ENGINEERING TRIPOS PART IIB

Monday 25 April 2005 9:00 to 10:30

Module 4B5

NANOTECHNOLOGY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the invigilator

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1 (a) The theory of quantum mechanics was devised at the turn of the Twentieth century in an attempt to explain a variety of physical phenomena which could not be explained by any other means. Discuss this statement, using three examples based on experimental evidence of the limitations of traditional physics. [30%]

(b) In quantum mechanics, what do wave-packets represent? Consider the wave-packet formed when two electromagnetic waves of the same amplitude but slightly different frequencies are combined. Write down a mathematical expression for such a wave-packet, and show how to extract from it the phase and group velocity of the wave-packet. What constraint is placed on the group velocity? How would we refine this wave-packet and further increase its degree of localisation? [20%]

(c) Consider the electromagnetic wave-packet described by:

$$E(x,t) = E_0\sqrt{\delta}e^{-\frac{\delta^2}{2}(x-ct)^2}\cos(\omega t - kx)$$

Where δ is the width of the spectral function, which is given by the Gaussian expression:

$$f(k) = \frac{1}{\sqrt{2\pi\delta}}e^{-\frac{k^2}{2\delta^2}}$$

Where c is the speed of light, ω is the angular frequency and k is the wavenumber. Outline the steps involved in deriving the form of $E(x,t)$. [30%]

(d) State the Heisenberg uncertainty principle and discuss its interpretation in the context of wave-packets. [20%]

2 Electrons at an energy E are incident from the left on the one-dimensional potential step of height V, which is 1 eV greater than E, as shown below in Fig. 1.

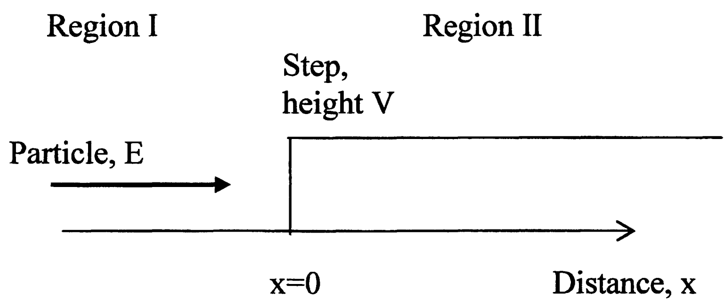

Fig. 1

(a) By considering Schrödinger's time-independent equation, write down the single-particle wave-functions for regions I and II. Hence calculate the reflection coefficient for this potential step. [30%]

(b) Define electronic probability density, and sketch it in both regions. [10%]

(c) Now consider the case where the potential step does not extend to infinity, but only to a certain length *d* as shown below in Fig. 2. Write down the wave-functions and sketch the probability densities in regions I, II and III. Find an *approximate* value of *d* at which the probability density is 10% of the value at x=0, stating any assumptions made. Briefly discuss how you would improve the precision of this calculation. [40%]

(d) What is the phenomenon described in part (c) above called? Briefly describe two examples where it is manifest. [20%]

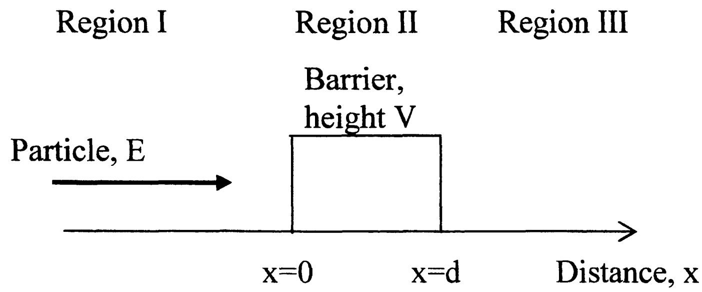

Fig. 2

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3 Consider a device with the potential profile as shown below in Fig. 3 where there are two bound states within the potential well, of energy E_1 and E_2. E_f is the Fermi energy of both sides.

(a) What is this device called? Sketch what happens to this potential profile as a voltage is applied to the right hand side (such that it lowers the potential there), whilst keeping the left hand side at ground potential. [20%]

(b) Describe what happens to the transmission probability, T as the applied voltage is increased. What is the relationship between T and the current through the device? Sketch the form of current vs voltage characteristic for the device, labelling the salient features. [40%]

(c) Sketch the wave-functions for an electron travelling through this device for the case where the applied voltage is such that the Fermi energy on the left hand side

(i) matches either E_1 or E_2, and

(ii) is mid-way between E_1 and E_2. [20%]

(d) Describe what would happen to the bound states if instead of having one potential well as in Fig. 3, we had two potential wells very close together. How many electrons could such a structure contain? How will the bound states evolve as

(i) we introduce more wells, or

(ii) we change the spacing between the two wells? [20%]

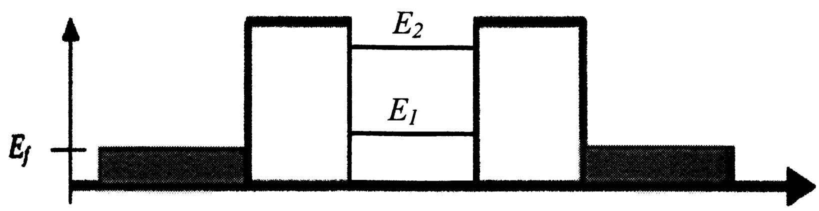

Fig. 3

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4 (a) Write down Schrödinger's equation for a quantum simple harmonic oscillator (QSHO), using the potential $V(x) = \frac{1}{2} kx^2$, where k is the stiffness of the "spring". By choosing an appropriate set of variables, show how this equation can be re-written as

$$\frac{\partial^2 \psi}{\partial y^2} + \left(\alpha - y^2\right)\psi = 0$$ [20%]

(b) Using this equation, derive the spectrum of allowed energy values for the QSHO. Why are only discrete values of energy allowed? How do you explain this apparent disagreement between classical and quantum mechanics? [40%]

(c) Sketch the probability density of the ground state solution, and compare it to that which you would expect from the classical simple harmonic oscillator. [20%]

(d) Discuss what the solution to the QSHO is relevant for. [20%]

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5 (a) Explain what is meant by Band Engineering. [10%]

(b) What is a heterojunction? Describe what happens to the electron bands when a piece of GaAs is brought into contact with a piece of GaAlAs. In what way is the resulting 2-dimensional electron gas (2-DEG) fundamentally different from the sea of electrons in a bulk semiconductor? Sketch the first three sub-bands of a 2-DEG, stating any assumptions made. [30%]

(c) We would like to fabricate a quantum well laser using a semiconductor heterostructure. The quantum well is 5 nm wide, and is formed in a semiconductor with a bandgap of 1.2 eV. Given that the electron and hole effective masses are 0.06 m_e and 0.3 m_e, respectively, where m_e is the free electron mass, estimate the emission wavelength of this laser. How accurate do you expect this answer to be? How would you refine your calculation? [50%]

(d) If we were to replace the quantum well with a quantum dot, in what ways would the laser's operation be different? [10%]

END OF PAPER