ENGINEERING TRIPOS PART IIB

Tuesday 26 April 2005 2.30 to 4

Module 4C2

DESIGNING WITH COMPOSITES

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The **approximate** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Module 4C2 datasheet (6 pages).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 A glass fibre-epoxy laminate is loaded as a beam in 3-point bending with a central load F. The span of the beam between inner and outer supports is ℓ, the width is w and the thickness is t.

(a) Identify likely failure modes and corresponding failure locations for a $[\pm 45]_S$ laminate? Sketch the expected load versus displacement response. Describe how you would calculate the expected failure load. [30%]

(b) A simple approach to design of the beam assumes that the stresses are uniform either side of the beam centre-line, with the top and bottom halves of the beam carrying equal and opposite line loads N_x, as illustrated in Fig. 1. Show that the line load N_x at the centre of the beam, using this approach, is given by:

$$N_x = \frac{F\ell}{2tw}$$ [10%]

(c) Find a suitable ply mix and thickness of laminate which minimises the weight of a beam of dimensions $\ell = 2$ m and $w = 0.1$ m, carrying a central load $F = 2$ kN. In addition the beam must have a shear modulus G_{xy} at least two-thirds of that for a $[\pm 45]_S$ laminate. Carpet plots for GFRP are given in Fig. 2. [40%]

(d) How might you improve and verify the suitability of your beam design found in part (c)? [20%]

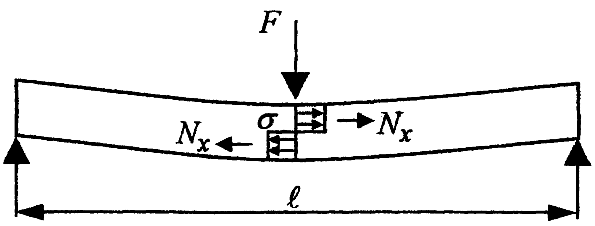

Fig. 1

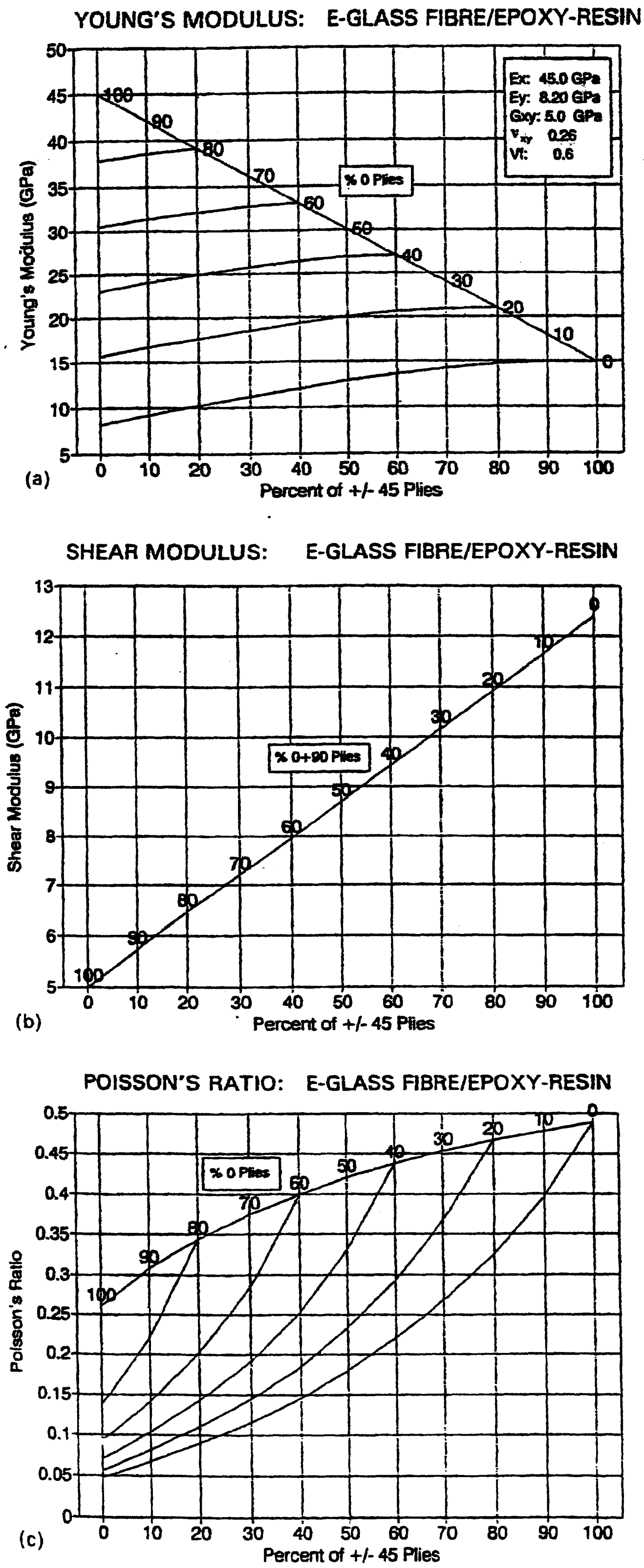

Fig. 2

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2 The hollow shaft of a canoe paddle is to be made from a $[\pm 30]_S$ glass fibre-epoxy pre-preg laminate, with elastic properties $E_2 = 0.2E_1$, $G_{12} = 0.2E_1$ and $\nu_{12} = 0.2$. Each ply is of thickness t.

(a) Obtain expressions for the laminate extensional matrix $[A]$ and the laminate coupling stiffness $[B]$ for the laminate in terms of the material properties. [65%]

(b) The $[\pm 30]_S$ laminate is made into a circular tube of mean diameter D, and wall thickness $4t$.

(i) Calculate the axial strain and hoop strain in the tube wall due to an axial force P. [20%]

(ii) Calculate the twist per unit length of the tube due to a torque Q. [15%]

3 A [0/90/0] cross-ply laminate is made from three plies, each of thickness 0.125 mm, of AS/3501 carbon fibre epoxy material (material data on the data sheet). The $[Q]$ matrix for a 0° lamina of this material is given by:

$$[Q] = \begin{bmatrix} 139 & 2.7 & 0 \\ 2.7 & 9.0 & 0 \\ 0 & 0 & 6.9 \end{bmatrix} \text{GPa}$$

(a) The laminate is subjected to biaxial line loading $N_y = 0.1\ N_x$, where the x axis coincides with the 0° direction.

(i) Use the Tsai-Hill failure criterion to estimate the line load N_x at first ply failure. What would you expect the failure mode to be? [65%]

(ii) Estimate the percentage reduction in the laminate stiffnesses E_x and E_y associated with first ply failure. [15%]

(b) Briefly describe how you might develop a micromechanical model to estimate the spacing of transverse cracks in the 90° ply. [20%]

4 (a) Explain briefly what is meant by "the simultaneous design of (i) an engineering composite structure and (ii) the configuration of the composite laminate from which it is to be made". [25%]

(b) Why are the majority of design problems associated with composite laminates attributed to their poor "secondary material properties"? Give three examples of design problems associated with these properties. [25%]

(c) Briefly describe one method of manufacturing large integrated composite structures for the automotive industry, identifying any additional requirements of this method, as compared with conventional methods using steel. [25%]

(d) A single section of fuselage of the new Boeing 7E7 Dreamliner aircraft is 7 m long, 6 m wide, and contains a door and windows. Propose a suitable composite *material system* from which to manufacture this part, giving your reasons. Outline a *manufacturing route* for making this part, and give an example of a critical issue involved in the manufacturing process. [25%]

END OF PAPER

ENGINEERING TRIPOS PART II B

Module 4C2 – Designing with Composites

DATA SHEET

The in-plane compliance matrix [S] for a transversely isotropic lamina is defined by

$$\begin{pmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{pmatrix}=[S]\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{pmatrix} \quad \text{where } [S]=\begin{bmatrix}1/E_1 & -\nu_{21}/E_2 & 0\\ -\nu_{12}/E_1 & 1/E_2 & 0\\ 0 & 0 & 1/G_{12}\end{bmatrix}$$

[S] is symmetric, giving $\nu_{12}/E_1 = \nu_{21}/E_2$. The compliance relation can be inverted to give

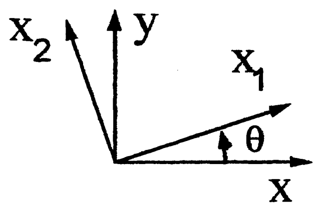
$$\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{pmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{pmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{pmatrix}$$

where

$$\begin{aligned} Q_{11} &= E_1/(1-\nu_{12}\nu_{21})\\ Q_{22} &= E_2/(1-\nu_{12}\nu_{21})\\ Q_{12} &= \nu_{12}E_2/(1-\nu_{12}\nu_{21})\\ Q_{66} &= G_{12}\end{aligned}$$

Rotation of co-ordinates

Assume the principal material directions (x_1, x_2) are rotated anti-clockwise by an angle θ, with respect to the (x, y) axes.



Then,

$$\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{pmatrix}=[T]\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{pmatrix}=[T]^{-T}\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{pmatrix}$$

where

$$[T]=\begin{bmatrix}\cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta\\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta\\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta-\sin^2\theta\end{bmatrix}$$

and

$$[T]^{-T}=\begin{bmatrix}\cos^2\theta & \sin^2\theta & \sin\theta\cos\theta\\ \sin^2\theta & \cos^2\theta & -\sin\theta\cos\theta\\ -2\sin\theta\cos\theta & 2\sin\theta\cos\theta & (\cos^2\theta-\sin^2\theta)\end{bmatrix}$$

The stiffness matrix $[Q]$ transforms in a related manner to the matrix $[\overline{Q}]$ when the axes are rotated from (x_1, x_2) to (x, y)

$$[\overline{Q}] = [T]^{-1}[Q][T]^{-T}$$

In component form,

$$[\overline{Q}] = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \quad \text{where} \quad \begin{aligned} \overline{Q}_{11} &= Q_{11}C^4 + Q_{22}S^4 + 2(Q_{12} + 2Q_{66})S^2C^2 \\ \overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})S^2C^2 + Q_{12}(C^4 + S^4) \\ \overline{Q}_{22} &= Q_{11}S^4 + Q_{22}C^4 + 2(Q_{12} + 2Q_{66})S^2C^2 \\ \overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})SC^3 - (Q_{22} - Q_{12} - 2Q_{66})S^3C \\ \overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})S^3C - (Q_{22} - Q_{12} - 2Q_{66})SC^3 \\ \overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})S^2C^2 + Q_{66}(S^4 + C^4) \end{aligned}$$

with $C = \cos\theta$ and $S = \sin\theta$.

The compliance matrix $[S] \equiv [Q]^{-1}$ transforms to $[\overline{S}] \equiv [\overline{Q}]^{-1}$ under a rotation of co-ordinates by θ from (x_1, x_2) to (x, y), as

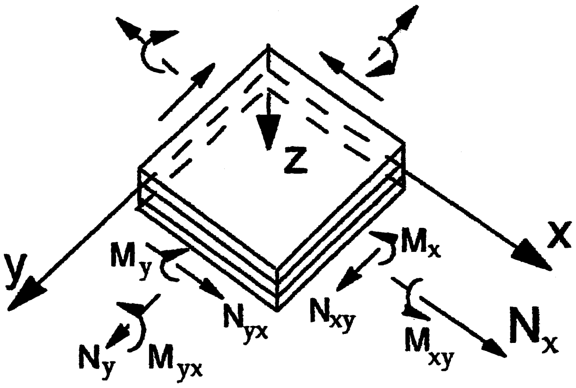
$$[\overline{S}] = [T]^T[S][T]$$

and in component form,

$$\begin{aligned} \overline{S}_{11} &= S_{11}C^4 + S_{22}S^4 + (2S_{12} + S_{66})S^2C^2 \\ \overline{S}_{12} &= S_{12}(C^4 + S^4) + (S_{11} + S_{22} - S_{66})S^2C^2 \\ \overline{S}_{22} &= S_{11}S^4 + S_{22}C^4 + (2S_{12} + S_{66})S^2C^2 \\ \overline{S}_{16} &= (2S_{11} - 2S_{12} - S_{66})SC^3 - (2S_{22} - 2S_{12} - S_{66})S^3C \\ \overline{S}_{26} &= (2S_{11} - 2S_{12} - S_{66})S^3C - (2S_{22} - 2S_{12} - S_{66})SC^3 \\ \overline{S}_{66} &= (4S_{11} + 4S_{22} - 8S_{12} - 2S_{66})S^2C^2 + S_{66}(C^4 + S^4) \end{aligned}$$

with $C = \cos\theta$, $S = \sin\theta$

Laminate Plate Theory



Consider a plate subjected to stretching of the mid-plane by $\left(\varepsilon_x^0, \varepsilon_y^0, \varepsilon_{xy}^0\right)^T$ and to a curvature $\left(\kappa_x, \kappa_y, \kappa_{xy}\right)^T$. The stress resultants $\left(N_x, N_y, N_{xy}\right)^T$ and bending moment per unit length $\left(M_x, M_y, M_{xy}\right)^T$ are given by

$$\begin{pmatrix} N \\ \cdots \\ M \end{pmatrix} = \begin{bmatrix} A & \vdots & B \\ \cdots & . & \cdots \\ B & \vdots & D \end{bmatrix} \begin{pmatrix} \varepsilon^0 \\ \cdots \\ \kappa \end{pmatrix}$$

In component form, we have,

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{pmatrix}$$

where the laminate extensional stiffness, A_{ij}, is given by:

$$A_{ij} = \int_{-t/2}^{t/2} \left(\overline{Q}_{ij}\right)_k dz = \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k - z_{k-1}\right)$$

the laminate coupling stiffnesses is given by

$$B_{ij} = \int_{-t/2}^{t/2} \left(\overline{Q}_{ij}\right)_k z dz = \frac{1}{2} \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right)$$

and the laminate bending stiffness are given by:

$$D_{ij} = \int_{-t/2}^{t/2} \left(\overline{Q}_{ij}\right)_k z^2 dz = \frac{1}{3}\sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right)$$

with the subscripts $i, j = 1, 2$ or 6.

Here,

$t =$ laminate thickness

$z_{k-1} =$ distance from middle surface to the inner surface of the k - th lamina

$z_k =$ distance from middle surface to the outer surface of the k - th lamina

Quadratic failure criteria.

For plane stress with $\sigma_3 = 0$, failure is predicted when

Tsai-Hill: $\frac{\sigma_1^2}{s_L^2} - \frac{\sigma_1\sigma_2}{s_L^2} + \frac{\sigma_2^2}{s_T^2} + \frac{\tau_{12}^2}{s_{LT}^2} \geq 1$

Tsai-Wu: $F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 \geq 1$

where $F_{11} = \frac{1}{s_L^+ s_L^-}$, $F_{22} = \frac{1}{s_T^+ s_T^-}$, $F_1 = \frac{1}{s_L^+} - \frac{1}{s_L^-}$, $F_2 = \frac{1}{s_T^+} - \frac{1}{s_T^-}$, $F_{66} = \frac{1}{s_{LT}^2}$

F_{12} should ideally be optimised using appropriate strength data. In the absence of such data, a default value which should be used is

$$F_{12} = -\frac{\left(F_{11}F_{22}\right)^{1/2}}{2}$$

Fracture mechanics

Consider an orthotropic solid with principal material directions x_1 and x_2. Define two effective elastic moduli E'_A and E'_B as

$$\frac{1}{E'_A}=\left(\frac{S_{11}S_{22}}{2}\right)^{1/2}\left(\left(\frac{S_{22}}{S_{11}}\right)^{1/2}\left(1+\frac{2S_{12}+S_{66}}{2\sqrt{S_{11}S_{22}}}\right)\right)^{1/2}$$

$$\frac{1}{E'_B}=\left(\frac{S_{11}S_{22}}{2}\right)^{1/2}\left(\left(\frac{S_{11}}{S_{22}}\right)^{1/2}\left(1+\frac{2S_{12}+S_{66}}{2\sqrt{S_{11}S_{22}}}\right)\right)^{1/2}$$

where S_{11} etc. are the compliances.

Then G and K are related for plane stress conditions by:

crack running in x_1 direction: $G_I E'_A = K_I^2; G_{II} E'_B = K_{II}^2$

crack running in x_2 direction: $G_I E'_B = K_I^2; G_{II} E'_A = K_{II}^2$.

For mixed mode problems, the total strain energy release rate G is given by

$G = G_I + G_{II}$

Approximate design data

	Steel	Aluminium	CFRP	GFRP	Kevlar
Cost C (£/kg)	1	2	100	5	25
E_1 (GPa)	210	70	140	45	80
G (GPa)	80	26	≈35	≈11	≈20
ρ (kg/m^3)	7800	2700	1500	1900	1400
e^+ (%)	0.1-0.8	0.1-0.8	0.4	0.3	0.5
e^- (%)	0.1-0.8	0.1-0.8	0.5	0.7	0.1
e_{LT} (%)	0.15-1	0.15-1	0.5	0.5	0.3

Table 1. Material data for preliminary or conceptual design. Costs are very approximate.

	Aluminium	Carbon/epoxy (AS/3501)	Kevlar/epoxy (Kevlar 49/934)	E-glass/epoxy (Scotchply/1002)
Cost (£/kg)	2	100	25	5
Density (kg/m^3)	2700	1500	1400	1900
E_1 (GPa)	70	138	76	39
E_2 (GPa)	70	9.0	5.5	8.3
ν_{12}	0.33	0.3	0.34	0.26
G_{12} (GPa)	26	6.9	2.3	4.1
s_L^+ (MPa)	300 (yield)	1448	1379	1103
s_L^- (MPa)	300	1172	276	621
s_T^+ (MPa)	300	48.3	27.6	27.6
s_T^- (MPa)	300	248	64.8	138
s_{LT} (MPa)	300	62.1	60.0	82.7

Table 2. Material data for detailed design calculations. Costs are very approximate.

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