot{z}_p$ is due entirely to bounce acceleration $\ddot{z}_s$ of the sprung mass.

V=30m/s, f=5Hz to 25Hz: With an increase in frequency the wavelengths are shorter and the left and right tracks are less correlated. The proportion of roll excitation increases. Both bounce and roll acceleration of the sprung mass contribute to the acceleration $\ddot{z}_p$.

(c)
V=1m/s, f=5Hz to 25Hz: The wavelengths (V/f) are very short and so the inputs from the left and right tracks are uncorrelated. Thus there is bounce and roll excitation from the road, and the bounce and roll response of the sprung mass contribute approximately equally to the acceleration $\ddot{z}_p$.

V=1m/s, f=0Hz to 5Hz: Again the wavelengths are short and there is bounce and roll excitation. One bounce resonance of the sprung mass is seen, at the same frequency as the bounce resonance at the higher speed. Two resonances of the sprung mass are evident in the $\ddot{\theta}_s$ response at low speed, whereas at the higher speed no resonances in $\ddot{\theta}_s$ are apparent. This is partly because of the minimal roll excitation at the higher speed, but also because of the effect of speed on c_{lat}. At low speed the c_{lat} term is very large and effectively locks up. At high speed the c_{lat} term is very small and provides little restraint. The effect is to make the natural frequencies of the sprung mass roll-lateral modes lower at the higher speed and thus less likely to contribute to the acceleration $\ddot{z}_p$.