UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Friday 29 April 2005 2.30 to 4

Module 4C8

APPLICATIONS OF DYNAMICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4C8 datasheet (3 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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1 A 'bicycle' model of a car, with freedom to sideslip with velocity v and yaw at rate Ω, is shown in Fig. 1. The car moves at steady forward speed u on a horizontal surface. It has mass m, yaw moment of inertia I, and lateral creep coefficients of C_f and C_r at the front and rear tyres. The lengths a and b and the steering angle δ are defined in the figure. A yawing moment N acts on the vehicle and a lateral force Y is applied at the centre of gravity G. The equations of motion in a coordinate frame rotating with the vehicle are given by:

$$m(\dot{v} + u\Omega) + (C_f + C_r)\frac{v}{u} + (aC_f - bC_r)\frac{\Omega}{u} = C_f\delta + Y$$

$$I\dot{\Omega} + (aC_f - bC_r)\frac{v}{u} + (a^2C_f + b^2C_r)\frac{\Omega}{u} = aC_f\delta + N$$

(a) State the assumptions needed to derive these equations of motion. [10%]

(b) Use the equations of motion to derive expressions for the steady state yaw rate and sideslip responses of the vehicle to a side force F applied a distance x forward of G, with $\delta = 0$. [30%]

(c) Define the terms *neutral steer point, static margin, oversteer* and *understeer.* Explain how the handling response of the car to a steady side force varies with the static margin. Sketch the paths of motion of a car with positive, negative and zero static margins. [30%]

(d) Derive an expression for the steering angle δ needed to turn the vehicle on a steady circular curve of radius R with $Y = N = 0$. Use this expression to explain the terms oversteer and understeer for a steadily turning car. [30%]

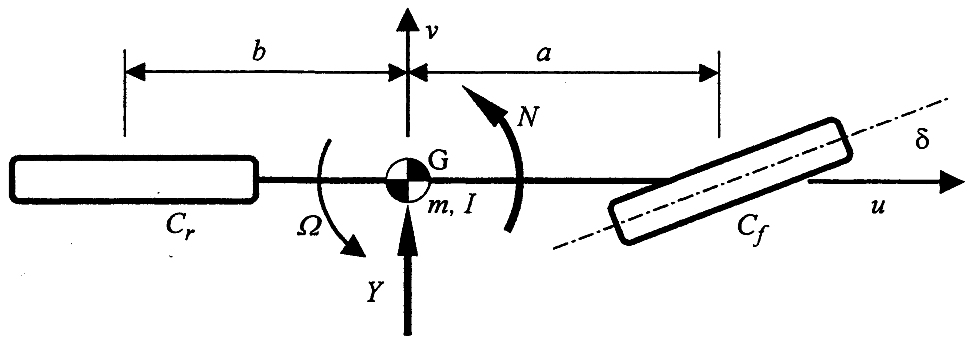

Fig. 1

2 Figure 2 shows a plan view of an idealized railway bogie which consists of two light, rigid wheelsets, connected to a light frame AB of length $2a$ by suspensions with yaw stiffnesses k. The wheels all have lateral and longitudinal creep coefficients C, effective conicity ε and average rolling radii r when running in the central position. The bogie runs on a straight track of gauge $2d$ at steady forward speed u, with a constant lateral force P applied at its mid point C. The wheelsets take up steady yaw angles θ_1 and θ_2 with constant lateral tracking errors y_1 and y_2 as shown (exaggerated) on the figure.

(a) Show that the net yawing moment acting on the front wheelset as a result of the wheel/rail contact forces is

$$N = \frac{2dC\varepsilon y_1}{r}$$

and derive an expression for the net lateral force acting on the front wheelset. State your assumptions. [40%]

(b) Derive a set of equations that could be used to find the steady state configuration of the bogie (y_1, y_2, θ_1, θ_2) and explain how you would solve these equations to find the yaw angle of the bogie frame relative to the track centre-line. [30%]

(c) Use your equations from part (b) to determine the lateral tracking error at C and the yaw angle of the bogie, for the case $k \to \infty$. Sketch the configuration of the bogie in this case. [30%]

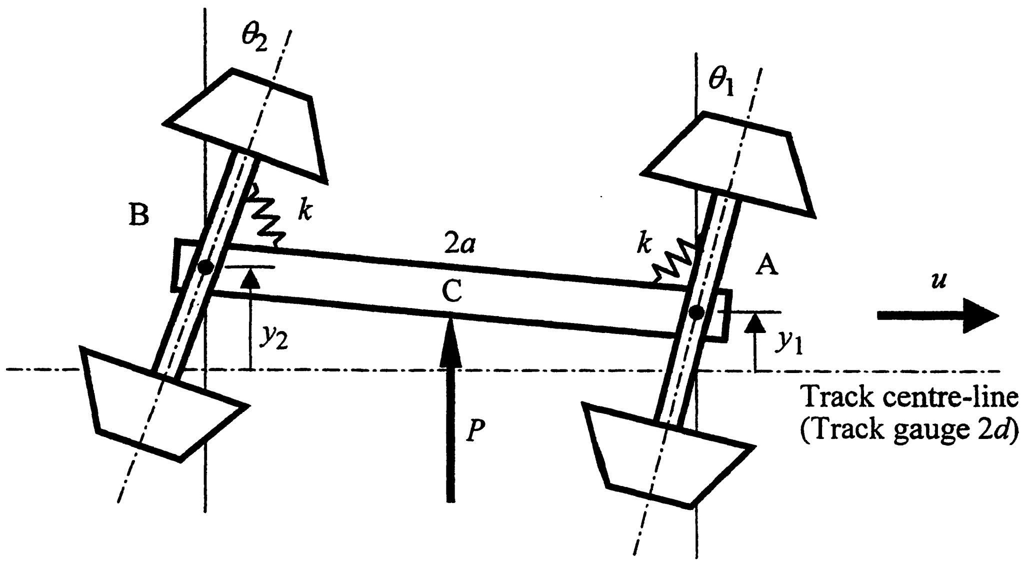

Fig. 2

3 Figure 3 shows a quarter-car model and Fig. 4 shows the corresponding conflict diagram. The conflict diagram shows how root mean square (RMS) tyre force, sprung mass acceleration and suspension working space vary with suspension stiffness k and damping c. The vertical input at the tyre-road contact is white-noise velocity with double-sided spectral density $S_0 = 10^{-4}$ (m s^{-1})^2(rad s^{-1})$^{-1}$.

(a) Explain the practical significance of the three response quantities shown in Fig. 4. [15%]

(b) With reference to the points A to E shown on Fig. 4, discuss the compromises involved in selecting the stiffness and damping for the suspension of a passenger car. Briefly explain why an active suspension can overcome some of the compromises. [50%]

(c) In Fig. 4 the mean square values of the sprung mass acceleration were calculated using:

$$\mathrm{E}\left[\ddot{z}_s^2\right] = \pi S_0 \left(\frac{(m_u + m_s)k^2 + k_t c^2}{m_s^2 c} \right)$$

Describe the steps that you would perform to derive this expression, starting from the model shown in Fig. 3. It is not necessary to include lengthy algebraic manipulation, but it should be possible for a Part I engineering student to derive the expression using your description. You may quote expressions from the data sheet and the following result:

$$\int_{-\infty}^{\infty} |H(j\omega)|^2 \, \mathrm{d}\omega = \frac{-\pi\left(A_0 A_1 A_4 B_2^2 + A_0 A_3 A_4 B_1^2\right)}{A_0 A_4 \left(A_0 A_3^2 + A_1^2 A_4 - A_1 A_2 A_3\right)}$$

where

$$H(j\omega) = \frac{B_0 + (j\omega)B_1 + (j\omega)^2 B_2 + (j\omega)^3 B_3}{A_0 + (j\omega)A_1 + (j\omega)^2 A_2 + (j\omega)^3 A_3 + (j\omega)^4 A_4}$$

[35%]

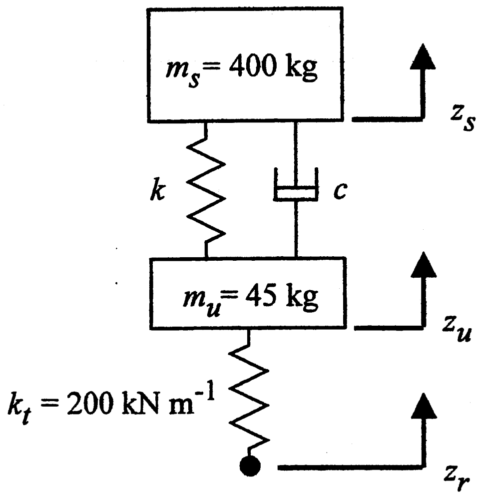

Fig. 3

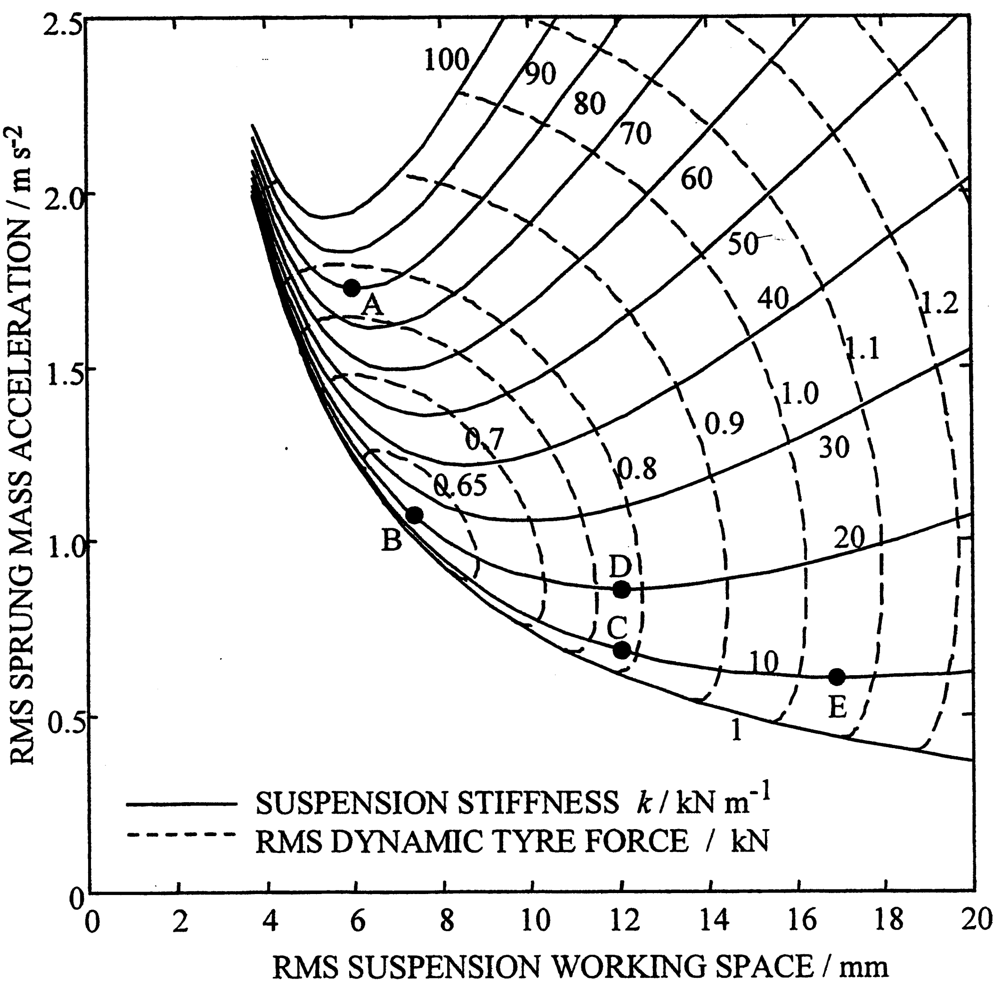

Fig. 4

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4 Figure 5 shows a roll-plane vehicle model. The model is used to calculate the vertical acceleration $\ddot{z}_s$ and roll acceleration $\ddot{\theta}_s$ at the centre of the sprung mass, and the vertical acceleration $\ddot{z}_p$ at a distance p from the centre of mass. The inputs are the road surface displacements z_L and z_R at the left and right tyres.

(a) If the road surface is assumed to be randomly rough, explain why the mean square spectral density (MSSD) of $\ddot{z}_p$ can be evaluated as

$$S_{\ddot{z}_p}(\omega) = S_{\ddot{z}_s}(\omega) + p^2 S_{\ddot{\theta}_s}(\omega).$$ [35%]

(b) Figure 6 shows the MSSDs of the sprung mass acceleration responses, calculated assuming that the input is a randomly rough road surface and the vehicle speed is 30 m s^{-1}. Explain the relative magnitudes of the spectral densities of $\ddot{z}_s$ and $p\ddot{\theta}_s$ in the range 0 Hz to 5 Hz and in the range 5 Hz to 25 Hz. [30%]

(c) Figure 7 shows the MSSD of the sprung mass acceleration responses, calculated assuming that the input is a randomly rough road surface and the vehicle speed is 1 m s^{-1}. Explain the relative magnitudes of the spectral densities of $\ddot{z}_s$ and $p\ddot{\theta}_s$ in the range 0 Hz to 5 Hz and in the range 5 Hz to 25 Hz. Compare the responses in Fig. 7 to the responses at the higher speed shown in Fig. 6. [35%]

Note: You do not need to perform detailed calculations for parts (b) and (c).

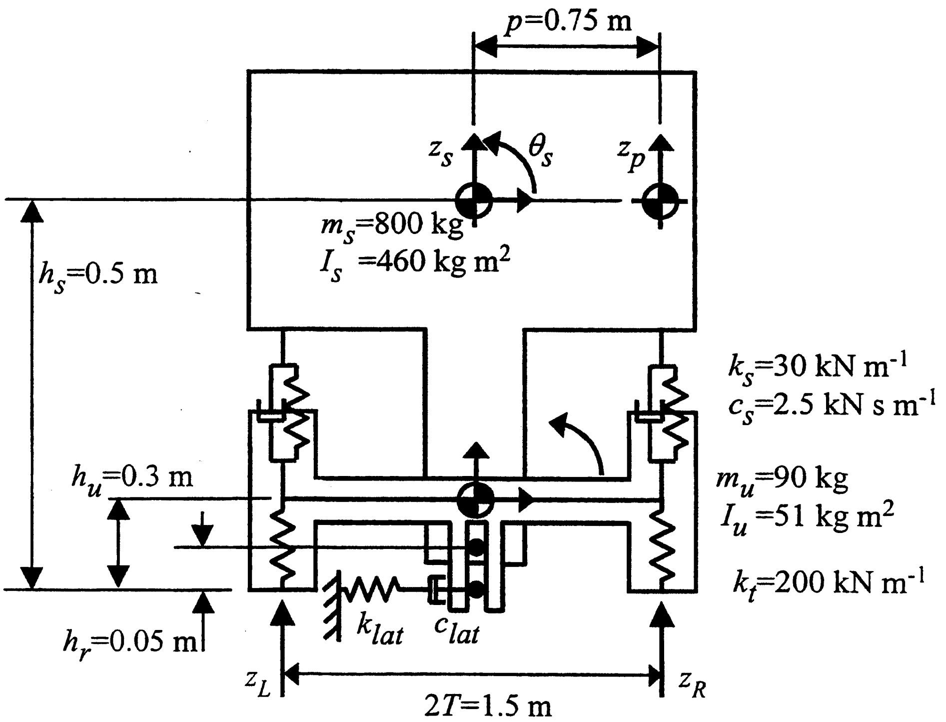

Fig. 5

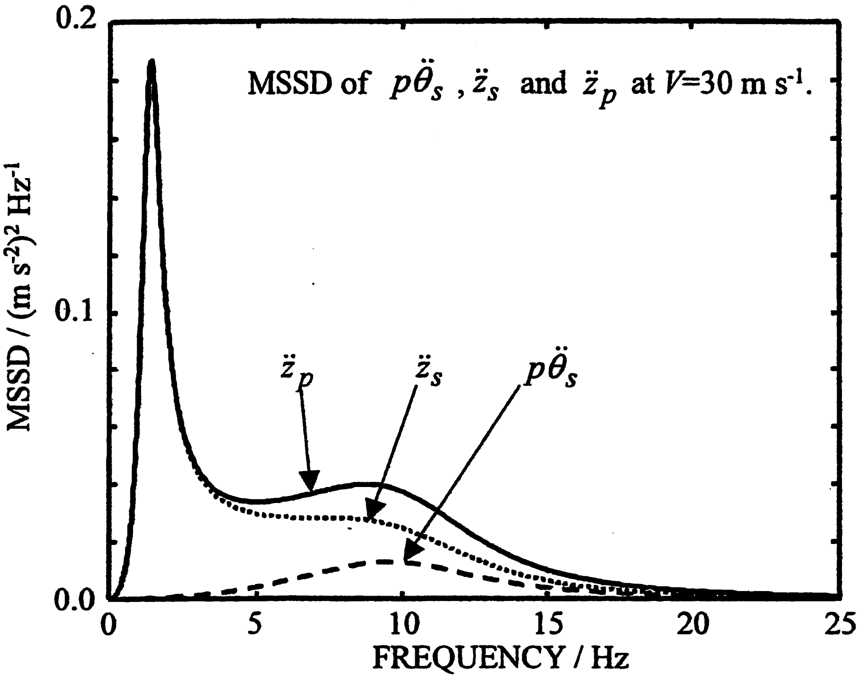

Fig. 6

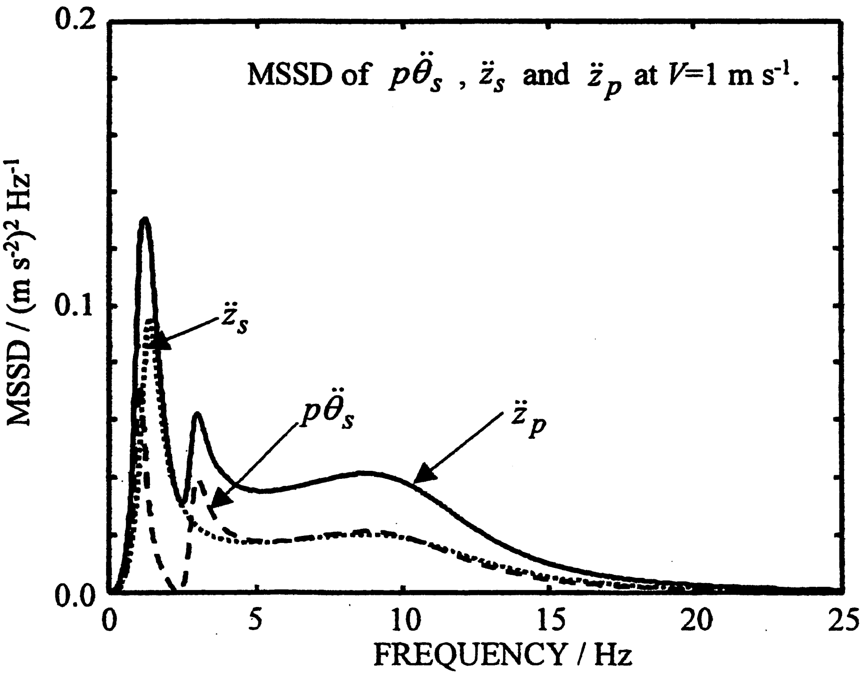

Fig. 7

END OF PAPER

Engineering Tripos Part IIB

Data sheet for Module 4C8: Applications of Dynamics

DATA ON VEHICLE VIBRATION

Random Vibration

$$\mathrm{E}\left[x(t)^2\right] = \frac{1}{T}\int_{t=0}^{t=T} x^2(t)\,dt = \int_{\omega=-\infty}^{\omega=\infty} S_x(\omega)\,d\omega \qquad \text{(or } \int_{\omega=0}^{\omega=\infty} S_x(\omega)\,d\omega \quad \text{if } S_x(\omega) \text{ is single sided)}$$

$$S_{\dot{x}}(\omega) = \omega^2 S_x(\omega)$$

Single Input – Single Output

$$S_y(\omega) = \left|H_{yx}(\omega)\right|^2 S_x(\omega)$$

$$y(\omega) = H_{yx}(\omega)x(\omega)$$

Two Input – Two Output

$$\begin{Bmatrix} y_1(\omega) \\ y_2(\omega) \end{Bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix} \begin{Bmatrix} x_1(\omega) \\ x_2(\omega) \end{Bmatrix}$$

$$\begin{bmatrix} S_{11}^y(\omega) & S_{12}^y(\omega) \\ S_{21}^y(\omega) & S_{22}^y(\omega) \end{bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix}^* \begin{bmatrix} S_{11}^x(\omega) & S_{12}^x(\omega) \\ S_{21}^x(\omega) & S_{22}^x(\omega) \end{bmatrix} \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix}^T$$

* means complex conjugate, T means transpose

If x_1 and x_2 are uncorrelated:

$$S_{(x_1+x_2)}(\omega) = S_{x_1}(\omega) + S_{x_2}(\omega)$$

$$S_{12}^x(\omega) = S_{21}^x(\omega) = 0$$

$$\mathrm{E}\left[(x_1(t) + x_2(t))^2\right] = \mathrm{E}\left[x_1(t)^2\right] + \mathrm{E}\left[x_2(t)^2\right]$$

DATA ON VEHICLE DYNAMICS

1. Creep Forces In Rolling Contact

1.1 Surface tractors

Longitudinal force $$X = \iint_A \sigma_x \, dA$$

Lateral force $$Y = \iint_A \sigma_y \, dA$$

Realigning Moment $$N = \iint_A \left(x\,\sigma_y - y\,\sigma_x\right) dA$$

where

σ_x, σ_y = longitudinal, lateral surface tractions

x, y = coordinates along, across contact patch

A = area of contact patch

1.2 Brush model

$\sigma_x = K_x q_x$, $\sigma_y = K_y q_y$ for $\sqrt{\sigma_x^2 + \sigma_y^2} \le \mu p$

where

q_x, q_y = longitudinal, lateral displacements of 'bristles' relative to wheel rim

K_x, K_y = longitudinal, lateral stiffness per unit area

μ = coefficient of friction

p = local contact pressure

1.3 Linear creep equations

$$X = -C_{11}\xi$$

$$Y = -C_{22}\alpha - C_{23}\psi$$

$$N = C_{32}\alpha - C_{33}\psi$$

where X, Y, N, are defined as in 1.1 above.

C_{ij} = coefficients of linear creep

ξ = longitudinal creep ratio = longitudinal creep speed/forward speed

α = lateral creep ratio = (lateral speed /forward speed) - steer angle

ψ = spin creep ratio = spin angular velocity/forward speed

2. Plane Motion in a Moving Coordinate Frame

$$\ddot{\mathbf{R}}_{O_1} = (\dot{u} - v\Omega)\mathbf{i} + (\dot{v} + u\Omega)\mathbf{j}$$

(i, j, k) axis system fixed to body at point O_1

where

u = speed of point O_1 in **i** direction

v = speed of point O_1 in **j** direction

$\Omega\mathbf{k}$ = absolute angular velocity of body

3. Routh-Hurwitz stability criteria

$$\left(a_2\frac{d^2}{dt^2} + a_1\frac{d}{dt} + a_0\right)y = x(t)$$

Stable if all $a_i > 0$

$$\left(a_3\frac{d^3}{dt^3} + a_2\frac{d^2}{dt^2} + a_1\frac{d}{dt} + a_0\right)y = x(t)$$

Stable if (i) all $a_i > 0$
and also (ii) $a_1a_2 > a_0a_3$

$$\left(a_4\frac{d^4}{dt^4} + a_3\frac{d^3}{dt^3} + a_2\frac{d^2}{dt^2} + a_1\frac{d}{dt} + a_0\right)y = x(t)$$

Stable if (i) all $a_i > 0$
and also (ii) $a_1a_2a_3 > a_0a_3^2 + a_4a_1^2$