ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Friday 6 May 2005 2.30 to 4

Module 4A1

NUCLEAR POWER ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4A1 data sheet (8 pages).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) Given that the net neutron current density $\underline{j}$ in a weakly absorbing (non-multiplying) material is given by $\underline{j} = -D\nabla\phi$, where ϕ is the neutron flux, show that in such a medium with spherical symmetry the steady-state, source-free diffusion equation for neutrons can be written as

$$\frac{1}{r}\left(\frac{d^2(\phi r)}{dr^2}\right) - \frac{\phi}{L^2} = 0$$

defining L^2 and stating any assumptions made. [25%]

(b) A very large spherical mass of this material has a central spherical cavity of radius R_1 at the centre of which is an isotropic source of neutrons causing the flux at R_1 to be ϕ_1. Deduce the neutron flux distribution throughout the material. [30%]

(c) Show how the steady-state, source-free diffusion equation should be modified if it applies to a material containing a uniform distribution of fissile material.

If the infinite multiplication factor k_∞ for the mixture is greater than 1, derive an expression for the critical radius of a solid (i.e. with no cavity) bare spherical reactor made of this material. Explain carefully what boundary conditions you have used at the outside of the reactor. [45%]

2 In a 'lumped' model of the kinetic behaviour in a reactor operating at low power, the equations for the neutron population $n(t)$ and the precursor population $c(t)$ may be written as

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where s is an independent source rate and the other symbols have their usual meanings.

(a) What major simplifying assumptions underlie this model? [10%]

(b) Estimate the ratio of precursors to neutrons in steady-state operation in a typical Pressurised Water Reactor for which $\beta = 0.0075$, $\lambda = 0.1\ \text{s}^{-1}$ and $\Lambda = 10^{-4}$ s. [10%]

(c) A critical, source-free Pressurised Water Reactor has been operating in steady state and is subject to a step increase of reactivity to $\rho = 0.005$. Find the dominant time constant for the resulting excursion predicted by this model, and compare it with that given using the *prompt jump approximation*, commenting on the safety implications of using the estimate given by the latter. [60%]

(d) If the same reactivity change was made in an Advanced Gas-Cooled Reactor and in a Fast Breeder Reactor, explain qualitatively how the dominant time constants of the excursions would differ from that calculated for the Pressurised Water Reactor. [20%]

(TURN OVER

3 (a) In a linear reactivity-versus-burnup model of a reactor using slightly enriched fuel, the reactivity ρ varies with burnup τ as

$$\rho = \rho_0 \left(1 - \frac{\tau}{T_1}\right)$$

where ρ_0 is the initial reactivity of the fuel and T_1 is the burnup at which the reactivity of the fuel falls to zero. In the planned refuelling scheme, one *M*-th of the fuel is to be changed at the beginning of each cycle, where *M* is an integer. The same fuel design (enrichment) will be used throughout.

Assuming an initial loading of all fresh fuel, find the cycle lengths (measured by burnup) for the first and second cycles, and derive an equation showing the dependence of the *M*-th cycle length on its predecessors. State any assumptions made. [25%]

(b) Derive an expression for the ratio of the long-term (steady-state) cycle length τ_∞ to the length of the first cycle, and sketch a plot showing how the steady-state discharge burnup *B* of the fuel (the accumulated burnup when fuel is removed from the reactor) varies with *M* for $M = 1,\ldots,5$.

Comment on the implications of this plot on the choice of a refuelling scheme for the reactor. [30%]

(c) In practice the number of fuel assemblies in a reactor may not divide into *M* equal batches. For instance, Sizewell B PWR has a 193 (a prime number) fuel assemblies.

Suppose now that *N* out of the N_0 fuel assemblies in the reactor are changed at the end of each cycle, where $N_0/N = m + f$, with *m* integer and $0 \le f < 1$. Show that for the same assumptions the steady-state cycle length is given by

$$\tau_\infty = \frac{2(m+f)T_1}{(m+1)(m+2f)}$$

and hence that the average discharge burnup is given by

$$B = \frac{2(m+f)^2 T_1}{(m+1)(m+2f)}$$

[30%]

(d) Superimpose on your plot in part (b) a further sketch showing the variation of *B* with *m* and *f*, and hence comment on the operational implications of non-integer batch refuelling. [15%]

4 (a) Describe the principal methods used in the treatment of radioactive wastes arising from power generation and discuss their relative advantages and disadvantages. What is the usual way of dealing with any solid residues arising from the various treatments? [60%]

(b) The liquid effluent from a civil Pressurised Water Reactor arises at a rate of $0.063\ m^3hr^{-1}$ and contains $24\ Bq\,g^{-1}$ of Co-60. Co-60 decays to Ni-60 (stable) with a half life of 5.26 yr. The effluent is collected in a hold-up tank with a working volume of $15\ m^3$. As soon as the tank is full, it passes through a further treatment with a decontamination factor of 10. The effluent density can be taken as $1000\ kg\,m^{-3}$.

Estimate the activity (in $Bq\,g^{-1}$) of Co-60 in the final effluent. [40%]

END OF PAPER

Data Sheet S/15

Revised January 2003

MODULE 4A1

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	299.792458×10^{6} m s^{-1}
Magnetic permeability in vacuum	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Planck constant	h	6.626176×10^{-32} J s
Boltzmann constant	k	1.380662×10^{-23} J K^{-1}
Elementary charge	e	$1.6021892 \times 10^{-19}$ C

Definitions

Unified atomic mass constant	u	$1.6605655 \times 10^{-27}$ kg (931.5016 MeV)
Electron volt	eV	$1.6021892 \times 10^{-19}$ J
Curie	Ci	3.7×10^{10} Bq
Barn	barn	10^{-28} m^{2}

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}Ne$	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}Ne$	20.99385 u
99.985%	$^{1}_{1}H$	1.00783 u	8.94%	$^{22}_{10}Ne$	21.99138 u
0.015%	$^{2}_{1}H$	2.01410 u	10.1%	$^{25}_{12}Mg$	24.98584 u
0%	$^{3}_{1}H$	3.01605 u	11.1%	$^{26}_{12}Mg$	25.98259 u
0.0001%	$^{3}_{2}He$	3.01603 u	0%	$^{32}_{15}P$	31.97391 u
99.9999%	$^{4}_{2}He$	4.00260 u	96.0%	$^{32}_{16}S$	31.97207 u
7.5%	$^{6}_{3}Li$	6.01513 u	0%	$^{60}_{27}Co$	59.93381 u
92.5%	$^{7}_{3}Li$	7.01601 u	26.2%	$^{60}_{28}Ni$	59.93078 u
0%	$^{8}_{4}Be$	8.00531 u	0%	$^{87}_{35}Br$	86.92196 u
100%	$^{9}_{4}Be$	9.01219 u	0%	$^{86}_{36}Kr$	85.91062 u
18.7%	$^{10}_{5}B$	10.01294 u	17.5%	$^{87}_{36}Kr$	86.91337 u
0%	$^{11}_{6}C$	11.01143 u	12.3%	$^{113}_{48}Cd$	112.90461 u
98.89%	$^{12}_{6}C$	12.00000 u		$^{226}_{88}Ra$	226.02536 u
1.11%	$^{13}_{6}C$	13.00335 u		$^{230}_{90}Th$	230.03308 u
0%	$^{13}_{7}N$	13.00574 u	0.72%	$^{235}_{92}U$	235.04393 u
99.63%	$^{14}_{7}N$	14.00307 u	0%	$^{236}_{92}U$	236.04573 u
0%	$^{14}_{8}O$	14.00860 u	99.28%	$^{238}_{92}U$	238.05076 u
99.76%	$^{16}_{8}O$	15.99491 u	0%	$^{239}_{92}U$	239.05432 u
0.04%	$^{17}_{8}O$	16.99913 u		$^{239}_{93}Np$	239.05294 u
0.20%	$^{18}_{8}O$	17.99916 u		$^{239}_{94}Pu$	239.05216 u
				$^{240}_{94}Pu$	240.05397 u

Simplified Disintegration Patterns

Isotope	$^{60}_{27}Co$	$^{90}_{38}Sr$	$^{90}_{39}Yt$	$^{137}_{55}Cs$	$^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

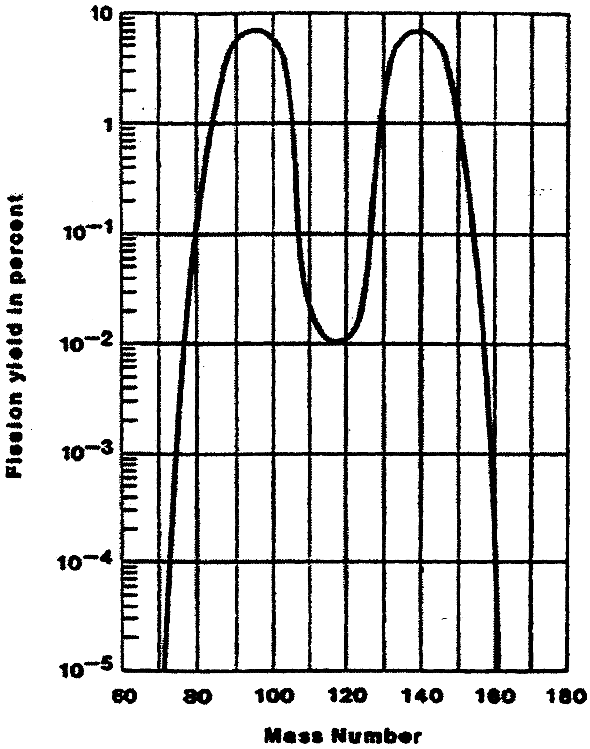
	"Nuclear" graphite	$^{16}_{8}O$	$^{113}_{48}Cd$	$^{235}_{92}U$	$^{238}_{92}U$	$^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half lives) from thermal-neutron fission of U-235 are:

Kr-90 $\xrightarrow{33\ \text{s}}$ Rb-90 $\xrightarrow{2.7\ \text{min}}$ Sr-90 $\xrightarrow{28\ \text{yr}}$ Y-90 $\xrightarrow{64\ \text{hr}}$ Zr-90

Sr-90 is a serious health hazard, because it is bone-seeking.

Sb-131 $\xrightarrow{23\ \text{min}}$ Te-131 $\xrightarrow{24\ \text{min}}$ I-131 $\xrightarrow{8\ \text{days}}$ Xe-131

I-131 is a short-lived health hazard. It is thyroid-seeking.

I-135 $\xrightarrow{6.7\ \text{hr}}$ Xe-135 $\xrightarrow{9.2\ \text{hr}}$ Cs-135 (nearly stable) Fission yield $\gamma = 0.064$

Xe-135 is a strong absorber of thermal neutrons, with $\sigma_a = 3.5$ Mbarn .

Pm-149 $\xrightarrow{54\ \text{hr}}$ Sm-149 Fission yield $\gamma = 0.014$

Sm-149 is a strong absorber of thermal neutrons, with $\sigma_a = 53$ kbarn .

Se-87 $\xrightarrow{17\ \text{s}}$ Br-87 $\xrightarrow{55\ \text{s}}$ either {Kr-86 + n} or {Kr-87 $\xrightarrow{78\ \text{min}}$ Rb-87}

This chain leads to a "delayed neutron".

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	
Mean life time of precursor ($1/\lambda_i$) / s	80	32	8.0	3.1	0.65	0.22	Total
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy ⇒ ~200 MeV / fission

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\frac{\text{production}}{\text{removal}} = \frac{P}{R}$
Excess multiplication factor	k_{ex}	$\frac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\frac{P-R}{P} = \frac{k_{ex}}{k_{eff}}$
Lifetime	l	$\frac{1}{R}$
Reproduction time	Λ	$\frac{1}{P}$

Reactor Kinetics Equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where n = neutron concentration

c = precursor concentration

β = delayed neutron precursor fraction $= \sum \beta_i$

λ = average precursor decay constant

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla.\underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$$D = \frac{1}{3\Sigma_s(1-\overline{\mu})}$$

with $\overline{\mu}$ = the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Cylindrical geometry: $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

Spherical geometry: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \psi^2}$

Bessel's Equation of 0th Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+R=0$$

Solution is: $$R(r)=A_1J_0(r)+A_2Y_0(r)$$

$J_0(0)=1$; $Y_0(0)=-\infty$;

The first zero of $J_0(r)$ is at $r=2.405$.

$J_1(2.405)=0.5183$, where $J_1(r)=\frac{1}{r}\int_0^r xJ_0(x)dx$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density $\mathrm{g\,cm^{-3}}$	Σ_a $\mathrm{cm^{-1}}$	D cm	$L^2=D/\Sigma_a$ $\mathrm{cm^2}$
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $$\theta=\frac{T_M}{T_1}=\frac{2}{M+1}$$

Enrichment of Isotopes

Value function: $$v(x)=(2x-1)\ln\left(\frac{x}{1-x}\right)\approx-\ln(x)\ \text{for small } x$$

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $$R_n\equiv\frac{L_n''}{P}=\frac{x_p-x_{n+1}'}{x_{n+1}'-x_n''}$$

Stripping section reflux ratio: $$R_n=\left[\frac{x_p-x_f}{x_f-x_w}\right]\left[\frac{x_{n+1}'-x_w}{x_{n+1}'-x_n''}\right]$$

Bateman's Equation

$$N_i = \lambda_1 \lambda_2 \ldots \lambda_{i-1} P \sum_{j=1}^{i} \frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j \tau)}{\lambda_j \prod_{\substack{k=1 \\ k\neq j}}^{i} (\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide i

λ_j = decay constant of nuclide j

P = parent nuclide production rate

T = filling time

τ = decay hold-up time after filling

Temperature Distribution

For axial coolant flow in a reactor with a chopped cosine power distribution, Ginn's equation for the non-dimensional temperature is:

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L = fuel half-length

L' = flux half-length

$T_{c1/2}$ = coolant temperature at mid-channel

T_{co} = coolant temperature at channel exit

$Q = \frac{\pi \dot{m} c_p}{UA} \frac{L}{L'}$

with $\dot{m}$ = coolant mass flow rate

c_p = coolant specific heat capacity (assumed constant)

$A = 4\pi r_o L =$ surface area of fuel element

and for radial fuel geometry:

$$\frac{1}{U} = \underbrace{\frac{1}{h}}_{\text{bulk coolant}} + \underbrace{\frac{1}{h_s}}_{\text{scale}} + \underbrace{\frac{t_c}{\lambda_c}}_{\text{thin clad}} + \underbrace{\frac{r_o}{h_b r_i}}_{\text{bond}} + \underbrace{\frac{r_o}{2\lambda_f}\left(1 - \frac{r^2}{r_i^2}\right)}_{\text{fuel pellet}}$$

with h = heat transfer coefficient to bulk coolant

h_s = heat transfer coefficient of any scale on fuel cladding

t_c = fuel cladding thickness (assumed thin)

λ_c = fuel cladding thermal conductivity

r_o = fuel cladding outer radius

r_i = fuel cladding inner radius = fuel pellet radius

h_b = heat transfer coefficient of bond between fuel pellet and cladding

λ_f = fuel pellet thermal conductivity

Q1 (a) -

(b) $\phi = \frac{\phi_1 R_1}{r} \exp\left(\frac{R_1 - r}{L}\right)$

(c) $R_2 = \frac{\pi}{B_m} - \delta$

Q2 (a) -

(b) 750

(c) 5.0594 s; 5.0 s

(d) -

Q3 (a) T_1; T_1/M; $\tau_M = T_1 - \frac{1}{M}\sum_{i=1}^{M-1} i\tau_i$

(b) $\frac{\tau_\infty}{T_1} = \frac{2}{M+1}$

(c) -

(d) -

Q4 (a) -

(b) 2.39 $\mathrm{Bq\,g^{-1}}$