ENGINEERING TRIPOS PART IIB

Thursday 5 May 2005 2.30 to 4pm

Module 4D2

LIGHTWEIGHT STRUCTURES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

(TURN OVER

1 A long uniform right-circular cylinder of thickness t, radius a, has open ends.

(a) Write down the governing equation of radial deformation, making clear the meaning of extra symbols you introduce. [10%]

(b) A moment stress resultant per unit length, M, and a shear force stress resultant per unit length, Q, are applied at one end, both uniform around the whole circumference and remote from the other end. Determine the radial deflection and the rotation in the longitudinal direction at the loaded end when subjected to M and Q. [60%]

(c) The cylinder is now heated, such that its radius uniformly expands by $a\lambda$, where $\lambda << 1$. One end of the cylinder is then rigidly connected to, and flush with, a rigid foundation. After the composite system is restored to ambient temperature conditions, determine the bending moment at the connected end, and the distance from this end at which the radial expansion *first* becomes zero. [30%]

2 A thin square plate, of side-length a, undergoes a small transverse deflection mode

$$w = \frac{B}{2}\left[x^2 - \nu y^2\right] + Cxy$$

The coordinate system, Oxy, is located in the centre of the plate with the x- and y axes parallel to adjacent sides. The Poisson's ratio is ν. By considering the stress resultants within the plate, determine the applied loading, and the physical meaning of the constants B and C. [100%]

3 A shell of revolution is formed by rotating about the downwards z-axis, a meridian generated by the parabola, $z = Ar^2$. The shell is subjected to an internal pressure p.

(a) Describe, without calculation, the nature of the Membrane Hypothesis on thin shell behaviour. [10%]

(b) In view of this hypothesis, determine the stress resultants, N_θ and N_ϕ, for the pressurised shell, where θ and ϕ are angular coordinates in the circumferential and meridional directions, respectively. Confirm that, for very small values of ϕ, N_θ and N_ϕ are approximately equal to each other. [70%]

(c) What is the nature of the relationship between N_θ and the rate of change of area enclosed by the meridian, $z = Ar^2$, the normal to it, and the z-axis? Perform a check for very small values of ϕ. [20%]

4 A circular plate of uniform thickness t, is clamped at its outer edge, $r = a$. A transverse pressure, p, is applied everywhere to the surface, and varies as

$$p_0 \left[1 - \left(\frac{r}{\beta a}\right)\right]$$

where p_0 is amplitude in the centre of plate and β is, as yet, an unknown constant.

(a) Write down the governing equation of transverse deformation, making clear the meaning of extra symbols you introduce. [10%]

(b) Determine an expression for the axi-symmetrical, transverse deflection, $w(r)$, under the specified pressure distribution. [60%]

(c) Compute the value of β which gives zero deflection in the centre of plate. [30%]

END OF PAPER