ENGINEERING TRIPOS PART IIB

Tuesday 26 April 2005 9 to 10.30

Module 4C6

ADVANCED LINEAR VIBRATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Candidates may bring their notebooks to the examination.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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1 A modal analysis is to be carried out on a structure using an impulse hammer. The head of the impulse hammer has a mass of 0.2 kg, the sensitivity of the force transducer is 4 pC/N and the velocity of the hammer on impact is 2 m/s. Three hammer tips are available with stiffnesses of 0.5 MN/m, 2 MN/m or 10 MN/m. The frequency range of interest is from 10 Hz to 1 kHz.

(a) (i) Which of the three hammer tips is most suitable for this application? Give reasons for your answer. [15%]

(ii) Sketch a graph of force against time for the impulse and indicate on your sketch values for the duration of the impulse and the peak force with your chosen hammer tip. [15%]

(iii) Design a simple charge amplifier to produce an output signal suitable for a data logger with an input range of ±5 V. Include in your design a suitable high-pass filter and explain why this is necessary. Show on a sketch how the high-pass filter influences the shape of the impulse. [15%]

(iv) Select a suitable sample rate for the data logger giving reasons for your answer. [5%]

(b) For a particular excitation point x and grid point y the modal analysis identifies four modes as follows:

Mode n	f_n (Hz)	$u_n(x)u_n(y)$	Q_n
1	47	0.43	28
2	56	-0.77	37
3	152	2.8	260
4	658	1.62	17

(i) Sketch the transfer function over the frequency range of interest with a dB vertical scale. [15%]

(ii) Discuss whether modes 1 and 2 are sufficiently well separated for the assumptions of circle-fitting analysis to be valid. [5%]

(iii) Sketch the four modal circles. How might the proximity of modes 1 and 2 affect their modal circles? [20%]

(iv) Sketch the response to the impulse in the time domain for a period of 1 s after the impulse. [10%]

2 "Damping tape" can be modelled as a constrained-layer system as analysed in the lecture notes, in which the thicknesses of the damping layer and constraining layer are much thinner than that of the structure to which the tape is attached: $h_2 << h_1$, $h_3 << h_1$. Under these conditions, it may be assumed that the complicated expression for the combined bending stiffness EI can be simplified to

$$EI \approx \frac{E_1 h_1^3}{12} - \frac{h_1^2}{8(1+g)}\left(E_2 h_2 + 2E_3 h_3\right)$$

where

$$g = \frac{G_2}{E_3 h_2 h_3 p^2}$$

with the wavenumber p determined by the underlying structure:

$$p^2 = \omega\sqrt{\frac{12m}{E_1 h_1^3}}$$

where ω is (angular) frequency, m is the mass per unit length of the beam, and all other notation is as in the lecture notes.

(a) Assuming that the only damping in the system comes from a complex shear modulus $G_2(1+i\eta_G)$, obtain an approximate expression for the effective loss factor of the damped beam. Do not assume that η_G is necessarily small. [40%]

(b) Sketch the effective loss factor as a function of frequency, assuming that the complex shear modulus is independent of frequency. Show that the loss factor reaches a maximum value at a certain frequency, and obtain an expression for this frequency and for the maximum value. [40%]

(c) Comment on the influence on damping performance of the value of the real part of the shear modulus of the damping material. Compare this with the influence of the value of the real part of Young's modulus of the damping material in a free-layer damping treatment. Comment on the significance of this comparison for the choice of possible damping materials for the two types of damping treatment. [20%]

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3 (a) A cylindrical cavity of radius a and height h is surrounded by rigid walls. For sound inside the cavity, acoustical pressure p is governed by a differential equation which can be written in cylindrical polar coordinates (r,θ,z) in the form

$$c^2\left[\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}+\frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2}+\frac{\partial^2 p}{\partial z^2}\right]=\frac{\partial^2 p}{\partial t^2}$$

where c is the speed of sound. The appropriate boundary condition on the walls is that the normal gradient of pressure is zero everywhere. Use the method of separation of variables to show that the acoustical normal modes in the cavity can be written

$$p = J_n(kr)\cos\frac{m\pi z}{h}\cos n\theta \quad \text{or} \quad J_n(kr)\cos\frac{m\pi z}{h}\sin n\theta, \quad m,n=0,1,2\cdots$$

where J_n is the Bessel function and

$$k^2=\left[\frac{\omega}{c}\right]^2-\left[\frac{m\pi}{h}\right]^2.$$ [40%]

Hence obtain an equation which determines the natural frequencies. [20%]

(b) Making use of the graphs of Bessel functions in Fig. 1, estimate the lowest non-zero natural frequency for the case h = 1 m, a = 0.5 m, c = 340 m/s. Describe the corresponding mode shape. [20%]

(c) Explain in qualitative terms what would happen to the natural frequencies if a small hole were made in one of the walls of the cavity. Can the interlacing theorem be applied to this modification? [20%]

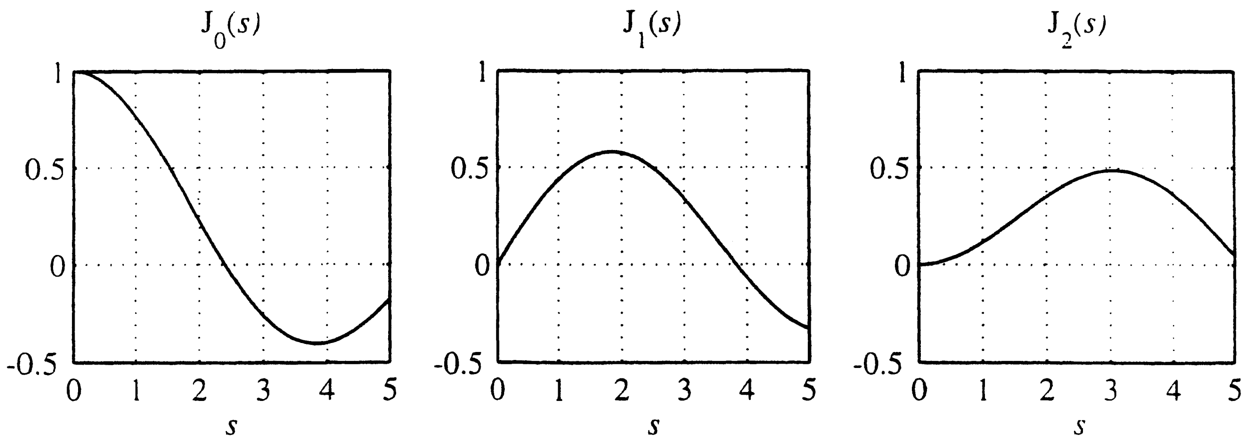

Figure 1

4 A rod of length L_1 with cross-sectional area A, Young's modulus E_1 and density ρ_1 is joined rigidly to the end of another rod of length L_2 with cross-sectional area A, Young's Modulus E_2 and density ρ_2. The combined rod of length $L_1 + L_2$ is built-in at both ends to fixed supports, and undergoes axial vibration, as shown in Fig. 2.

(a) Show that harmonic axial displacement y at frequency ω can be written in the form

$$y(x,t) = e^{i\omega t}\begin{cases} \alpha_1 \sin k_1(x + L_1) & -L_1 \le x \le 0 \\ \alpha_2 \sin k_2(x - L_2) & 0 \le x \le L_2 \end{cases}$$

where k_1, k_2 should be defined. [25%]

Specify the boundary conditions at the join between the two sections of rod, and hence show that the natural frequencies are determined by the equation

$$\frac{\tan k_1 L_1}{\sqrt{E_1 \rho_1}} + \frac{\tan k_2 L_2}{\sqrt{E_2 \rho_2}} = 0 \, .$$ [25%]

(b) Sketch a graphical solution of this equation. [25%]

(c) Use the solution to (b) to demonstrate the application of the interlacing theorem to this problem. [25%]

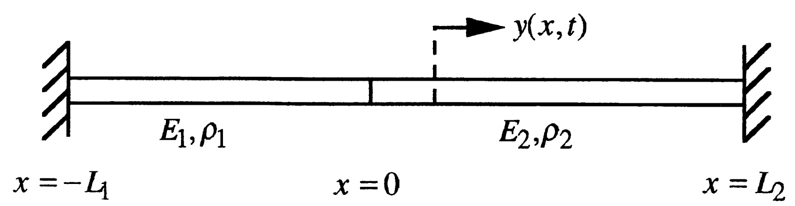

Fig. 2

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