ENGINEERING TRIPOS PART IIB

Monday 25 April 2005 2:30 to 4

Module 4C15

MEMS DESIGN

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The **approximate** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. A top view of a resonator is shown in Fig. 1. The device is implemented in a polysilicon surface micromachining process and the thickness of the structural layer is 3μm. The length of the clamped-clamped beam portion of the resonator is 150μm and the width of the beam is 1.5μm. A single electrode is attached to the beam and a potential difference is applied between a fixed electrode spaced at a gap of 1μm from the movable electrode as shown. The overlap length of the electrodes is 50μm and the width of the movable electrode is 1.5μm.

(a) Spring constant and Damping estimates.

(i) Estimate the spring constant for static motion. The spring constant for lateral motion of a clamped-clamped beam structure is given by:

$$k = \frac{\pi^4}{6} EH \left(\frac{W}{L}\right)^3$$

where E is Young's Modulus, H is the out-of-plane thickness, and W and L are the width and length of the beam respectively. [15%]

(ii) Estimate the damping for the resonant structure along the driven mode assuming operation in air. The damping constant is calculated based on the assumption that squeeze film damping dominates. The damping constant is given as:

$$b = \frac{96 \eta L W^3}{\pi^4 g^3}$$

where η is the viscosity of air (value: 1.8×10^{-5} kg/m.s at room temperature and atmospheric pressure), L and W are the length and width of the plates respectively and g is the gap between the plates. [15%]

(b) Estimate the undamped resonant frequency of the structure for operation in the first mode. Use the Rayleigh-Ritz method to estimate the primary resonant frequency for in-plane motion. A trial function $\zeta(x)$ for the mode shape can be written as:

$$\zeta(x) = \frac{c}{2}\left(1 + \cos\left(\frac{2\pi x}{L}\right)\right)$$

where L is the length of the beam and the origin of the coordinate x is taken to be at the midpoint of the beam as shown in Fig. 1.

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The kinetic energy KE and potential energy PE densities are given by:

$$KE = \frac{\omega^2}{2}\rho_m \zeta^2(x)$$

$$PE = \frac{E}{2}\left(y\frac{d^2\zeta}{dx^2}\right)^2$$

where ρ_m is the mass density of an element of the beam, y is the distance of the element from the neutral axis, E is the Young Modulus of the material and ω is the resonant frequency. [30%]

(c) Construct a lumped-element equivalent electrical circuit model for this system and hence derive the transfer function relating displacement-to-force by the application of Kirchhoff's voltage and current laws to the circuit. [20%]

(d) Estimate the deflection of the proof mass for the application of a potential difference between the fixed electrode and the movable structure with an AC component at the primary resonant frequency of the structure with amplitude 50mV and a DC component of 5V.

[20%]

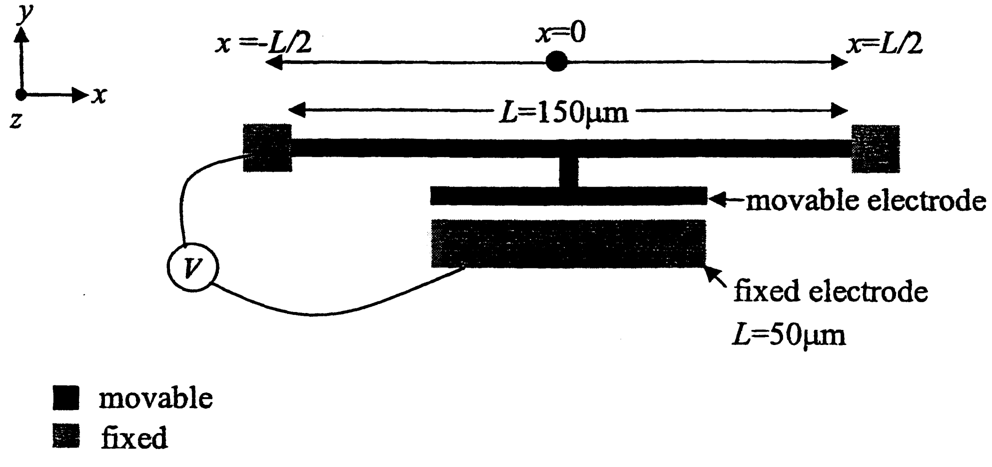

Fig. 1: Top view of the resonator drawn as a clamped-clamped beam. V is the applied potential between the fixed and movable electrodes.

2. A membrane pressure sensor is fabricated in a silicon micromachining process. A top view and cross-section is shown in Fig. 2. The dimensions of the square diaphragm are 1000μm x 1000μm with a thickness of 5μm. Capacitive detection is employed to monitor the deflection of the membrane in response to an applied external pressure. A fixed electrode is placed at a gap of 5μm from the movable plate. The overlap area of the fixed electrode is 20μm x 20μm and is placed symmetrically with respect to the movable membrane as shown. The membrane is modeled as a plate, the deflection *w(x,y)* of which in response to an external pressure load P is given by:

$$w(x,y)=\frac{c_1}{4}\left[1+\cos\left(\frac{2\pi x}{L}\right)\right]\left[1+\cos\left(\frac{2\pi y}{L}\right)\right]$$

$$\text{where } c_1=\frac{6P(1-\nu^2)L^4}{\pi^4 EH^3}$$

and P is the applied pressure load on the membrane, L is the diaphragm size, ν is Poisson ratio, E is Young's Modulus for the membrane material and H is the thickness of the membrane. The Young's Modulus for silicon is 160 GPa and the Poisson ratio is 0.27.

(a) Calculate the capacitance between the fixed electrode and the movable membrane as a function of applied pressure load, stating any assumptions made. Note that the dimensions of the electrode are much smaller than the dimensions of the membrane. [40%]

(b) Estimate the scale factor or sensitivity of the device to applied pressure load. The scale factor is defined as the ratio of the fractional change in capacitance to the fractional change in pressure. Starting from a relaxed position, calculate the fractional change in capacitance for an applied pressure load of 500Pa. [40%]

(c) Estimate the damping constant for membrane motion assuming that the membrane cavity is backfilled with nitrogen at atmospheric pressure. The viscosity of nitrogen at atmospheric pressure and room temperature is 1.8×10^{-5} kg/m.s. Squeeze-film damping can be assumed with the damping constant given by:

$$b=\frac{96\eta L^4}{\pi^4 g^3}$$

where η is the viscosity of the medium between the plates, L is the diaphragm size and g is the gap between the plates. [20%]

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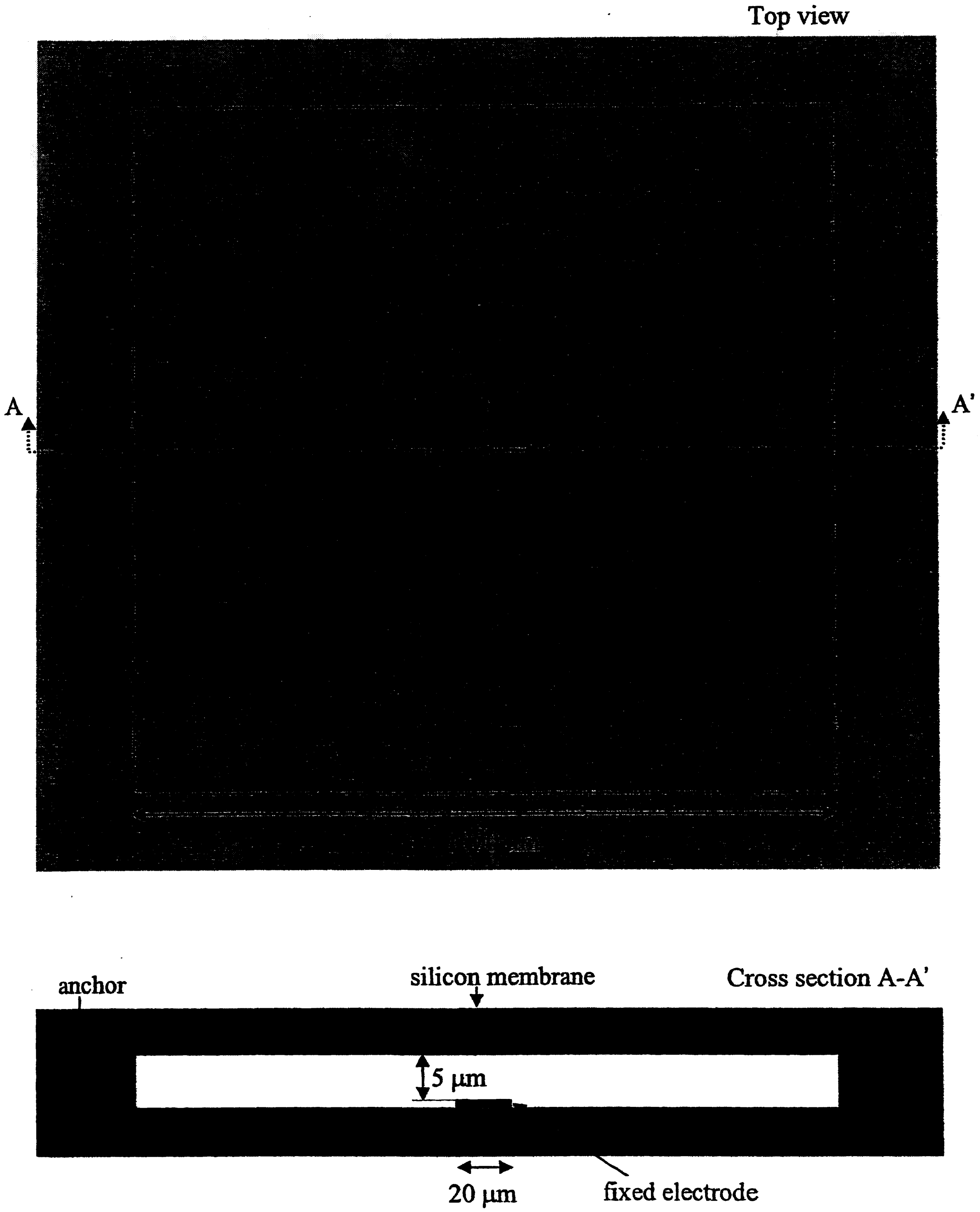

Fig. 2: Top view and cross-section of a micromachined pressure sensor (not drawn to scale). Note that the dimensions of the membrane are much larger than the dimensions of the electrode.

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3. A biomolecular separation device is constructed on a glass substrate. The critical part of the device is a separation column where differentially charged species separate via electrophoresis. The separation column is created as a microfluidic channel etched into the glass substrate with fluidic ports and electrodes suitably patterned. A top view of the device is shown in Fig. 3. The width of the channel is $30\mu m$, the depth of the channel is $50\mu m$ and the length of the channel is $10cm$. An electrical double layer is formed at the channel surface with an associated Debye length of $0.5nm$. The channel is filled with an electrolyte buffer solution. The viscosity of the solution is found to be $1.5x10^{-3}kg/m.s$ and the density of the solution is approximately $1200kg/m^3$.

(a) The simplified form of the Navier Stokes equation for plug flow is valid for electroosmosis:

$$\frac{d^2U}{dx^2} = \frac{\sigma_w E_x}{\eta L_D} e^{\frac{-z}{L_D}}$$

σ_w is the channel surface charge per unit area, η is the viscosity of the solution, L_D is the Debye length, E_x is the applied electric field along the length of the channel and z is the distance away from the surface along the width of the channel. Using the no-slip condition, show that the flow velocity U along the direction can be written in the form:

$$U = U_0\left(1 - e^{\frac{-z}{L_D}}\right)$$

Write down an expression for U_0 as a function of σ_w, η, L_D and E_x. [20%]

(b) The glass wall potential ϕ_w is found to be equal to $80mV$. The channel surface charge density σ_w is related to the glass wall potential as follows:

$$\sigma_w = \frac{\phi_w \varepsilon}{L_D}$$

Calculate the channel surface charge density. You may assume that the dielectric constant for the solution is 80. [20%]

(c) Estimate the volumetric flow rate for an applied electric field of $500V/cm$ between ports 1 and 2. Calculate the time required for a plug of solution to traverse a distance of $1cm$ under these conditions. [20%]

(d) Two differentially charged biomolecular species of roughly equal mass are required to be separated in this flow column. The two biomolecular species differ by a charge equivalent to 4 electrons. The electrophoretic mobility μ_{ep} is related to the charge on the molecule Q and the viscosity η of the solution as:

$$\mu_{ep} = \frac{2\times10^5 Q}{\eta}$$

Calculate the time required for the two species to separate through a distance of $10\mu\text{m}$. What is the distance traversed by the bulk solution in this time? [20%]

(e) If the diffusion constant for the biomolecular species is roughly equal to $10\mu\text{m}^2/\text{s}$, estimate the width of each band comprising the two species after they have separated through a distance of $10\mu\text{m}$. Comment on the device parameters required to be optimised to obtained the sharpest separated bands for a given biomolecular species. [20%]

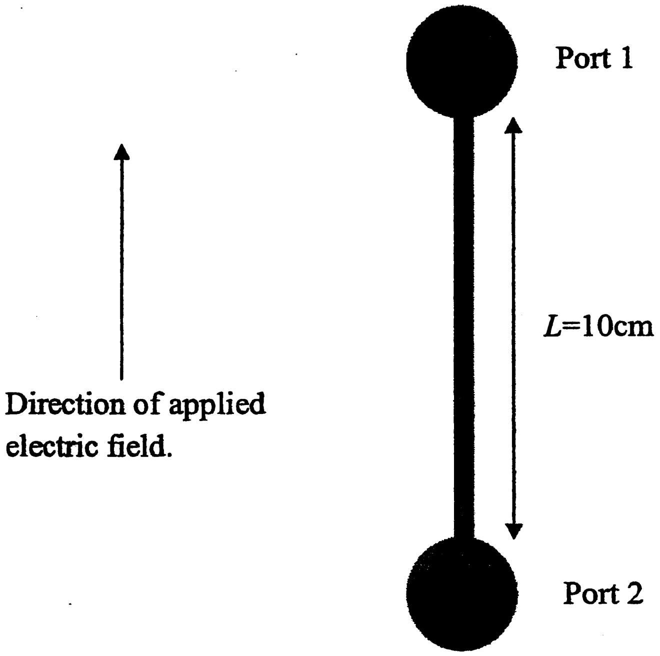

Fig. 3: Top view of a biomolecular separation device (not drawn to scale). The device comprises of an input and output fluidic port and a separation column. Continuous flow is assumed.

4. A symmetrical wedge shaped actuator is designed for an application in an electrical switch. A cross-section of the device is shown in Fig. 4. The movable plate is supported by springs and is in the shape shown. The plate is driven by an electrostatic force when a voltage is applied between an electrode attached to the substrate and the moving plate. The minimum nominal gap between the two electrodes is g and the fixed electrode is positioned to fully overlap with the suspended plate. The fixed electrode has a width w into the page and length $2L$ as shown. Assume that the electric field in the gap is uniform throughout and neglect fringing field effects.

(a) Find an expression for the displacement of the movable plate as a function of voltage difference V between the electrodes. [40%]

(b) The actuator has an associated pull-in instability. Derive an expression for the critical actuator displacement at which pull-in occurs as a function of θ, L and g. Show that the expression reduces to that for the parallel plate case as $\theta \to 0$. [50%]

(c) Comment on why the geometry shown below may be more suitable for an application in an electrical switch compared to the parallel plate actuator. [10%]

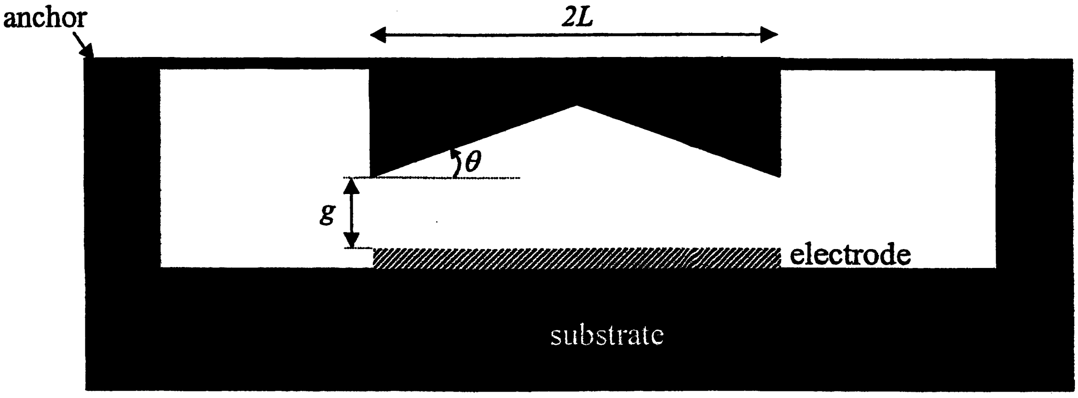

Fig. 4

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