

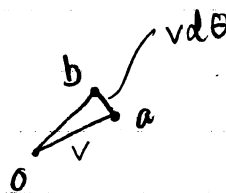
4D5 foundation Engineering

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1) a) Consider 2 infinitesimal elements (rigid)



Space

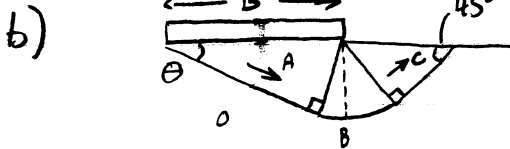


Velocity

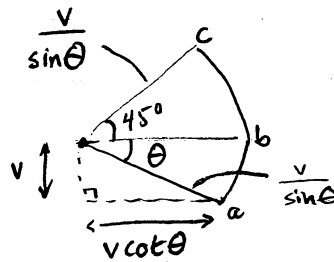
$$\dot{D} = \sum (s_u \times \text{length} \times \text{relative velocity}) \quad (\text{summed over all slip planes})$$

$$= \underbrace{s_u r d\theta v}_{\text{length on circumference}} + \underbrace{s_u r d\theta v}_{\text{length on radial plane}}$$

$$= 2s_u r v d\theta$$



Space



Velocity

Call v, vertical component of foundation velocity.

$$\text{Work dissipated} = \underbrace{s_u B \cos \theta \frac{v}{\sin \theta}}_{OA} + \underbrace{2s_u B \sin \theta \frac{v}{\sin \theta} \left(\frac{\pi}{4} + \theta \right)}_{\text{fan}} + \underbrace{s_u B \sin \theta \frac{v}{\sin \theta}}_{OB}$$

$$= s_u B v \left(\cot \theta + \frac{\pi}{2} + 2\theta + 1 \right)$$

$$\text{Work input} = H v \cot \theta + V v$$

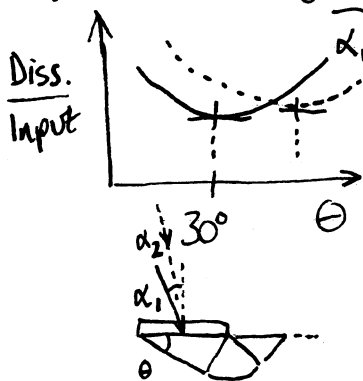
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1) b) $\Rightarrow \frac{\text{Dissipated}}{\text{Input}} = \frac{s_u B (\cot \theta + \frac{\pi}{2} + 2\theta + 1)}{H \cot \theta + V}$

c) for a given load inclination, optimal mechanism will occur. (i.e. value of θ for which $\frac{\text{Dissipated}}{\text{Input}}$ is minimum).

To find minimum:

Numerator of $\frac{d(\frac{\text{Dissipated}}{\text{Input}})}{d\theta}$ is (by quotient differentiation): (ignoring constant $s_u B$).



Higher θ optimal for smaller α . Given a value of θ , corresponding α is found from $d(\text{Diss}/\text{Input})/d\theta = 0$.

$$= \left(-\frac{1}{\sin^2 \theta} + 2\right) (H \cot \theta + V) - (\cot \theta + \frac{\pi}{2} + 2\theta + 1) \left(\frac{-H}{\sin^2 \theta}\right)$$

$$= 2H \cot \theta - \frac{V}{\sin^2 \theta} + 2V + \left(\frac{\pi}{2} + 2\theta + 1\right) \frac{H}{\sin^2 \theta}$$

$$= V (2\sin^2 \theta - 1) + H \left(2\sin \theta \cos \theta + \frac{\pi}{2} + 2\theta + 1\right)$$

$$= V (2\sin^2 \theta - 1) + H \left(\sin 2\theta + \frac{\pi}{2} + 2\theta + 1\right)$$

Set equal to zero and substitute for $\theta = 30^\circ$

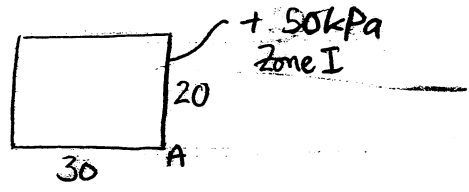
$$\frac{H}{V} = \frac{(1 - 2\sin^2 \theta)}{(\sin 2\theta + \frac{\pi}{2} + 2\theta + 1)} = \frac{1/2}{\sqrt{3}/2 + \pi/2 + \pi/4 + 1}$$

$$\frac{H}{V} = 0.118 \Rightarrow \alpha = \tan^{-1}\left(\frac{H}{V}\right) = 6.75^\circ$$

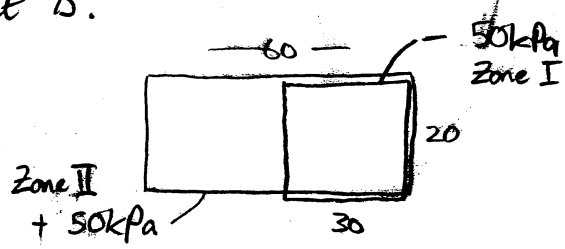
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2) a) Divide soil into 3 layers, thinner close to ground surface, where $d(\Delta\sigma_v)/dz$ is highest.

Consider two loaded regions, using superposition to calculate settlement at point B.



Load case for settlement at A



Load cases for settlement at B (using superposition of Zone I and II)

Settlement of each layer, ΔH , estimated as: ($k=0.02$)

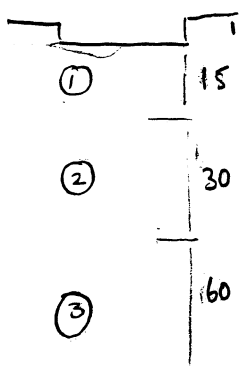
$$V_0 = 1.5 - 0.02 \ln \sigma'_{v0}$$

$$\Delta V = 0.02 \ln \left(\frac{\sigma'_{v0} + \Delta\sigma_v}{\sigma'_{v0}} \right)$$

$$\Delta H = H \frac{\Delta V}{V_0}$$

- Using Fadum
- 1) Divide soil into layers, thinner at top.
 - 2) Calculate position of mid-depth normalised by L and B, hence m, n
 - 3) Use Fadum chart to find influence factor, and hence stress increment at each layer
 - 4) Use expression on left to determine settlement of each layer

Using Fadum chart to calculate influence factor for each layer:



Layer	Mid-depth	below fndn base		Zone I		Zone II	
		Zone I n x m	Zone II n x m	Zone I I (Fadum)	Zone II I (Fadum)	Zone I	Zone II
①	7.5	4 x 2.67	8 x 2.67	0.24	0.24	12	12
②	30	1 x 0.67	2 x 0.67	0.145	0.16	7.25	8
③	75	0.4 x 0.27	0.8 x 0.27	0.045	0.07	2.25	3.5

Layer	Thickness, H	$\bar{\sigma}'_{v0}$	ΔV		V_0	ΔH (mm)	
			Zone I	Zone II		Zone I	Zone II
①	15	85	2.64×10^{-3}	2.64×10^{-3}	1.41	28	28
②	30	310	4.62×10^{-4}	5.09×10^{-4}	1.39	9.97	10.99
③	60	760	5.91×10^{-5}	9.19×10^{-5}	1.37	2.59	4.02

[consider ~ 2-3 B below foundation]
- see elastic stress distribution in databook.

Sum for settlement at A \nearrow
Sum difference for settlement at B \nearrow

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2) a) continued.

Settlement @ A = 41 mm

Settlement @ B = 2.5 mm

Note: different layer thicknesses will lead to different answers: method is sensitive to chosen thicknesses. due to non-linear $\Delta\sigma'_v$ and v_0 with depth.

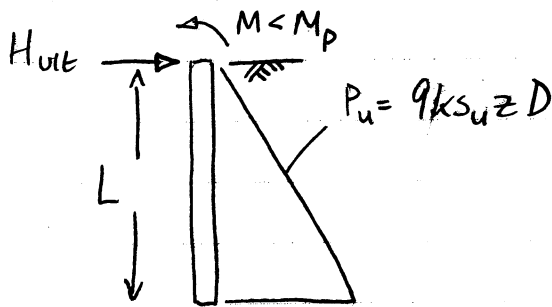
[In practice, spreadsheet is used, so number of layers $\gg 3$.]

b) Settlement at B may be lower due to:

- higher initial soil stiffness close to existing building since $\sigma'_{v0} > \gamma'z$ due to consolidation under load due to existing building
- volume reduction during consolidation may be counteracted by dilation during shear deformation. Hence, a correction factor (e.g. Skempton - Bjerrum) may be necessary. Typical values for OC clay $\sim 0.5-0.7$.
- other building may be founded on piles, so less affected by movements close to ground surface.

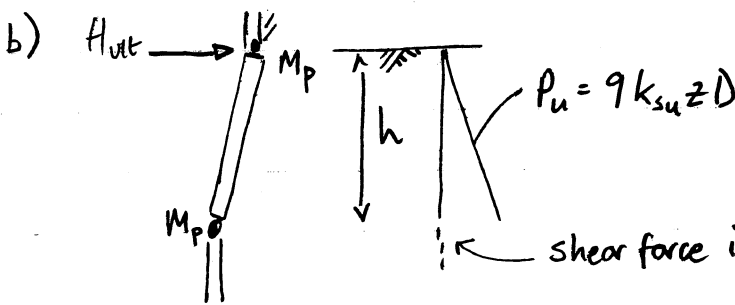
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3) a) Horizontal resistance / unit pile length = $9s_u D = 9k_{su} z D$ databook, can be quoted



in n.c. clay

Horizontal equilibrium: $H_{ult} = \bar{P}_u L = \frac{1}{2} 9k_{su} L D \cdot L = 5400kN$



First consider 2-hinge mechanism

$\frac{dM}{dz} = S$
 shear force is zero where moment is a maximum.

Horizontal equilibrium: $H_{ult} = \bar{P}_u h = \frac{1}{2} 9k_{su} h D \cdot h \quad \text{--- (1)}$

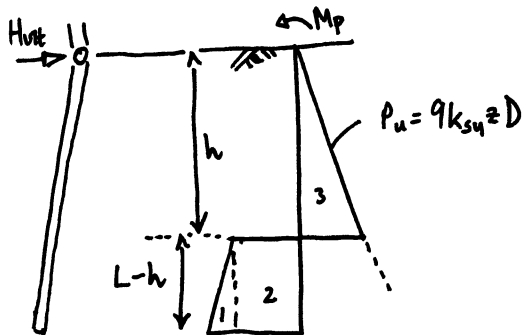
Moment eq^m about head: $2M_p = \frac{1}{2} 9k_{su} h D \cdot h \cdot \frac{2}{3} h \quad \text{--- (2)}$
 lever arm

from (2), $h^3 = \frac{6M_p}{9k_{su} D} \Rightarrow h^2 = \left(\frac{2M_p}{3k_{su} D}\right)^{2/3}$

Sub into (1) $H_{ult} = \frac{9}{2} k_{su} D \left(\frac{2M_p}{3k_{su} D}\right)^{2/3}$

$H_{ult} = 3.43 (k_{su} D)^{1/3} M_p^{2/3} = 4.95 M_p^{2/3} \text{ (kN)}$

3 b) continued



Consider 1-hinge mechanism

Horizontal equilibrium:
$$H_{ult} = \underbrace{\frac{1}{2}(9k_{su}hD)}_{\bar{p}_u} \underbrace{h}_{\text{length}} - \underbrace{\frac{1}{2}(9k_{su}(h+L)D)}_{\bar{p}_u} \underbrace{(L-h)}_{\text{length}}$$

$$H_{ult} = \frac{1}{2} 9k_{su} D (2h^2 - L^2) = 27h^2 - 5400 \text{ kN}$$

Moment equilibrium:
$$M_p = \underbrace{\frac{1}{2}(9k_{su}hD)}_{\bar{p}_u} \underbrace{h}_{\text{length}} \underbrace{\frac{2}{3}h}_{\text{lever}} - \underbrace{9k_{su}Dh}_{\text{region 2}} \underbrace{(L-h)}_{\text{length}} \underbrace{\left(\frac{h}{2} + \frac{L}{2}\right)}_{\text{lever}}$$

$$- \underbrace{9k_{su}D \frac{L-h}{2}}_{\text{region 1}} \underbrace{(L-h)}_{\text{length}} \underbrace{\left(\frac{2L}{3} + \frac{h}{3}\right)}_{\text{lever}}$$

simplifies to:

$$M_p = \frac{1}{2} 9k_{su} D \left(\frac{4}{3} h^3 - \frac{2}{3} L^3 \right) = 18h^3 - 72000 \text{ kN}$$

Combining equations to link H_{ult} and M_p :
$$H_{ult} = 27 \left(\frac{M_p + 72000}{18} \right)^{2/3} - 5400$$

Hence, capacity is minimum of offered by 3 mechanisms:

$$H_{ult} = \min \left(\underbrace{5400}_{\text{no hinge}}, \underbrace{4.95 M_p^{2/3}}_{\text{1 hinge}}, \underbrace{27 \left(\frac{M_p + 72000}{18} \right)^{2/3} - 5400}_{\text{2 hinges}} \right)$$

c) To prevent bending failure mechanisms, M_p must be sufficiently large that no-hinge mechanism is critical. To find this value, equate capacities of mechanisms to find required M_p .

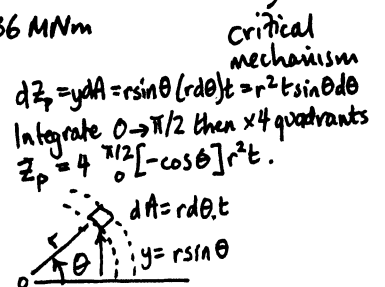
no hinge vs. 1-hinge: $5400 \leq 27 \left(\frac{M_p + 72000}{18} \right)^{2/3} - 5400 \Rightarrow M_p \geq 72 \text{ MNm}$

no hinge vs. 2-hinge: $5400 \leq 4.95 M_p^{2/3} \Rightarrow M_p \geq 36 \text{ MNm}$

Plastic section modulus, $Z_p = D^2 t$

$$\Rightarrow t = \frac{M_p}{D^2 \sigma_y} = \frac{72}{2^2 \cdot 200} = 90 \text{ mm}$$

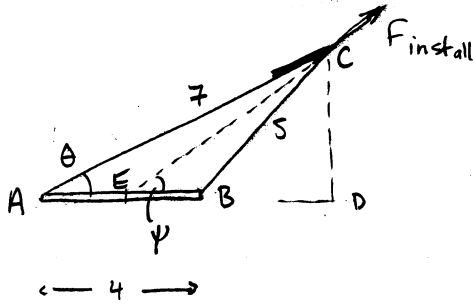
bookwork, integration of elements:



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4) a) Installation force:

- Calculate angle of installation line to plate, Ψ



Cosine rule

$$\cos \theta = \frac{5^2 - 7^2 - 4^2}{2 \times 7 \times 4}$$

$$\Rightarrow \theta = 44.4^\circ$$

$$CD = 7 \sin 44.4 = 4.90$$

$$BD = \sqrt{5^2 - 4.9^2} = 1$$

$$\Rightarrow \Psi = \tan^{-1} \frac{4.90}{3} = 58.5^\circ$$

- Calculate resistance in direction parallel to plate:

$$F_{\text{parallel}} = \underbrace{2 \alpha S_u (A_{\text{plan}} + A_{\text{edge}})}_{\text{'shaft friction' on top, bottom and 2 edges}} + \underbrace{2 N_c S_u A_{\text{edge}}}_{\substack{\text{+ve and -ve} \\ \text{'bearing capacity' on front and back}}}$$

$$S_u / \sigma'_{vo} = \frac{50}{5 \times 20} = 0.5 \Rightarrow \alpha = 1/\sqrt{2} \quad (\text{API / Randolph / Murphy})$$

$$N_c = \text{say } 7: \quad \text{deeply embedded strip} \quad \uparrow \text{ databook.}$$

$$\text{Skempton depth factor} \rightarrow 1.33$$

$$1.33 \times (2 + \pi) \sim 7$$

$$F_{\text{parallel}} = 2 \frac{1}{\sqrt{2}} 50 (4^2 + 4 \times 0.05) + 2 \cdot 7 \cdot 50 (4 \times 0.05)$$

$$= 1145 + 140 = 1285 \text{ kN}$$

$$\text{for equilibrium, } F_{\text{install}} = \frac{F_{\text{parallel}}}{\cos \Psi} = 1799 \text{ kN.}$$

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4) Mooring capacity

flow-round mechanism: $s_c N_c = 2(2 + \pi) 1.18$ (2 square surface footings back to back)

$$f_{\text{mooring}} = s_c N_c s_u A_{\text{plan}}$$

(ignore shear on thickness)

$$f_{\text{mooring}} = 9707 \text{ kN.}$$

$$\text{Hence, anchor efficiency} = \frac{\text{mooring}}{\text{installation}} = \frac{9707}{1799} = 5.4$$

b) Crack-free water at anchor

- infinitesimal drainage distance to edge
- 'zero' nominal consolidation time
- hence, tension via $-ve \Delta u$ unsustainable
- so, no 'negative' bearing capacity.

$$f_{\text{install}} = 1701 \text{ kN} \quad \downarrow 5\% \quad (\text{ignoring } N_c s_u \text{ on rear edge.})$$

$$f_{\text{mooring}} = 4853.5 \text{ kN} \quad \downarrow 50\% \quad (\text{ignoring tension on back face})$$

$$\text{efficiency} = 2.85.$$

Also, crack could lead to softening of soil around anchor in long term.

Note: vertical shear mechanism in part a) is not optimal.

$$F = A s_u = (4 \times 4 \times 20) 50 = 16000 \text{ kN}$$