ENGINEERING TRIPOS PART IIA
ENGINEERING TRIPOS PART IIB

Thursday 28 April 2005 9 to 10.30

Module 4C4

DESIGN METHODS

*Answer not more than **three** questions.*

All questions carry the same number of marks..

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin*

Attachments: Special Datasheet (6 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 (a) Discuss a range of approaches that may be taken to design safe systems. [30%]

(b) Describe the method of Fault Tree Analysis and explain its role in the analysis of system behaviour. [20%]

(c) A butane gas supply is being designed for use in a fire-fighter training unit (FFTU) (Fig. 1). A *butane tank* is connected to a feed line via two *manual valves* connected in series. The pipe traverses 100 m underground to the FFTU where it is connected to a pair of *shutdown valves*, again connected in series.

The shutdown valves are designed to fail-safe and each contains two position sensors monitoring their open and closed positions. A control system monitors the position of the shutdown valves and will close them if there is a persistent discrepancy between the sensor states and valve position. The *shutdown valve control signal* is connected via a pair of fail-safe *butane sensors* wired in series. If butane gas is detected the shutdown valves will be automatically closed.

Butane flows from the shutdown valves to a pressure *regulator* and then to a proportional *burner valve*. The valve is controlled by the *burner valve control signal* and has two position sensors monitoring its open and closed positions. The burner valve control signal is connected via a pair of *guard rail* sensors which prevent the valve opening if the guard rail is not in position. The guard rail prevents fire-fighters from falling on the burner. The height of the *burner* flame is controlled by the burner valve control signal.

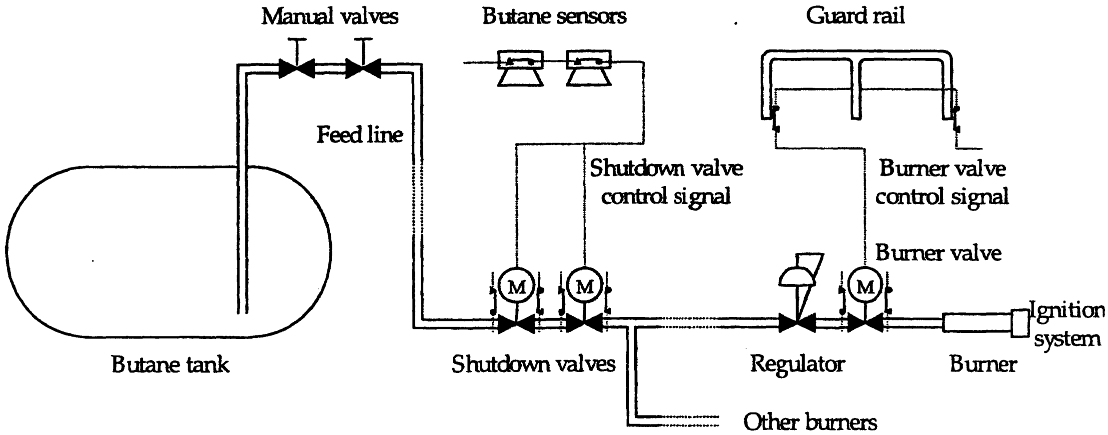

Fig. 1

(i) Draw a fault tree to show the events that may lead to a fire-fighter falling on the burner when it is lit. [15%]

(ii) Draw a fault tree to show the events that may lead to a butane gas leak. [35%]

2 A leading pharmaceutical company wishes to develop a new Dry Powder Inhaler (DPI). The device is to deliver no fewer than 200 doses. The powder is to be stored in bulk in the inhaler, i.e. all the powder is to be stored in a single compartment.

The base material, which is used to provide bulk to the active ingredient, is lactose which is known to be hygroscopic. Particles with sizes in the range 30 to 70 μm are known to be acceptable for inhalation. Each dose, including the lactose, nominally occupies a volume of 5 mm^3.

The company would like to investigate two different powder formulations. The first comprises lightly compressed powder in the form of a cylindrical pellet. The second comprises loose uncompressed powder.

(a) Use a solution neutral problem statement to describe the overall function of the new inhaler. [10%]

(b) List the key requirements for the new inhaler. [20%]

(c) Define a process function structure for the inhaler. [10%]

(d) Detail two inhaler concepts, one for each powder formulation, that will ensure effective administration of the powder. [50%]

(e) Discuss the relative merits of the two concepts presented in (d). [10%]

3 An engineer wishes to determine the optimum hub height h and blade tip radius r of a windmill for a particular site.

The wind velocity U at a height h above the ground may be given by the following power law:

$$\frac{U(h)}{U_r} = \left(\frac{h}{h_r}\right)^{\alpha}$$

where U_r is the wind velocity at a reference height h_r and the parameter $\alpha = 1/3$ for this particular site.

The mechanical power P extracted from the wind by the windmill is given by:

$$P = \frac{1}{2}\rho U^3 C_P A$$

where ρ is the density of air, U is the wind velocity at the windmill hub, C_p is a fixed coefficient of performance and A is the area swept by the blades.

(a) (i) Given that no part of the windmill may exceed a height limit h_{max} make a formal optimisation statement for maximising the mechanical power P. Include all obvious constraints and distinguish between constants, parameters and variables. [30%]

(ii) Find the values for the variables h and r that give an optimum for the maximum power and comment on the activity of any constraints. [20%]

(b) A simple financial model for the windmill gives the construction cost of the blades and generator as £$800r^3$ and the cost of the tower and foundations as £$3h^3r^2$ where h and r are in metres. The efficiency and lifetime of the windmill is such that it has a current financial value of £kP where P is the mechanical power of the windmill in Watts and:

$$k = \frac{900}{\left[\dfrac{\pi\rho U_r{}^3 C_P}{2h_r}\right]}$$

(i) Find an objective function for the net cost of the windmill. [15%]

(ii) Using $h = 16$ m and $r = 7$ m as a starting point use one step of the gradient (steepest descent) method to find an improved design. [35%]

4 During use the chain and chainring on a bicycle wear. This leads to a lengthening of the chain and ultimately the chain will *jump* off the chainring under load. A simple model has been found to be useful in estimating the effect of chain and chainring wear by focussing on the *effective* pitch of each part.

The mean effective pitch of the chain P_c in mm and its deviation σ_c are given by:

$$P_c = P_0(1 + \alpha t) \quad \text{and} \quad \sigma_c = \sigma_0(1 + 10\alpha t)$$

where α represents the effect of chain wear, t is the time in years, P_0 is the mean pitch of a new chain and σ_0 is the standard deviation in pitch of a new chain. Similarly, the mean pitch P_r and deviation σ_r of the chainring are given as:

$$P_r = P_0(1 + \beta t) \quad \text{and} \quad \sigma_r = \sigma_0(1 + 10\beta t)$$

where β represents the effect of chainring wear.

If the difference between the chain and chainring pitches exceeds 0.5 ± 0.1 mm the chain will be prone to *jump*. For a particular chain and chainring, P_0=12 mm, σ_0=0.01 mm, α=0.04 and β=0.02.

(a) Estimate the time in years when the *safety factor* for the chain jumping reaches unity using:

(i) mean pitch values; [20%]

(ii) worst case pitch values. [30%]

(b) Estimate the time in years when the *safety margin* for the chain jumping predicts that 5% of chains will jump. [40%]

(c) If chain lubrication can reduce wear by a factor of two (α=0.02 and β=0.01), estimate the revised time in years before 5% of chains jump. [10%]

Assume that all probability distributions are normal and that the range from minimum to maximum is equivalent to two standard deviations.

END OF PAPER

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1995

(Revised 2001)

(Revised 2002)

(Revised 2003)

MODULE 4C4

DATA BOOK

1.	OPTIMIZATION	Page 2
2.	STATISTICS	Page 5

1.0 OPTIMIZATION

DATA SHEET

1.1 Series

Taylor Series

For a function of one variable:

$$f(x_k+\delta)=f(x_k)+\delta f'(x_k)+\frac{1}{2}\delta^2 f''(x_k)+\ldots \quad \text{where} \quad x_{k+1}=x_k+\delta$$

For a function of several variables:

$$f(\underline{x}_k+\underline{\delta x})=f(\underline{x}_k)+\{\nabla f(\underline{x}_k)\}^t\underline{\delta x}+\frac{1}{2}\underline{\delta x}^t\mathbf{H}(\underline{x}_k)\underline{\delta x}+\ldots \quad \text{where} \quad \underline{x}_{k+1}=\underline{x}_k+\underline{\delta x}$$

where $\{\nabla f(\underline{x}_k)\}^t$ is the Grad of the function at $\underline{x}_k$:

$$\begin{bmatrix} \frac{\partial f(\underline{x}_k)}{\partial x_1} & \frac{\partial f(\underline{x}_k)}{\partial x_2} & \cdots & \frac{\partial f(\underline{x}_k)}{\partial x_n} \end{bmatrix}$$

and $\mathbf{H}(\underline{x}_k)$ is the Hessian of the function at $(\underline{x}_k)$:

$$\begin{bmatrix} \frac{\partial^2 f(\underline{x}_k)}{\partial x_1^2} & \frac{\partial^2 f(\underline{x}_k)}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f(\underline{x}_k)}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f(\underline{x}_k)}{\partial x_2 \partial x_1} & & & \\ \vdots & & & \\ \frac{\partial^2 f(\underline{x}_k)}{\partial x_n \partial x_1} & \frac{\partial^2 f(\underline{x}_k)}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f(\underline{x}_k)}{\partial x_n^2} \end{bmatrix}$$

Note:
1. $\nabla f(\underline{x}_k)$ is defined as a column vector.
2. The Hessian is symmetric.
3. If $f(\underline{x})$ is a quadratic function the elements of the Hessian are constants and the series has only three terms.

1.2 Line searches

Golden Section Ratio = $\frac{\sqrt{5}-1}{2} \approx 0.6180$

Newton's Method (1D)

When derivatives are available: $x_{k+1} = x_k - \{f'(x_k)\}/\{f''(x_k)\}$

When derivatives are unavailable:

$$x_4 = \frac{1}{2}\frac{\left(x_2^2 - x_3^2\right)f(x_1) + \left(x_3^2 - x_1^2\right)f(x_2) + \left(x_1^2 - x_2^2\right)f(x_3)}{(x_2 - x_3)f(x_1) + (x_3 - x_1)f(x_2) + (x_1 - x_2)f(x_3)}$$

1.3 Multidimensional searches

Conjugate Gradient Method

To find the minimum of the function

$f(\underline{x}) = f(\underline{x}_0) + \nabla f(\underline{x}_0)^t\, \partial\underline{x} + \frac{1}{2}\partial\underline{x}^t \mathbf{H} \partial\underline{x}$, where $\partial\underline{x} = \underline{x} - \underline{x}_0$ and $\underline{x}$ has n dimensions:

First move is in direction $\underline{s}_0$ from $\underline{x}_0$ where:

$$\underline{s}_0 = -\nabla f(\underline{x}_0)$$

Then $\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{s}_k$

where $\alpha_k = \frac{-\underline{s}_k{}^t \nabla f(\underline{x}_k)}{\underline{s}_k{}^t H \underline{s}_k}$ (which minimises $f(\underline{x})$ along the defined line)

Then $\underline{s}_{k+1} = -\nabla f(\underline{x}_{k+1}) + \beta_k \underline{s}_k$

where $\beta_k = \frac{\nabla f(\underline{x}_{k+1})^t \mathbf{H} \underline{s}_k}{\underline{s}_k{}^t \mathbf{H} \underline{s}_k}$

For a quadratic function, the method converges at $\underline{x}_n$.

Fletcher-Reeves Method

To find the minimum of the function $f(\underline{x})$ where $\underline{x}$ has n dimensions:

First move is in direction $\underline{s}_0$ from $\underline{x}_0$ where:

$$\underline{s}_0 = -\nabla f(\underline{x}_0)$$

Then $\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{s}_k$ such that $f(\underline{x})$ is minimised along the defined line.

Then $\underline{s}_{k+1} = -\nabla f(\underline{x}_{k+1}) + \beta_k \underline{s}_k$

where $\beta_k = \dfrac{(\nabla f(\underline{x}_{k+1}))^2}{(\nabla f(\underline{x}_k))^2}$

For quadratic functions, the method will converge at $\underline{x}_n$. For higher order functions, the method should be restarted when $\underline{x}_n$ is reached.

1.4 Constrained Minimisation

Penalty and Barrier functions

The most common Penalty function is:

$$q(\mu, \underline{x}) = f(\underline{x}) + \frac{1}{\mu}\sum_{i=1}^{p} (\max[0, g_i(\underline{x})])^2$$

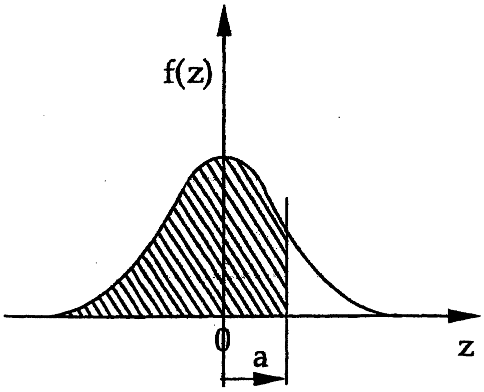
where $f(\underline{x})$ is subject to the constraints $g_1(\underline{x}) \le 0, \ldots, g_p(\underline{x}) \le 0$

A typical Barrier function for the same problem is:

$$q(\mu, \underline{x}) = f(\underline{x}) - \mu\sum_{i=1}^{p} g_i(\underline{x})^{-1}$$

2.0 STATISTICS DATA SHEET

2.1 Standardised normal probability density function



$$P(z<a)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{-\frac{z^2}{2}}\,dz$$

$$z=\frac{x-\mu}{\sigma}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9349	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9723	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABULATED VALUES

2.2 Combining distributed variables

For the function

$$y = f(x_1, x_2, \dots x_n)$$

where x_1, x_2 etc. are <u>independent</u> and defined by their respective distributions:

	y	μ_y	σ_y^2
1	$x + a$	$\mu_x + a$	σ_x^2
2	ax	$a\mu_x$	$a^2\sigma_x^2$
3	$a_1x_1 + a_2x_2$	$a_1\mu_1 + a_2\mu_2$	$a_1^2\sigma_1^2 + a_2^2\sigma_2^2$
4	x_1x_2	$\mu_1\mu_2$	$\mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2$
5	x_1/x_2	μ_1/μ_2	$\frac{1}{\mu_2^4}\left(\mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2\right)$

Where: μ = mean; σ = standard deviation; a = constant.