ENGINEERING TRIPOS PART IIB

Tuesday 26 April 2005 2.30 to 4

Module 4A6

FLOW INDUCED SOUND AND VIBRATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment
Data sheet for 4A6 (2 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) In three dimensions the acoustic pressure on $r = a$ is $Ae^{i\omega t}$, where A is a constant.

(i) Find the acoustic pressure everywhere in the case in which the far field is composed only of outgoing waves. [10%]

[Hint: Use the solution of the wave equation,

$$\frac{\alpha}{r}e^{i\omega(t-r/c)} + \frac{\beta}{r}e^{i\omega(t+r/c)},$$

where α and β are arbitrary constants].

(ii) Now suppose instead that the pressure at $r = 0$ is finite. Show that in this case the pressure everywhere is given by

$$\frac{Aa}{r}\frac{\sin(\omega r/c)}{\sin(\omega a/c)}e^{i\omega t}.$$ [15%]

Calculate the acoustic velocity field in this case, and show that the time averaged acoustic energy flux is zero [20%]

Explain what happens to the pressure in $r < a$ when the wave length of sound is much larger than a. [5%]

(b) Low-frequency sound propagates in the positive x direction along a duct with circular cross-section of radius a_1. At $x = 0$ the radius of the duct abruptly changes to a_2. Determine the amplitude of the pressure in the transmitted and reflected waves in terms of the amplitude of the incident wave. [30%]

Under what circumstances are the total time-averaged reflected energy and the total time-averaged transmitted energy equal? [20%]

2 (a) One face of a small cube of side a moves with velocity $u_0 \cos(\omega t)$ in air of density ρ_0. The other faces are stationary. For the case when ωa is much smaller than the speed of sound c, determine the sound power radiated to the far field if the cube is

(i) in unbounded space, [35%]

(ii) at one end of a tube of square cross-section and length ℓ as shown in Fig. 1. [35%]

You may assume that the frequency ω is not a tube resonance.

(b) If, instead of being uniform, the velocity varies over the moving face of the cube with the velocity on the surface with mean normal in the 1-direction being equal to $u_0 \cos(\omega t) \sin(\pi x_2 / a)$ for $-\frac{1}{2}a \le x_2 \le \frac{1}{2}a$, $-\frac{1}{2}a \le x_3 \le \frac{1}{2}a$. The other faces are again stationary and $\omega a \ll c$. Describe qualitatively the form of the sound field if the cube is

(i) in unbounded space, [15%]

(ii) at one end of a tube of length ℓ. [15%]

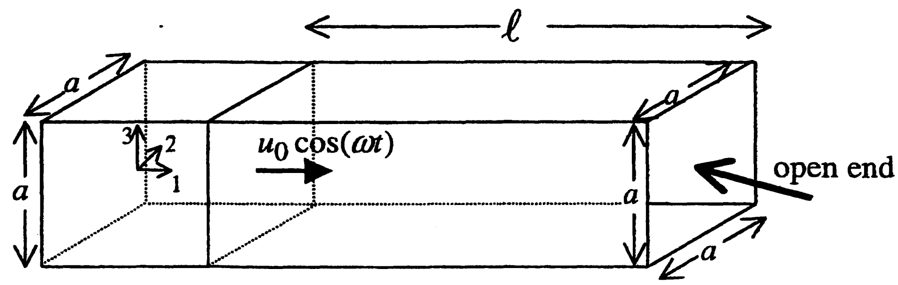

Fig. 1

3 (a) (i) Consider the sound speed profile $c_0(x) = x + \beta$, which varies linearly with x where β is a positive constant. Show that the paths of rays launched in the positive x direction from the origin are circles. [25%]

(ii) The sound-speed profile in the ocean can be modelled as

$$c_0(x) = x + \beta \quad \text{when} \quad 0 \le x \le h$$

$$c_0(x) = x + h \quad \text{when} \quad x \ge h,$$

where h is a positive constant and the x axis points vertically downwards from the surface where $x = 0$. A sonar device is placed on the surface and emits sound in all downward directions. For what range of initial ray angles will a ray from the sonar device return to the surface? Sketch a graph of a typical ray trajectory in this case. [25%]

(b) A Helmholtz resonator is formed by a large bulb of volume V, with a straight neck of length L. The air inside the resonator is in unsteady motion, with frequency ω. Show that:

(i) The mass flux, Q, into the Helmholtz resonator is given by

$$Q = iV\omega\rho'$$ [10%]

where ρ' is the acoustic density fluctuation inside the bulb.

(ii) The acoustic pressure inside the vessel p_2' is given by

$$p_2' = \frac{c_0^2 Q}{iV\omega},$$ [10%]

where c_0 is the speed of sound.

(CONT

(iii) The pressure difference between the outside and the inside is

$$i\omega\frac{QL}{S}$$ [20%]

where S is the cross-sectional area of the neck.

Hence, show that the resonant frequency of the Helmholtz resonator is

$$c_0\sqrt{\frac{S}{VL}}.$$ [10%]

(TURN OVER

4 (a) Derive Lighthill's eighth-power law for jet noise. Formulae on the data card may be used without proof. [50%]

(b) Designs for a 'silent' aircraft aim to reduce the jet noise by 25dB. What change in jet exhaust area is necessary for this to be achieved at static conditions (i.e. when the aircraft has negligible forward speed) while maintaining constant thrust. [30%]

(c) Give three ways in which you would expect the answer to (b) to be influenced by forward flight of the aircraft. [20%]

END OF PAPER

Module 4A6 FLOW INDUCED SOUND AND VIBRATION DATA CARD

USEFUL DATA AND DEFINITIONS

Physical Properties

Speed of sound in an ideal gas $\sqrt{\gamma RT}$, where T is temperature in Kelvins
Speed of sound in sea water, c, is a function of temperature, T

T °C	-4	0	5	10	15	20	25	30
c ms^{-1}	1430.2	1449.5	1471.1	1490.2	1507.1	1521.9	1543.7	1545.9

Units of sound measurement

SPL (sound pressure level) $= 20\log_{10}\left(\dfrac{p'_{rms}}{2.10^{-5}\,\mathrm{Nm}^{-2}}\right)$ dB

IL (intensity level) $= 10\log_{10}\left(\dfrac{\text{intensity}}{10^{-12}\,\text{watts m}^{-2}}\right)$ dB

PWL (power level) $= 10\log_{10}\left(\dfrac{\text{sound power output}}{10^{-12}\,\text{watts}}\right)$ dB

Definitions

Surface impedance, Z_n, relates the pressure perturbation applied to a surface, p', to its normal velocity v_n: $p' = Z_n v_n$.

Characteristic impedance of a fluid $\rho_0 c$

Nondimensional surface impedance of a surface $Z/\rho_0 c$

Transmission loss $= 10\log_{10}\left(\dfrac{\text{incident sound power}}{\text{transmitted sound power}}\right)$

Absorption coefficient of a sound absorber $= \dfrac{\text{sound power absorbed}}{\text{incident sound power}}$

Sound absorption (in metric sabins) $= \sum \alpha_i S_i$ where S_i is surface area (in metres^2) with absorption coefficient α_i.

Reverberation time of a room = time taken for the sound intensity level in the room to drop from 60dB to the threshold of hearing.

Wavelength, λ, for sound waves with angular frequency ω, $\lambda = 2\pi c/\omega$

Wave-number, $k = 2\pi/\lambda$

Phase speed $= \omega/k$

Group velocity $= \dfrac{\partial\omega}{\partial k}$

Helmholtz number (or compactness ratio) $= kD$ where D is a typical dimension of the source.

Strouhal number $= \omega D/(2\pi U)$ for sound of frequency ω produced in a flow with speed U, length scale D.

BASIC EQUATIONS FOR LINEAR ACOUSTICS

Conservation of mass $\dfrac{\partial\rho}{\partial t} + \rho_0\,\nabla\cdot\mathbf{v} = 0$

Conservation of momentum $\rho_0\dfrac{\partial\mathbf{v}}{\partial t} + \nabla p' = 0$

Isentropic $c^2 = \left.\dfrac{dp}{d\rho}\right|_s$

These equations combine to give the wave equation $\dfrac{1}{c^2}\dfrac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = 0$

Energy density $e = \frac{1}{2}\rho_0 v^2 + \frac{1}{2}p'^2/\rho_0 c^2$

Intensity $\mathbf{I} = p'\mathbf{v}$

$\mathrm{div}\,\bar{\mathbf{I}} = 0$ for statistically stationary (in time) sound fields.

Velocity potential $\phi(\mathbf{x},t)$ satisfies the wave equation and $p' = -\rho_0\dfrac{\partial\phi}{\partial t}$, $\mathbf{v} = \nabla\phi$.

SIMPLE WAVE FIELDS

1D or plane wave

The general solution of the 1D wave equation is

$$p'(x_1,t) = f(x_1 - ct) + g(x_1 + ct)$$

where f and g are arbitrary functions.

In a plane wave propagating to the right $p' = \rho_0 c u$. In a plane wave propagating to the left $p' = -\rho_0 c u$, u being the particle velocity.

Spherically symmetric sound fields

The general spherically symmetric solution of the 3D wave equation is

$$p'(r,t) = \frac{f(r-ct)}{r} + \frac{g(r+ct)}{r}$$

where $r = |\mathbf{x}|$; f and g are arbitrary functions.

cos θ dependence

The general solution of the 3D wave equation with $\cos\theta$ dependence is

$$p'(\mathbf{x},t) = \frac{\partial}{\partial x_1}\left[\frac{f(r-ct)}{r} + \frac{g(r+ct)}{r}\right] = \cos\theta\,\frac{\partial}{\partial r}\left[\frac{f(r-ct)}{r} + \frac{g(r+ct)}{r}\right]$$

SOURCES

Point sources

monopole of strength $Q(t)$ at the origin generates a pressure field

$$p'(\mathbf{x},t) = \frac{Q(t-|\mathbf{x}|/c)}{4\pi|\mathbf{x}|}.$$

dipole of strength $\mathbf{F}(t)$ at the origin generates a pressure field

$$p'(\mathbf{x},t) = -\frac{\partial}{\partial x_i}\left[\frac{F_i(t-|\mathbf{x}|/c)}{4\pi|\mathbf{x}|}\right] = \frac{x_i}{4\pi}\left[\frac{1}{|\mathbf{x}|^3}F_i(t-|\mathbf{x}|/c) + \frac{1}{|\mathbf{x}|^2 c}\frac{\partial F_i}{\partial t}(t-|\mathbf{x}|/c)\right]$$

Distributed sources

Monopole, strength $q(\mathbf{x},t)$, wave equation $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p' = q$, pressure field $p'(\mathbf{x},t) = \int \frac{q(\mathbf{y},t-|\mathbf{x}-\mathbf{y}|/c)}{4\pi|\mathbf{x}-\mathbf{y}|}d^3\mathbf{y}$

Dipole, strength $\mathbf{f}(\mathbf{x},t)$, wave equation $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p' = -\nabla\cdot\mathbf{f}$, $p'(\mathbf{x},t) = -\frac{\partial}{\partial x_i}\int \frac{f_i(\mathbf{y},t-|\mathbf{x}-\mathbf{y}|/c)}{4\pi|\mathbf{x}-\mathbf{y}|}d^3\mathbf{y}$.

Quadrupole, strength $T_{ij}(\mathbf{x},t)$, wave equation $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p' = \frac{\partial T_{ij}}{\partial x_i \partial x_j}$, $p'(\mathbf{x},t) = \frac{\partial^2}{\partial x_i \partial x_j}\int \frac{T_{ij}(\mathbf{y},t-|\mathbf{x}-\mathbf{y}|/c)}{4\pi|\mathbf{x}-\mathbf{y}|}d^3\mathbf{y}$.

Far-field form $|\mathbf{x}| >> |\mathbf{y}|$, $\mathbf{y}$ near origin

$$|\mathbf{x}-\mathbf{y}| \sim |\mathbf{x}| - \frac{\mathbf{x}\cdot\mathbf{y}}{|\mathbf{x}|} + O(|\mathbf{x}|^{-2})$$

$$\frac{1}{|\mathbf{x}-\mathbf{y}|} \sim \frac{1}{|\mathbf{x}|} + O(|\mathbf{x}|^{-2})$$

$$\frac{\partial}{\partial x_i} \sim -\frac{x_i}{|\mathbf{x}|c}\frac{\partial}{\partial t} + O(|\mathbf{x}|^{-1}).$$

Physical sources

Heat addition at a rate $w(\mathbf{x},t)$/unit volume is equivalent to a monopole source of strength $\frac{(\gamma-1)}{c^2}\frac{\partial w}{\partial t}$.

Lighthill's acoustic analogy shows that jet noise is generated by quadrupoles of strength $T_{ij} = \rho v_i v_j + \left(p' - c^2\rho'\right)\delta_{ij} - \tau_{ij}$.

The Ffowcs-Williams Hawkings equation shows that foreign bodies in linear motion generate far-field sound

$$p'(\mathbf{x},t) = \frac{1}{4\pi|\mathbf{x}|}\frac{\partial}{\partial t}\int_S \rho_0 \mathbf{n}\cdot\mathbf{u}\left(\mathbf{y},t-\frac{|\mathbf{x}|}{c}+\frac{\mathbf{x}\cdot\mathbf{y}}{|\mathbf{x}|c}\right)dS + \frac{x_i}{4\pi|\mathbf{x}|^2 c}\frac{\partial}{\partial t}\int n_i p\left(\mathbf{y},t-\frac{|\mathbf{x}|}{c}+\frac{\mathbf{x}\cdot\mathbf{y}}{|\mathbf{x}|c}\right)dS$$

USEFUL MATHEMATICAL FORMULAE

In spherical polar coordinates (r,θ,ϕ)

$$\nabla p' = \left(\frac{\partial p'}{\partial r}, \frac{1}{r}\frac{\partial p'}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial p'}{\partial \phi}\right)$$

For $\mathbf{v} = (v_r, v_\theta, v_\phi)$

$$\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\nabla^2 p' = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p'}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial p'}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 p'}{\partial\phi^2}.$$

Heaviside function $H(t-\tau) = 1 \quad t > \tau \; ; \; = 0 \quad t < \tau$

δ-functions

Kronecker delta $\delta_{ij} = 1$ if $i = j$; 0 if $i \neq j$

1D δ-function: $\delta(t) = 0$ for $t \neq 0$; $f(t)\delta(t-\tau) = f(\tau)\delta(t-\tau)$

$\int_{-\infty}^{\infty}\delta(t)dt = 1$ and $\int_{-\infty}^{\infty}\delta(t-\tau)f(t)dt = f(\tau)$ for any function $f(t)$.

3D δ-function: $\delta(\mathbf{x}) = \delta(x_1)\,\delta(x_2)\,\delta(x_3)$;
$\delta(\mathbf{x}) = 0$ for $|\mathbf{x}| \neq 0$;
$f(\mathbf{x})\delta(\mathbf{x}-\mathbf{y}) = f(\mathbf{y})\delta(\mathbf{x}-\mathbf{y})$
$\int_V \delta(\mathbf{x})dV = 1$ for any volume V that includes the origin

and

$\int_V \delta(\mathbf{x}-\mathbf{y})f(\mathbf{x})d^3\mathbf{y} = f(\mathbf{x})$ for any function $f(\mathbf{x})$ and volume V that includes $\mathbf{x}$.

$$\nabla^2\left(\frac{1}{|\mathbf{x}|}\right) = -4\pi\delta(\mathbf{x}).$$

Autocorrelation

$F(\xi)$, the autocorrelation of $f(y) = \overline{f(y)f(y+\xi)}$

$$F(0) = \overline{f^2}$$

Integral lengthscale ℓ $\quad \ell\overline{f^2} = \int_{-\infty}^{\infty}F(\xi)d\xi$.