Module 4D8 – Prestressed Concrete

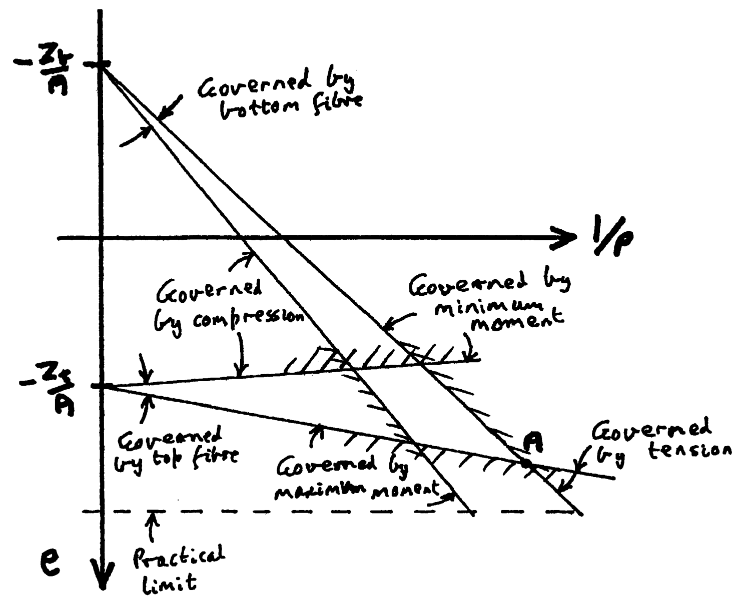
Examination 2005 – Solutions and examiner's comments

1 (a) The stress limits can all be expressed in the form $\sigma = \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} > f_t$

which can be rearranged into the form $e > \text{or} < -\frac{Z}{A} + \frac{Zf_t}{P} + \frac{M}{P}$

(inequality direction depends on sign of Z)

After the section has been designed these limits can all be expressed as linear relationships between e & $1/P$.



(b) Section properties

Area = 1500.500 + 2000.400 + 1000.500 = $2.05.10^6$ mm^2

$A\bar{y}$ = 1500.200.2750 + 2000.400.1500 + 1000.500.250 = $3387.5.10^6$ mm^3

$$\text{so } \bar{y} = \frac{3387.5}{2.05} = 1652 \text{ mm}$$

I about bottom (in m^4)

$$= \frac{1.5(0.5)^3}{12} + 1.5 \times 0.5 \times (2.75)^2 + \frac{2^3 \times 0.4}{12} + 2 \times 0.4 \times (1.5)^2 + \frac{1 \times 0.5^3}{12} + 1 \times 0.5 \times (0.25)^2$$
$$= 7.791 \text{ m}^4$$

$\therefore$ I about centroid = $7.791 - 2.05.(1.652)^2 = 2.196$ m^4

$$Z_b = \frac{2.196}{1.652} = 1.33\ \text{m}^3 \quad \text{so} \Rightarrow -\frac{Z_b}{A} = -0.648\ \text{m}$$

$$Z_t = -\frac{2.196}{(3.0-1.652)} = -1.63\ \text{m}^3 \qquad \text{so} \Rightarrow -\frac{Z_t}{A} = +0.795\ \text{m}$$

Choose a prestress force to calculate the Magnel diagram (arbitrary).

$$\frac{A.f_c}{2} = \frac{2.05.10^6.20}{2} = 20500\ \text{kN} \qquad \text{(say 20 MN)}$$

Under a moment of 20,000 kNm.

Tension in bottom fibre $e \geq -\frac{Z_b}{A} + \frac{0.Z_b}{P} + \frac{M_b}{P} = -0.648 + \frac{20000}{20000} = +0.352\ \text{m}$ (Point 1 on figure)

Compression in top fibre

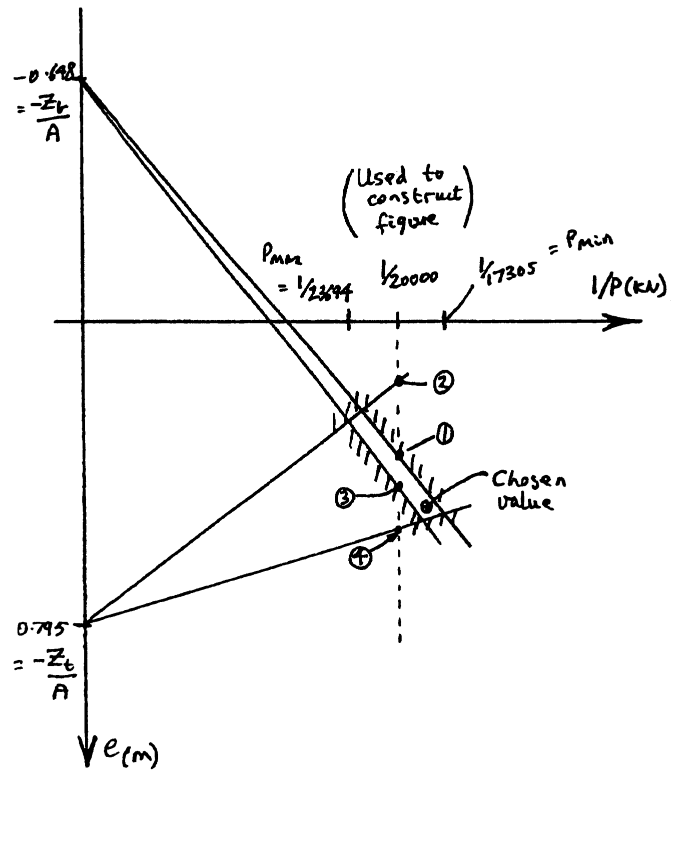
$$e \geq -\frac{Z_t}{A} + \frac{f_c.Z_t}{P} + \frac{M_b}{P} = +0.795 - \frac{20 \times 1.63.10^3}{20000} + \frac{20000}{20000} = +0.165\ \text{m} \qquad \text{(Point 2)}$$

Under moment of −5000 kNm

Compression in bottom fibre

$$e \geq -\frac{Z_b}{A} + \frac{f_c.Z_b}{P} + \frac{M_a}{P} = -0.648 + \frac{20 \times 1.33.10^3}{20000} - \frac{5000}{20000} = +0.432\ \text{m} \qquad \text{(Point 3)}$$

Tension in top $e \geq -\frac{Z_t}{A} + \frac{0.Z_t}{P} + \frac{M_a}{P} = +0.795 - \frac{5000}{20000} = +0.545\ \text{m}$ (Point 4)

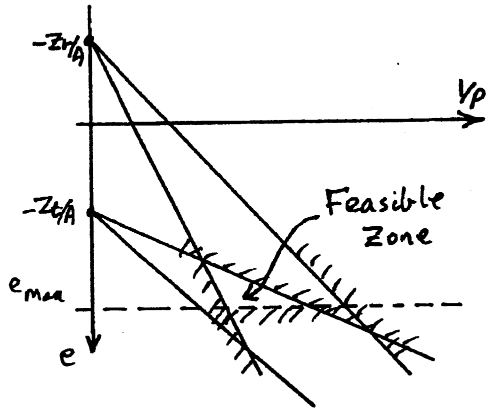


From Magnel diagram find $P_{\min} = 17305$ kN at $e = 0.5$ m
(This value by calculation – accept ± 2% by drawing)

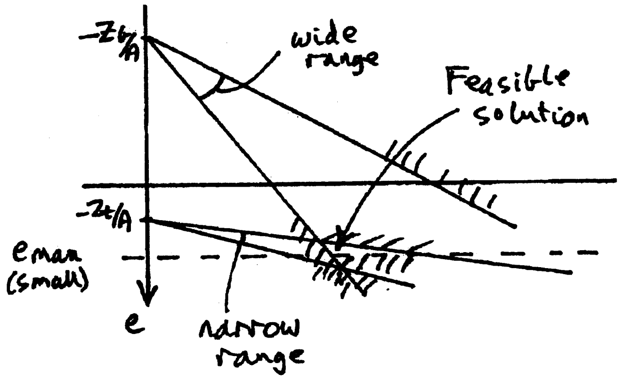
Choose practical value where feasible zone is wider to allow for variations of P and e on site
(say $P = 18000$ kN, $e = 0.48$ m)

(c) Two common cases where compressive stresses govern:-

(i) where moment range is small by comparison with maximum moment. Leads to feasible zone largely outside the section.



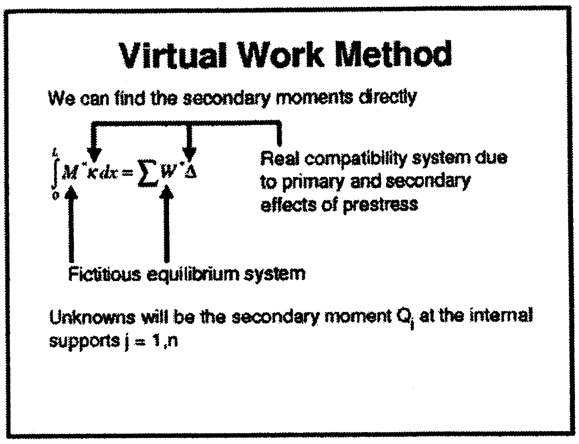
(ii) When top flange small but bottom flange large, centroid is low and difficult to get eccentricity required.



Examiner's Comments

Attempted by all candidates. Many candidates got virtually full marks on parts (a) and (b); most errors were numerical and the students had clearly learnt the importance of the Magnel diagram for design purposes; only two candidates lost significant marks here. One candidate couldn't work out the second moment of area of an I-beam. However, part (c), which asked for particular loading cases which could cause compressive stresses to govern, was done badly by all but a few candidates; they appeared unable to reason beyond the merely putting numbers into equations.

2 (a) Bookwork. Should reproduce method here.



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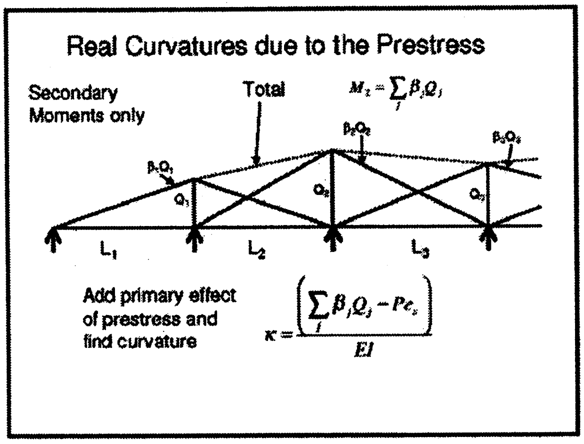

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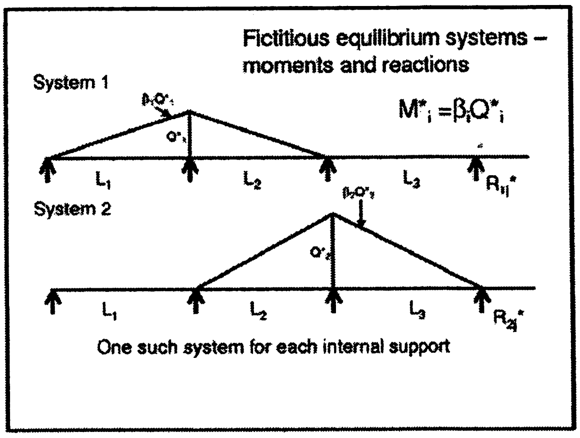

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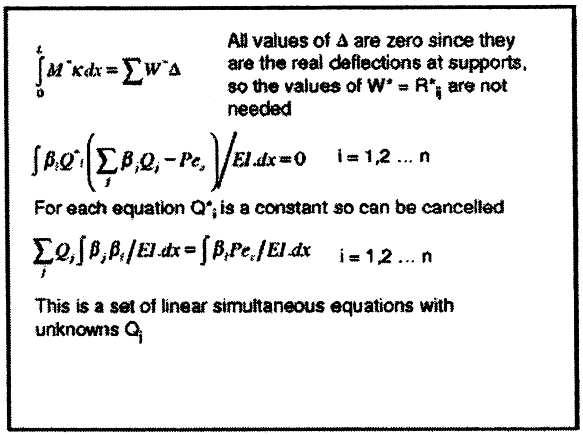

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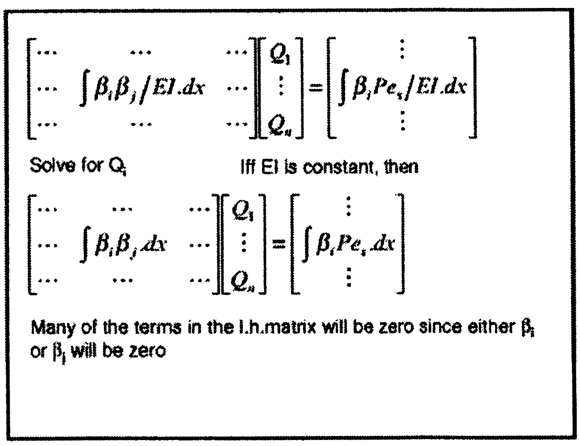

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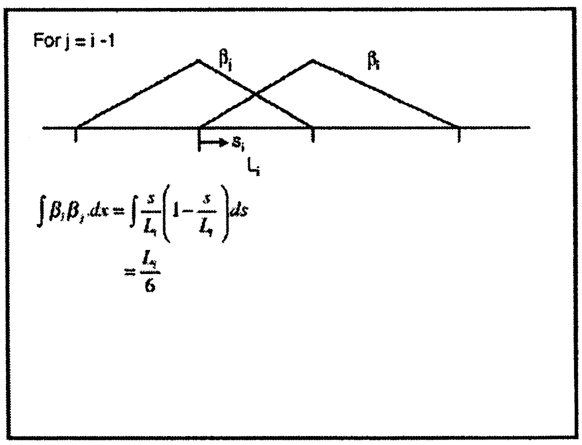

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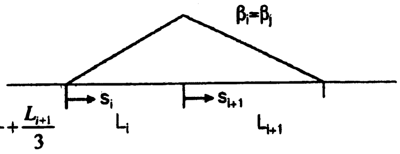
For j = i



$\int \beta_i \beta_j . dx = \frac{L_i}{3} + \frac{L_{i+1}}{3}$

So, finally, we get a simple result

$$\frac{1}{6}\begin{bmatrix} 2(L_1+L_2) & L_2 & 0 & 0 \\ L_2 & 2(L_2+L_3) & L_3 & 0 \\ 0 & L_3 & 2(L_3+L_4) & L_4 \\ 0 & 0 & L_4 & etc. \end{bmatrix} . Q_j = \int \beta_i P e_s dx$$

The integrals on the r.h.s. can be found numerically

7

For 2-span beam, only one indeterminacy, so only one equation to solve

$$\frac{2(L_1 + L_2)}{6} . Q = \int \beta P e . dx = P \int \beta e . dx$$

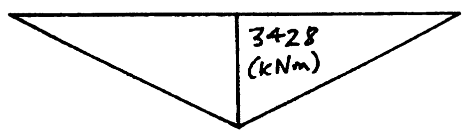
Use Simpson's rule with $h = 4$ m.

Chainage	e	Simpson's Coeff.(c)	β	$e.c.\beta$
0	–0.3	1	0	0
4	0.3	4	0.25	0.30
8	0.6	2	0.5	0.60
12	0.2	4	0.75	0.60
16	–1	2	1.0	–2.00
20	0.2	4	0.83	0.67
24	0.7	2	0.67	0.93
28	0.8	4	0.50	1.60
32	0.7	2	0.33	0.47
36	0.4	4	0.16	0.27
40	0	1	0	0
			Total	3.43

$\int \beta e . dx = \frac{3.43h}{3} = 4.57 \text{ m}^2$ so $\frac{2(L_1 + L_2)}{6} Q = 10000 \times 4.57 \text{ kNm}^2$

$\Rightarrow Q = 3428$ kN (sagging)

∴ Secondary moments



Eccentricity of cable line of thrust at internal support will be shifted *down* by $\frac{3428}{10000} = 0.3428$ m

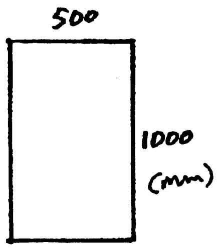
e at internal support will be –0.657 m.

(c) Designers choose cables that deliberately introduce secondary moments since these can reduce moments over piers, which are often difficult to take into account in structures which have a large top flange (and therefore high centroid).

Examiner's comments.

Not attempted by any candidates! This was perhaps less obviously straightforward than Qu.1. but it was probably shorter overall.

3 Find the elastic strain in concrete at level of tendon due to prestress.



$$I = \frac{1.(0.5)^3}{12} = 0.0104\,\text{m}^4; \quad e = 0.5 - 0.3 = 0.2\ \text{m};$$

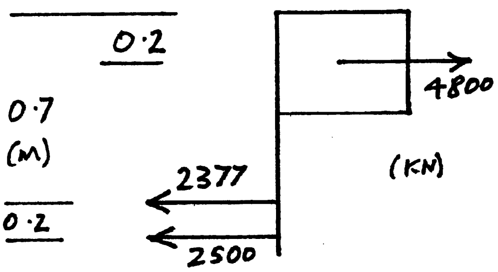
Moment due to prestress = 2000.800.10^{-3}.0.2 = 320 kNm

Elastic stress due to prestress at level of tendon $\sigma = \frac{Me}{I} = \frac{320 \times 0.2}{0.0104} = 0.62$ MPa

Thus elastic strain = 20.10^{-6} so negligible

Strain in steel when σ = 800 MPa = 0.00362

		1st iteration	2nd iter.	3rd iter.	4th iter.
Depth of n.a.		500	250	446	400
Strain in p.s. steel	=	$.0035\frac{200}{500} = 0.0014$	.0063	.0020	.0026
Total strain in p.s.	=	.00502	.00992	.00561	.00622
Stress in p.s.	=	1025 MPa	1563	1109	1189
Force in p.s.	=	5050 kN	3127	2218	2377
Force in rebar	=	2500 kN	2500	2500	2500
Tensile force	=	4500 kN	5627	4718	4877
Coeff. Force	=	6000 kN	3000	5352	4800
$\therefore$ T/C	=	0.758	1.87	0.881	(close enough)



M about centroid of compression zone

= 2377 . 0.5 + 2500 . 0.7 = 2939 kNm

Examiner's comments

Popular, but probably the last question tackled by some, since several candidates failed to finish. Several candidates got the answer completely right, and most of the others had the logic basically correct.

4 (a) Flexibility of the ducts should not affect friction losses, but the duct may tend to move more when concrete is cast, so wobble losses can be expected to be higher . So False.

(b) There is no relationship between secondary moments and friction, so False.

(c) False. There is no link between them.

(d) False. Friction occurs before grout is introduced $\therefore$ no effect.

(e) True. The untensioned rebar does not creep so picks up much of the prestress. $\therefore$ Concrete prestress loss much higher.

(f) False. The opposite is true. Wedge slip affects area near the anchorage only. $\therefore$ Important in short tendons.

(g) False. Force transfer affects any tendon that is already bonded. $\therefore$ It affects *all* tendons in pretensioned beams.

(h) Relaxation effects not altered by pre or post tensioning. $\therefore$ False..

Examiner's comments

Most had the logic correct in most cases; the most frequent mistake was the assumption that non-concordant cables had more curvature than concordant cables (which is not true), so forces were higher, so friction losses would be higher.

5 (a) Creep causes bending moment distribution to change if the elastic modulus of concrete differs within the structure.

In phased construction the production of concrete in the newer portion will be lower than that in the older portions, so the second part will creep more than the first part. This will lead to a redistribution of moments.

The effect of temperature variation across the depth of the section will alter the creep rates since creep is thermally activated. This will lead to creep taking place more quickly in the warmer part of the beam (usually the top). Since creep is affected primarily by prestress and dead loads, which are permanent, we can use an effective modulus method. Thus, if the modulus varies over the depth we can use transformed section properties.

(b) Due to the effect of creep, the bending moment after creep will tend to the monolithic distribution, so any trapped moments will tend to reduce.

Daily temperature variation will not cause moment variations since they are too short to cause the core of the concrete to change, but the annual temperature effects will affect the core and will affect moments. They will also alter the creep rates through the depth, and hence the transformed section.

Examiner's comments

Only tackled by a few of the weaker candidates, none of whom wrote sensibly in the subject.