ENGINEERING TRIPOS PART IIA AND PART II, 2005
Engineering Tropis Part IIA and Part IIB 2005
Crib for Paper 4C14: Mechanics of Biological Systems

1. (a) The cytoskeleton is the internal soft skeleton of the cell, comprising 3 types of protein. The *microtubules* are stiff, hollow tubes made from the protein tubulin, with a persistence length of 3mm. They emanate from the centrosome, and small molecular motors (dyneins and kinesins) transport their cargo of proteins to various parts of the cytoplasm. The motors move from off the microtubules onto the *actin filaments*. Microtubules aid cell division via the stored internal energy.

The *actin cortex* sits mainly below the plasma membrane and forms a 2D network, providing support for the membrane, and allowing transport of protein cargo on molecular motors.

Intermediate filaments are passive rope-like filaments of keratin for example. They anchor to cell junctions, and link adjacent cells together (eg in the intestine). Connective tissue between cells also contains a 3D network of intermediate filaments. [30%]

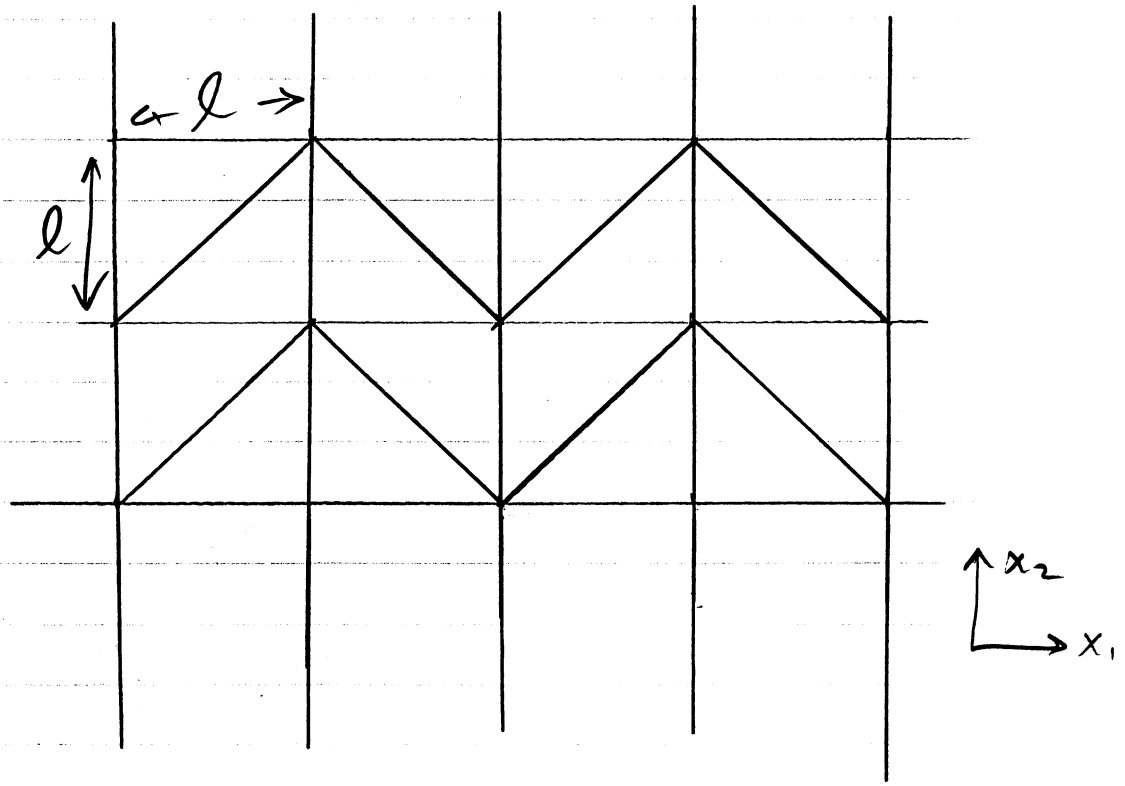
1 (b). Myofibrils are the functional units of skeletal muscles, containing protein filaments that make up the contractile unit. Each myofibril is segmented into numerous individual contractile units called sarcomeres, each about 2.5 μm long. The sarcomere is made up primarily of two types of parallel filaments, designated as thin and thick filaments. Viewed end-on, six thin filaments are positioned around each central thick filament in a hexagonal arrangement. Viewed along its length, there are regions where thin or thick filaments are overlapping or non-overlapping. At the end of the sarcomere is a region, called the Z- line (or disc), where the thin filaments are anchored. Thin filaments extend from the Z-lines at each end toward the centre, where they overlap with the thick filaments. The regions where there is no overlap, containing thin filaments only are called the I-bands, and the regions containing thick filaments (with some overlap with the thin filaments) are called A-bands. The central region of the sarcomere, containing only thick filaments is called the H-zone. During contraction, both the H-zone and the I-bands shorten as the overlap between the thin and thick filaments increases.
Thick filaments contain the protein Myosin while the thin filaments contain the three proteins, actin, tropomyosin and troponin. The actin and the myosin together form the contractile machinery of the muscle while the tropmomyosin acts as a mask for the actin binding sites. [30%]

1. (c) The persistence length $\xi_p \equiv D/kT$ is the length along a molecule at which directional correlation is lost. D = bending stiffness, k = Boltzmann's constant, T = temperature. It gives the chain length between links, when representing the molecule by a chain. When the contour length L of the molecule is much greater than the persistence length, the fibre behaves in an entropic, rubber-like manner (eg spectrin, elastin, DNA). Alternatively, when the contour length L of the molecule is much less than the persistence length, the fibre behaves as a stiff, deterministic fibre (eg. Actin and microtubules, and collagen fibres in skin). [20%]

1. (d) Consider an elastic vessel of cross-sectional area A_0 in the unpressurised state. The area increases to a value A under an internal pressure P according to $A = A_0 + cP$ where c is the compliance of the cell wall. The higher the compliance the smaller the pressure drop required to drive a given fluid flow. Veins are much more compliant than arteries, and so the pressure drop across them is much less.

[20%]

Q2.



(a) $$\bar{\rho} = \frac{t\,l\,(1+\sqrt{2})}{l^2} = (1+\sqrt{2})\,\frac{t}{l}$$ [15%]

(b) $E_2 = ?$ wall stress $\sigma_2 = \varepsilon_2 \cdot \frac{l}{t}$

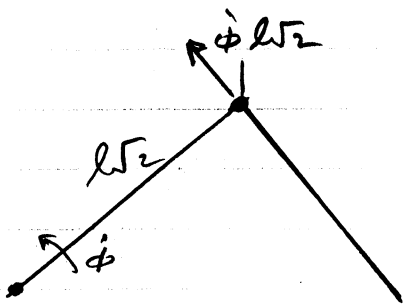
$$\varepsilon_2 = \frac{\sigma_2}{E_s}$$

$$\Rightarrow \quad E_2 = \frac{\sigma_2}{\varepsilon_2} = \frac{t}{l}\,\frac{\sigma_2}{\varepsilon_2} = E_s \cdot \frac{t}{l}$$

$$\Rightarrow \quad E_2 = \frac{\bar{\rho}}{1+\sqrt{2}}\,E_s$$

$$\sigma_{2y} = \sigma_{ys}\,\frac{\bar{\rho}}{1+\sqrt{2}} = \frac{t}{l}\cdot\sigma_{ys}$$ [30%]

Q2. (c) (i)



Resolve velocity $\dot{\phi} l\sqrt{2}$ horizontally to give $\dot{\phi} l\sqrt{2}/\sqrt{2} = \dot{\phi} l$

So $\dot{\varepsilon}_1 = \dfrac{\dot{\phi} l}{l} = \dot{\phi}$ [15%]

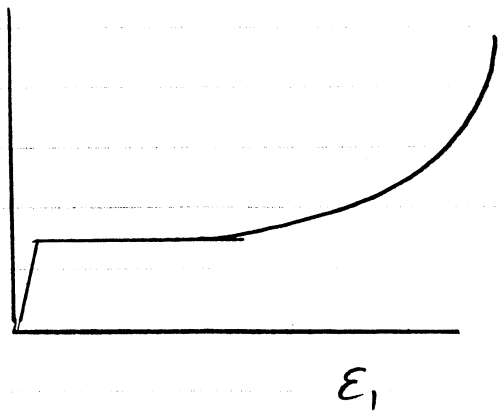
(ii) Work Calculation.

$\Sigma_1 \cdot l\,(\dot{\varepsilon}_1 l) = 2 M_p \dot{\phi}$ where $M_p = \frac{1}{4} t^2 \sigma_{YS}$

$\Rightarrow \quad \Sigma_1 = \frac{1}{2}\left(\dfrac{t}{l}\right)^2 \sigma_{YS} = \frac{1}{2} \dfrac{A^2}{(1+\sqrt{2})^2} \sigma_{YS}$

[20%]

(iii)



Contour length $= l\sqrt{2}$.

$\Rightarrow$ nominal strain $= \dfrac{l\sqrt{2} - l}{l} = \sqrt{2} - 1$

$= 41\%$ [20%]

3. (a)

(i) $(T+a)\nu = b(T_0 - T)$

$$\nu = \frac{b(T_0 - T)}{T+a}$$

$$\text{Power} = \nu T = \frac{bT(T_0 - T)}{T+a}$$

(ii) max. power at $\frac{d}{dT}(\nu T) = 0$

$$\nu T = P = \frac{b(T_0 - T)}{1 + \frac{a}{T}}$$

$$\frac{dP}{dT} = 0$$

$$\Rightarrow \left(\frac{1}{a} + \frac{1}{T}\right) = \left(\frac{T_0}{T^2} - \frac{1}{T}\right)$$

$$\Rightarrow T = \frac{-2a \pm \sqrt{4a^2 + 4aT_0}}{2}$$

$$T = -a + \sqrt{a^2 + aT_0}$$

$$\frac{T}{T_0} = -\frac{a}{T_0} + \sqrt{\frac{a^2}{T_0^2} + \frac{a}{T_0}}$$

$$\nu_{opt} = \frac{b\left[1 + \frac{a}{T_0} - \sqrt{\frac{a^2}{T_0^2} + \frac{a}{T_0}}\right]}{\sqrt{\frac{a^2}{T_0^2} + \frac{a}{T_0}}}$$

3. Thus, in order to better climb a hill, adjust the gears so that the pedalling speed $\Rightarrow$ muscle speed $= v_{opt}$ as this will maximise muscle power.

(b)

(i) Number of crossbridges in ½ a sarcomere $= \frac{mAs}{2}$

Allow the sarcomere to shorten a length l. Since $l >> h$, all crossbridges have had an opportunity to go through one cycle. If T is tension per unit area, work done during this shortening is (assuming T constant over small length l)

$$Tl A = \int_{-\infty}^{\infty} \left[n(x) \frac{mAs}{2} \right] x \lambda dx$$

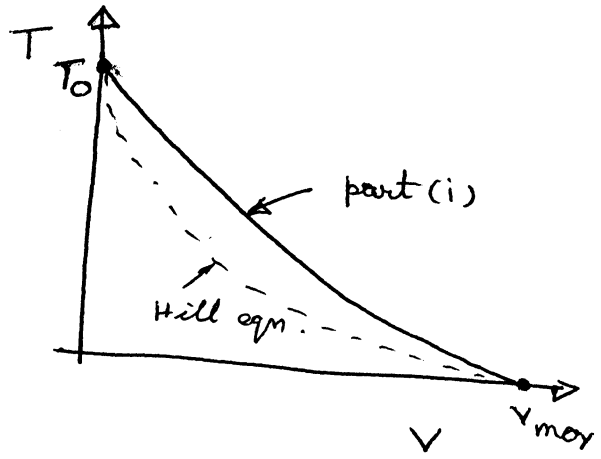
$$T = \frac{ms\lambda}{2l} \left[\int_{-\infty}^{0} n_0 e^{\frac{kx}{v}} x dx + \int_{0}^{h} n_0 x \, dx \right]$$

$$= \frac{ms\lambda n_0}{2l} \left[\frac{v^2}{k^2} \left[\frac{k}{v} x e^{\frac{kx}{v}} - e^{\frac{kx}{v}} \right]_{-\infty}^{0} + \frac{h^2}{2} \right]$$

$$T = \frac{n_0 s\lambda m}{2l} \left[\frac{h^2}{2} - \frac{v^2}{k^2} \right]$$

3.

(ii)



$$T_0 = \frac{n_0 S \lambda m h^2}{4\ell}$$

$$V_{max} = \frac{bh}{\sqrt{2}}$$

The Hill eqn. is hyperbolic while the equation derived in (i) predicts that T decreases quadratically ~~as~~ with v. ~~Hence~~ However, by adjusting parameters it is possible to get reasonable agreement with the Hill equation ~~and~~ over the range of interest.

4. (a) The primary role of the respiratory system is the gas transfer between tissues and outside air. Its main working principles are:

* movement of oxygen-containing medium so it contacts a moist membrane overlying blood vessels;
* diffusion of oxygen from medium into blood;
* transport of oxygen to tissues and cells of the body;
* diffusion of oxygen from blood into cells;
* carbon dioxide follows a reverse path.

(b) When leaving the alveolar capillaries, the partial pressure of O_2 increases compared with that when entering the alveolar capillaries, and the partial pressure of CO_2 decreases. The variations are due to the diffusion of gases occuring in the alveolar capillaries: O_2 diffuses from alveoli into blood while CO_2 diffuses from blood into the alveoli.

(c) In the alveoli capillaries, bicarbonate combines with a hydrogen ion (proton) to form carbonic acid, which breaks down into CO_2 and water. CO_2 then diffuses into the alveoli and out of the body with the next exhalation.

4. (d)

Because U_{O_2} is independent of time, we have

$$v \frac{dU_{O_2}}{dx} = D_{O_2}(\sigma_{O_2} P_{O_2} - U_{O_2})$$

Since $U_{O_2} = U_0$ at $x=0$, the solution of the above is

$$U_{O_2}(x) = \sigma_{O_2} P_{O_2} + (U_0 - \sigma_{O_2} P_{O_2}) e^{-D_{O_2} x / v}$$

The total flux of O_2 is then

$$Q_{O_2} = P \int_0^L q(x,t)\, dx$$

$$= vA U_{O_2}(L) - vA U_{O_2}(0)$$

$$= vA \left\{ \sigma_{O_2} P_{O_2} + (U_0 - \sigma_{O_2} P_{O_2}) e^{-D_{O_2} L / v} - U_0 \right\}$$

But $U_0 = \sigma_{O_2} P_0$ ~ partial pressure at $x=0$

$$\Rightarrow \quad Q_{O_2} = vA \sigma_{O_2} (P_{O_2} - P_0)(1 - e^{-D_{O_2} L / v})$$

in the limit $L \to \infty$ $= vA \sigma_{O_2} (P_{O_2} - P_0)$

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