UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

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Tuesday 10 May 2005 2.30 to 4.00

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Module 4D5

FOUNDATION ENGINEERING

*Answer not more than* ***three*** *questions*

*All questions carry the same number of marks*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*Attachments:*

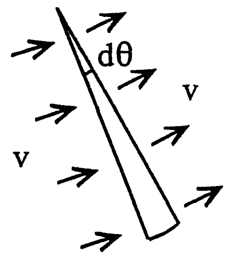
*i) Special datasheet (14 pages).*

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

(TURN OVER

1 (a) Show that the rate of work dissipation in the element of plane strain circular shear fan (radius r) shown below is:

$$\dot{D} = 2 s_u r v d\theta$$




Tresca material, $s_u$

[20%]

(b) The upper bound failure mechanism of a plane strain surface foundation (width B) on uniform undrained clay (treat as a Tresca material) under combined vertical and horizontal (V-H) loading is shown below. The mechanism comprises two rigid zones and a shear fan. Slip at the foundation-soil interface can be ignored.

Show that the ratio of work dissipated in this mechanism to work input is:

$$\frac{\text{Work dissipated}}{\text{Work input}} = \frac{s_u B(\cot\theta + 1 + \pi/2 + 2\theta)}{H\cot\theta + V}$$

[50%]

(c) Calculate the load inclination, $\alpha = \tan^{-1}(H/V)$, that would cause failure by the mechanism shown, with $\theta = 30°$.

[30%]

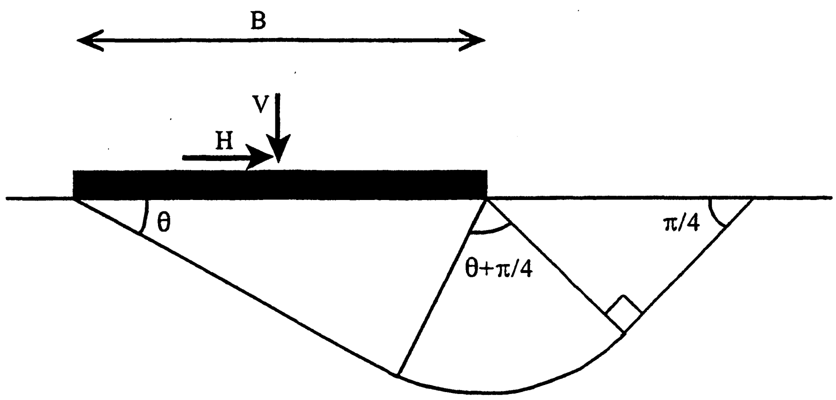



Fig. 1. Strip footing upper bound mechanism under V-H loading.

2 A proposed building is shown as shaded below. The foundation will be an impermeable raft at 1 m below the ground surface, exerting a net bearing pressure of 50 kPa (allowing for the weight of the excavated soil). The ground comprises over-consolidated clay with effective unit weight, $\gamma' = 10$ kN/m$^3$. The one-dimensional compressibility of the clay is given by:

$$v = 1.5 - \kappa \ln \sigma'_v$$

where v is the specific volume, $\sigma'_v$ is the vertical effective stress, and the compressibility, $\kappa$, is equal to 0.02.

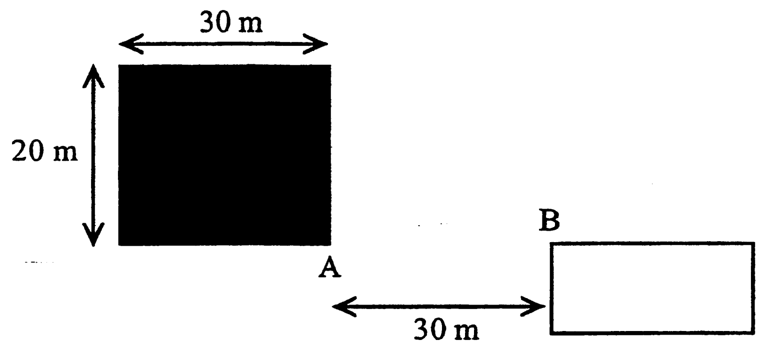



Fig. 2. Proposed building.

(a) Estimate the drained settlement at the corner of the new building (A) and at the nearest point of the adjacent building (B), by dividing the subsoil into three layers.

[70%]

(b) Discuss why the actual settlement at A and B may differ from these estimated values.

[30%]

3 An offshore jacket structure is supported by 2 m diameter steel tubular piles driven to a depth of 20 m in normally consolidated clay. The strength profile is $s_u = kz$, where z is the depth below the mudline and the strength gradient, k= 1.5 kPa/m. The piles are restrained at the head and are sufficiently far apart for interaction to be ignored.

(a) Derive an expression for the horizontal capacity of a single pile, ignoring the possibility of bending failure.

[25%]

(b) Derive an expression for the horizontal capacity of a single pile, if failure is by bending.

[50%]

(c) Calculate the required wall thickness to prevent bending failure. Assume a steel yield stress of 200 MPa.

[25%]

4 The drag-embedded vertically loaded plate anchor (VLA) shown below is 4 m square in plan, and is installed in a horizontal position at a depth of 20 m in uniform clay with shear strength 50 kPa and effective unit weight 5 $kN/m^3$. The harness, when pulled by the mooring line, has the dimensions (in the vertical plane) shown below.

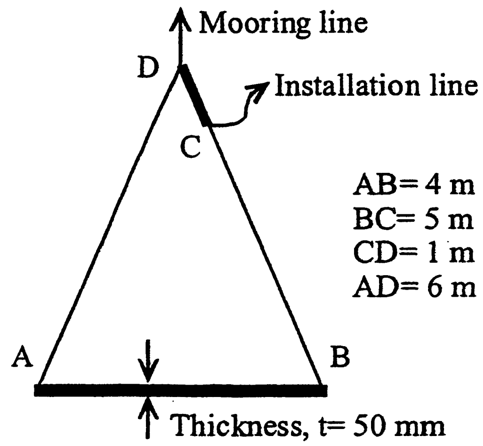



Fig. 3. Vertically loaded plate anchor (VLA).

(a) Estimate the anchor efficiency, defined as the ratio of mooring capacity to installation force. Assume that installation and loading are fast compared to consolidation, and that the installation force acts through the centroid of the plate.

[70%]

(b) How might the anchor efficiency be changed if a crack extending from the back of the anchor to the seabed remained open after installation?

[30%]

**END OF PAPER**

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# Cambridge University Engineering Department

# Supplementary Databook

# Module 4D5: Foundation Engineering

DJW. February 2005

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## Section 1: Plasticity theory

This section is common with the Soil Mechanics Databook supporting modules 3D1 and 3D2. Undrained shear strength ('cohesion' in a Tresca material) is denoted by $s_u$ rather than $c_u$.

**Plasticity: Tresca material, $\tau_{max} = s_u$**

**Limiting stresses**

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2s_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3}q_u^2 = 2s_u^2$

$q_u$= undrained triaxial compression strength; $s_u$= undrained plane shear strength.

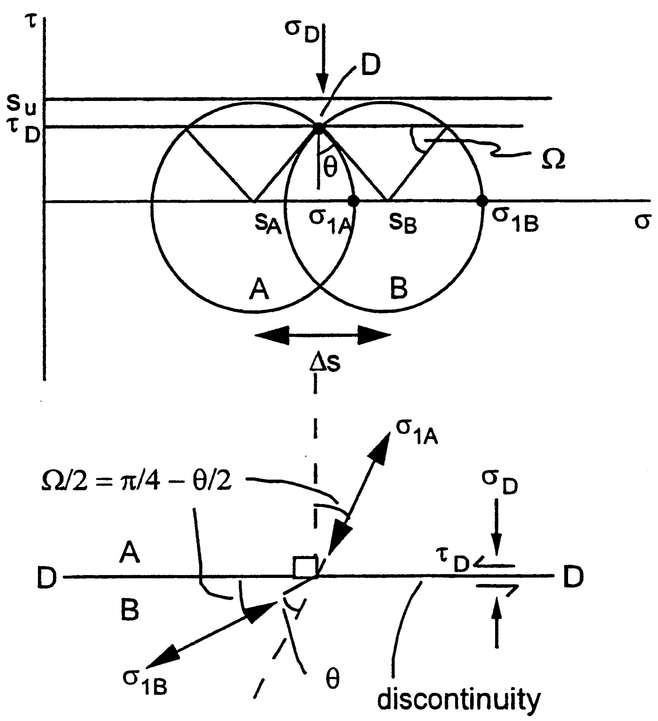
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = s_u \delta\varepsilon_\gamma$$

For a relative displacement $x$ across a slip surface of area $A$ mobilising shear strength $s_u$, this becomes

$$D = A s_u x$$

**Stress conditions across a discontinuity:**




Rotation of major principal stress

$$\theta = \pi/2 - \Omega$$

$$s_B - s_A = \Delta s = 2s_u \sin\theta$$
$$\sigma_{1B} - \sigma_{1A} = 2s_u \sin\theta$$

In limit with $\theta \to 0$

$$ds = 2s_u \, d\theta$$

Useful example:

$$\theta = 30°$$

$$\sigma_{1B} - \sigma_{1A} = s_u$$

$$\tau_D / s_u = 0.87$$

$\sigma_{1A}$ = major principal stress in zone A

$\sigma_{1B}$ = major principal stress in zone B

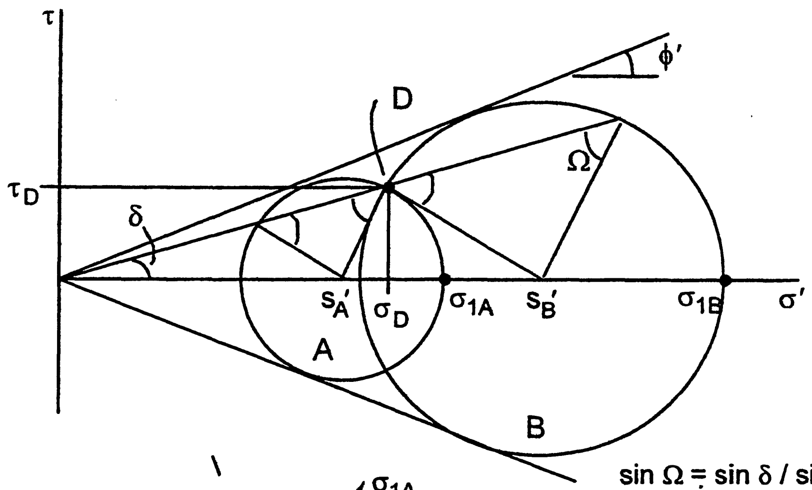
**Plasticity: Coulomb material $(\tau/\sigma')_{max} = \tan\phi$**

**Limiting stresses**

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u)$$

where $\sigma'_{1f}$ and $\sigma'_{3f}$ are the major and minor principal effective stresses at failure, $\sigma_{1f}$ and $\sigma_{3f}$ are the major and minor principal total stresses at failure, and u is the pore pressure.

**Stress conditions across a discontinuity**



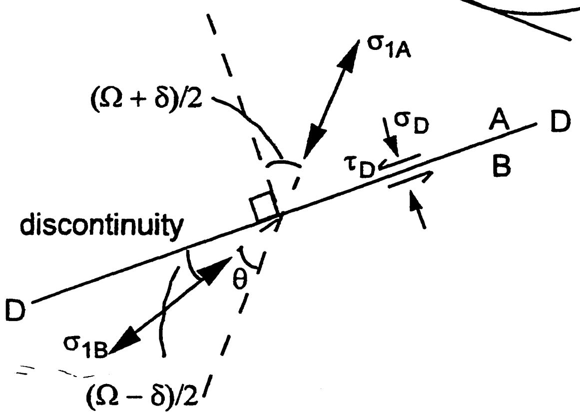

Rotation of major principal stress

$\theta = \pi/2 - \psi$

$\sigma_{1A}$ = major principal stress in zone A

$\sigma_{1B}$ = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$




$\sin\Omega = \sin\delta / \sin\phi'$

$s'_B/s'_A = \sin(\Omega+\delta) / \sin(\Omega - \delta)$

In limit, $\delta\theta \rightarrow 0$ and $\delta \rightarrow \phi'$

$ds' = 2s'.\ \delta\theta \tan\phi'$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi')$

# Section 2: Bearing capacity of shallow foundations

### 2.1 Tresca soil, with undrained strength $s_u$.

**Vertical loading**

The vertical bearing capacity, $q_f$, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

$V_{ult}$ and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and $\gamma$ (or $\gamma'$) is the appropriate density of the overburden.

The exact bearing capacity factor $N_c$ for a plane strain surface foundation (zero embedment) on uniform soil is

$N_c = 2 + \pi$ (Prandtl, 1921)

***Shape correction factor:***
For a rectangular footing of length L and breadth B (Eurocode 7):

$s_c = 1 + 0.2\, B / L$

The exact solution for a rough circular foundation (B/L=1) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 0.2$.

***Embedment correction factor:***
A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is

$d_c = 1 + 0.33 \tan^{-1}(h/D)$ (or h/B for a strip or rectangular foundation)

**Combined V-H loading**

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}}$ or $\frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$

If $V/V_{ult} < 0.5$: $H = H_{ult} = B s_u$

**Combined V-H-M loading**

With lift-off: combined Green-Meyerhof

Without lift-off: $\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0$ (Taiebet & Carter 2000)

## 2.2 Frictional (Coulomb) soil, with friction angle $\phi$.

### Vertical loading

The vertical bearing capacity, $q_f$, of a shallow foundation under drained loading is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + N_\gamma \frac{\gamma' B}{2}$$

The bearing capacity factors $N_q$ and $N_\gamma$ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. $\sigma'_{v0}$ is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for $N_q$ is:

$$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)} \qquad \text{(Prandtl 1921)}$$

An empirical relationship to estimate $N_\gamma$ from $N_q$ is (Eurocode 7):

$$N_\gamma = 2\,(N_q - 1)\tan\phi$$

Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054 e^{9.6\phi}$

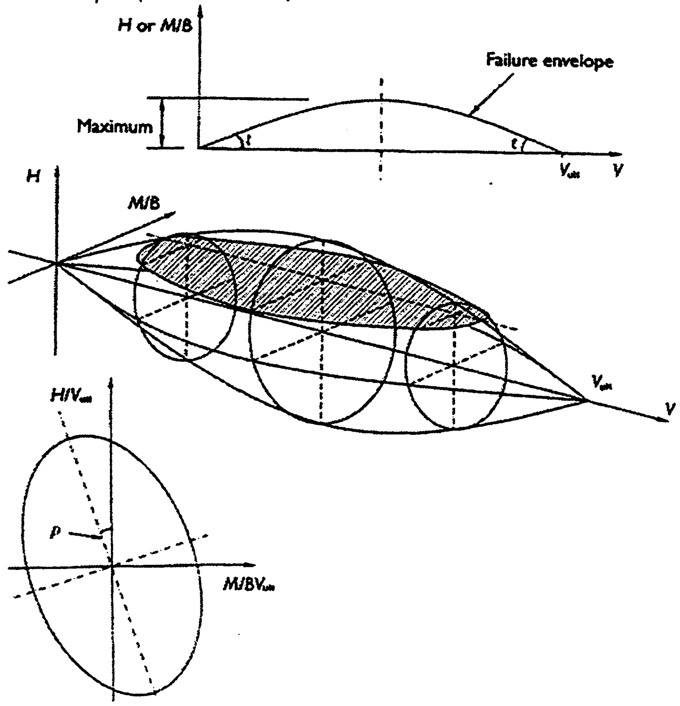
Smooth base: $N_\gamma = 0.0663 e^{9.3\phi}$

***Shape correction factors:***

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B \sin\phi)/L$$
$$s_\gamma = 1 - 0.3\,B/L$$

For circular footings assume L = B.




### Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

**Combined V-H-M loading** (with lift-off- drained conditions- see failure surface shown above)

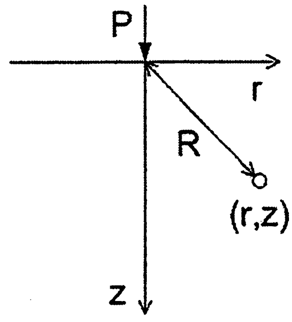
$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi 1994)

## Section 3: Settlement of shallow foundations

### 3.1 Elastic stress distributions below point, strip and circular loads

**Point loading (Boussinesq solution)**



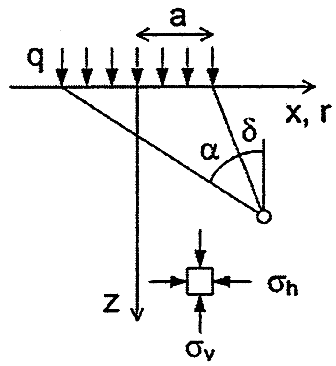

Vertical stress $\sigma_z = \dfrac{3Pz^3}{2\pi R^5}$

Radial stress $\sigma_r = \dfrac{P}{2\pi R^2}\left[\dfrac{3r^2 z}{R^3} - \dfrac{(1-2\nu)R}{R+z}\right]$

Tangential stress $\sigma_\theta = \dfrac{P(1-2\nu)}{2\pi R^2}\left[\dfrac{R}{R+z} - \dfrac{z}{R}\right]$

Shear stress $\tau_{rz} = \dfrac{3Prz^2}{2\pi R^5}$

**Uniformly-loaded strip**




Vertical stress $\sigma_v = \dfrac{q}{\pi}[\alpha + \sin\alpha\cos(\alpha + 2\delta)]$

Horizontal stress $\sigma_h = \dfrac{q}{\pi}[\alpha - \sin\alpha\cos(\alpha + 2\delta)]$

Shear stress $\tau_{vh} = \dfrac{q}{\pi}\sin\alpha\sin(\alpha + 2\delta)$

Principal stresses

$$\sigma_1 = \frac{q}{\pi}(\alpha + \sin\alpha) \qquad \sigma_3 = \frac{q}{\pi}(\alpha - \sin\alpha)$$

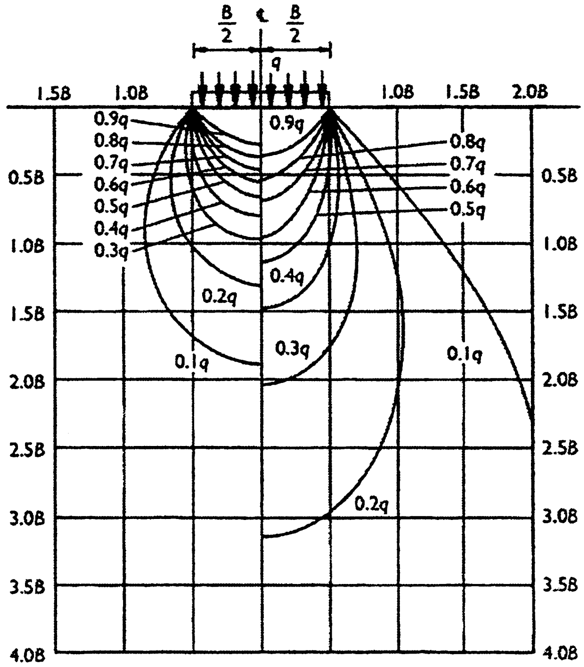
**Uniformly-loaded circle**
**(on centerline, r=0)**

Vertical stress

$$\sigma_v = q\left[1 - \left(\frac{1}{1+(a/z)^2}\right)^{\frac{3}{2}}\right]$$

Horizontal stress

$$\sigma_h = \frac{q}{2}\left[(1+2\nu) - \frac{2(1+\nu)z}{(a^2+z^2)^{1/2}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$




Contours of vertical stress below uniformly-loaded strip and circular areas

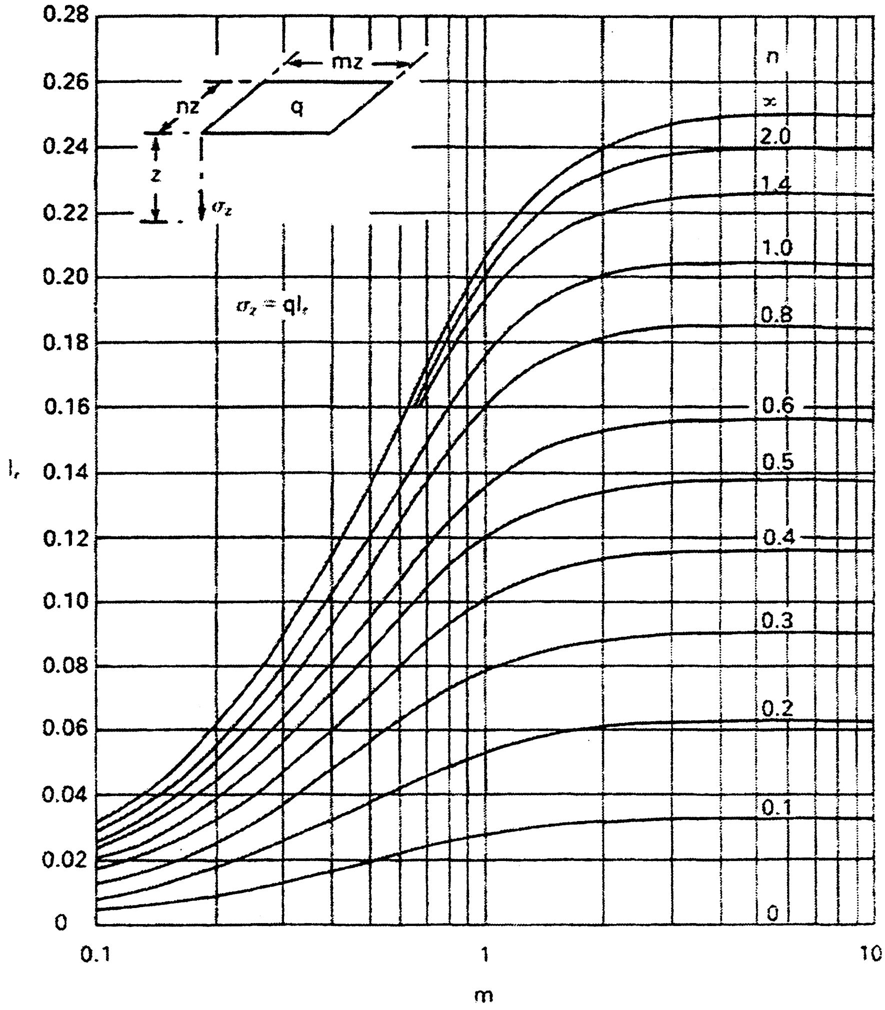
### 3.2 Elastic stress distribution below rectangular area

The vertical stress, $\sigma_z$, below the corner of a uniformly-loaded rectangle (L × B) is:

$$\sigma_z = I_r q$$

$I_r$ is found from m (=L/z) and n (=B/z) using Fadum's chart or the expression below (L and B are interchangeable), which are from integration of Boussinesq's solution.

$$I_r = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right)+\tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$




Influence factor, $I_r$, for vertical stress under the corner of a uniformly-loaded rectangular area (Fadum's chart)

## 3.3 Elastic solutions for surface settlement

**Point load (Boussinesq solution)**

Settlement, w, at distance s: $w(s) = \frac{1}{2\pi}\frac{(1-\nu)}{G}\frac{P}{s}$

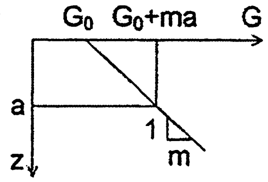
**Circular area (radius a), uniform soil**

Uniform load: central settlement: $w_o = \frac{(1-\nu)}{G} qa$

edge settlement: $w_e = \frac{2}{\pi}\frac{(1-\nu)}{G} qa$

Rigid punch: ($q_{avg} = V/\pi a^2$) $w_r = \frac{\pi}{4}\frac{(1-\nu)}{G} q_{avg} a$

**Circular area, heterogeneous soil**



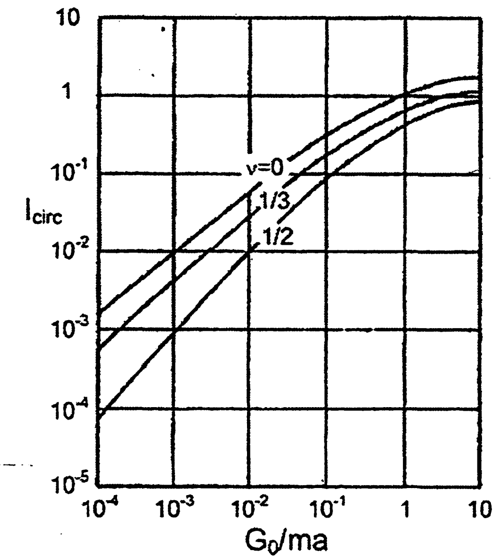

For $G_0 = 0$, $\nu = 0.5$:

w= q/2m under loaded area of any shape
w= 0 outside loaded area

For $G_0 > 0$, central settlement:

$$w_o = \frac{qa}{2G_0} I_{circ}$$

For $\nu = 0.5$, $w_o \approx \frac{qa}{2(G_0 + ma)}$




**Rectangular area, uniform soil**

Uniform load, corner settlement:

$$w_c = \frac{(1-\nu)}{G}\frac{qB}{2} I_{rect}$$

Where $I_{rect}$ depends on the aspect ratio, L/B:

| L/B | $I_{rect}$ | L/B | $I_{rect}$ | L/B | $I_{rect}$ | L/B | $I_{rect}$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.561 | 1.6 | 0.698 | 2.4 | 0.822 | 5 | 1.052 |
| 1.1 | 0.588 | 1.7 | 0.716 | 2.5 | 0.835 | 6 | 1.110 |
| 1.2 | 0.613 | 1.8 | 0.734 | 3 | 0.892 | 7 | 1.159 |
| 1.3 | 0.636 | 1.9 | 0.750 | 3.5 | 0.940 | 8 | 1.201 |
| 1.4 | 0.658 | 2 | 0.766 | 4 | 0.982 | 9 | 1.239 |
| 1.5 | 0.679 | 2.2 | 0.795 | 4.5 | 1.019 | 10 | 1.272 |

Rigid rectangle: $w_r = \frac{(1-\nu)}{G}\frac{q_{avg}\sqrt{BL}}{2} I_{rgd}$ where $I_{rgd}$ varies from 0.9→0.7 for L/B = 1-10.

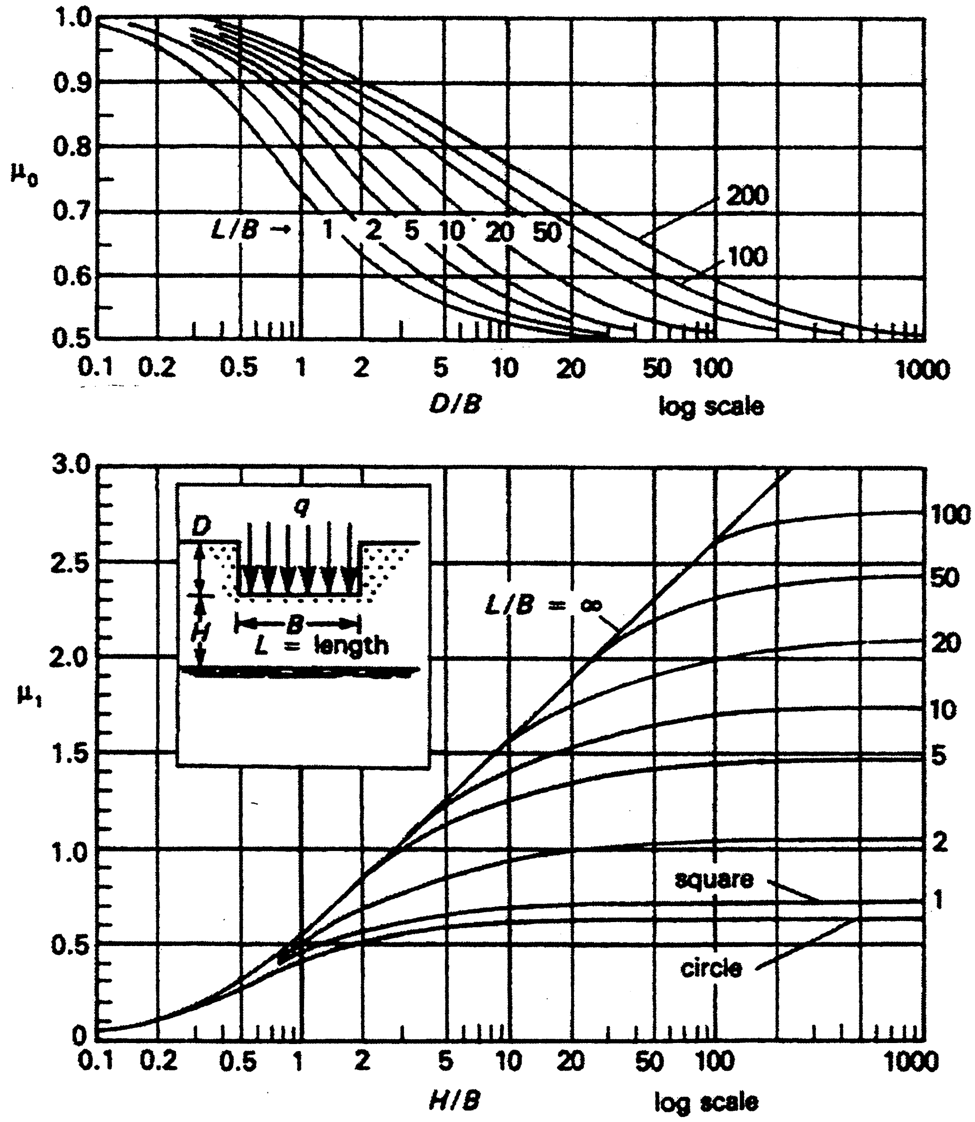
Note: $G = \frac{E}{2(1+\nu)}$ where ν= Poisson's ratio, E= Young's modulus.

**Elastic layer of finite thickness**

The mean settlement of a uniformly loaded foundation embedded in an elastic layer of finite thickness can be found using Janbu's charts, for $\nu \sim 0.5$.

$$w_{avg} = \mu_0 \mu_1 \frac{(1-\nu)}{G} \frac{qB}{2}$$

The influence factor $\mu_1$ accounts for the finite layer thickness. The influence factor $\mu_0$ accounts for the embedded depth.




Average immediate settlement of a uniformly loaded finite thickness layer

# Section 4: Bearing capacity of deep foundations

## 4.1 Axial capacity: API (2000) design method for driven piles

### Sand

***Unit shaft resistance:*** $\tau_{sf} = \sigma'_{hf} \tan\delta = K\sigma'_{v0} \tan\delta \le \tau_{s,lim}$

Closed-ended piles: $K = 1$
Open-ended piles: $K = 0.8$

***Unit base resistance:*** $q_b = N_q\, \sigma'_{vo} < q_{b,limit}$

| Soil category | Soil density | Soil type | Soil-pile friction angle, δ (°) | Limiting value $\tau_{s,lim}$ (kPa) | Bearing capacity factor, $N_q$ | Limiting value, $q_{b,lim}$ (MPa) |
|---|---|---|---|---|---|---|
| 1 | Very loose<br>Loose<br>Medium | Sand<br>Sand-silt<br>Silt | 15 | 50 | 8 | 1.9 |
| 2 | Loose<br>Medium<br>Dense | Sand<br>Sand-silt<br>Silt | 20 | 75 | 12 | 2.9 |
| 3 | Medium<br>Dense | Sand<br>Sand-silt | 25 | 85 | 20 | 4.8 |
| 4 | Dense<br>Very dense | Sand<br>Sand-silt | 30 | 100 | 40 | 9.6 |
| 5 | Dense<br>Very dense | Gravel<br>Sand | 35 | 115 | 50 | 12 |

API (2000) recommendations for driven pile capacity in sand

### Clay

American Petroleum Institute (API) (2000) guidelines for driven piles in clay.

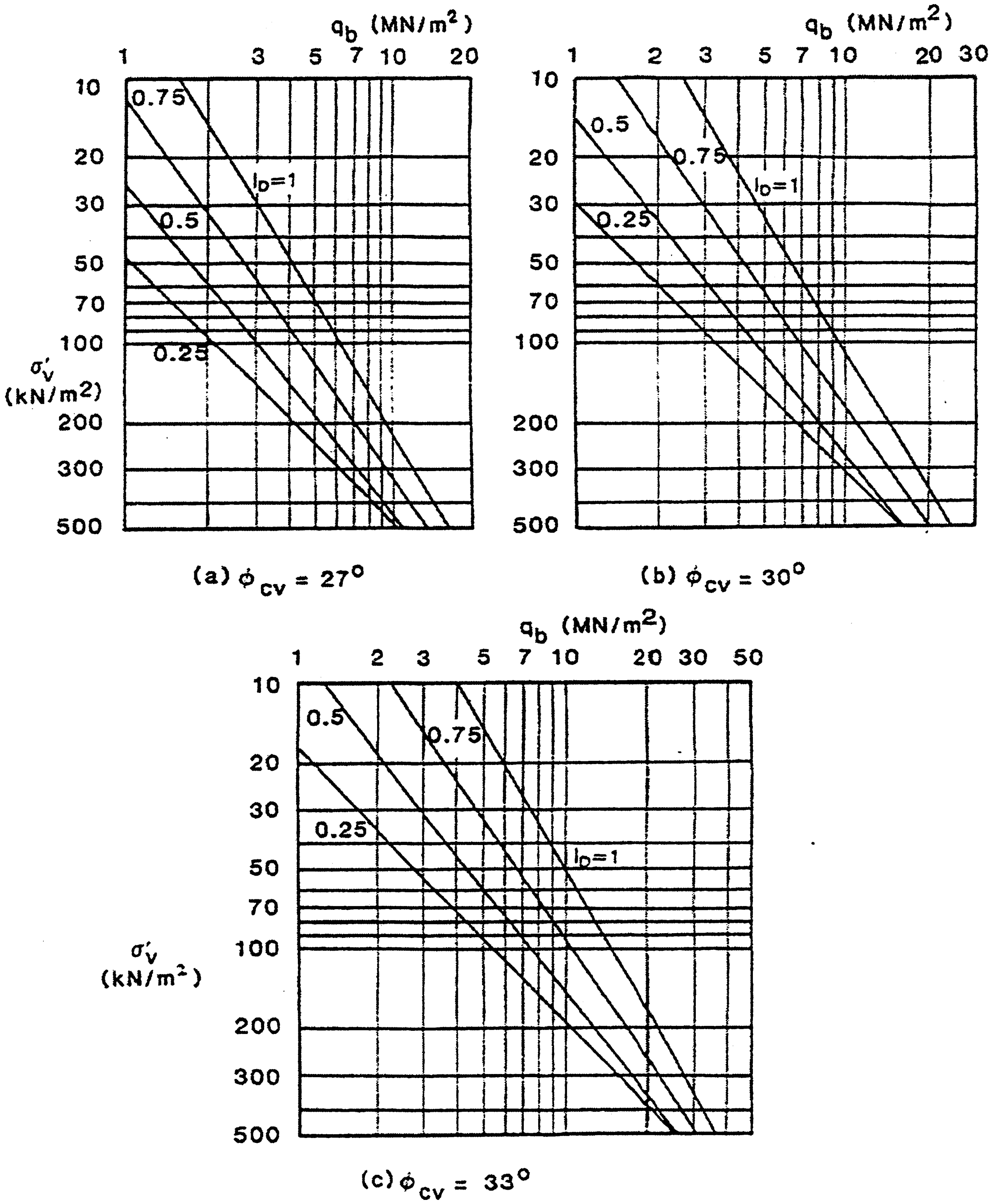
***Unit shaft resistance:*** $\alpha = \frac{\tau_s}{s_u} = 0.5 \cdot \mathrm{Max}\left[\left(\frac{\sigma'_{vo}}{s_u}\right)^{0.5}, \left(\frac{\sigma'_{vo}}{s_u}\right)^{0.25}\right]$

It is assumed that equal shaft resistance acts inside and outside open-ended piles.

***Unit base resistance:*** $q_b = N_c\, s_u$ $\qquad N_c = 9.$

## 4.2 Axial capacity: base resistance in sand using Bolton's stress dilatancy

Unit base resistance, $q_b$, is expressed as a function of relative density, $I_D$, constant volume (critical state) friction angle, $\phi_{cv}$, and in situ vertical effective stress, $\sigma'_v$.




Design charts for base resistance in sand
(Randolph 1985, Fleming et al 1992)

# Section 5: Settlement of deep foundations

## 5.1 Settlement of a rigid pile

**Shaft response:**

Equilibrium:

$$\tau = \tau_s \frac{R}{r}$$

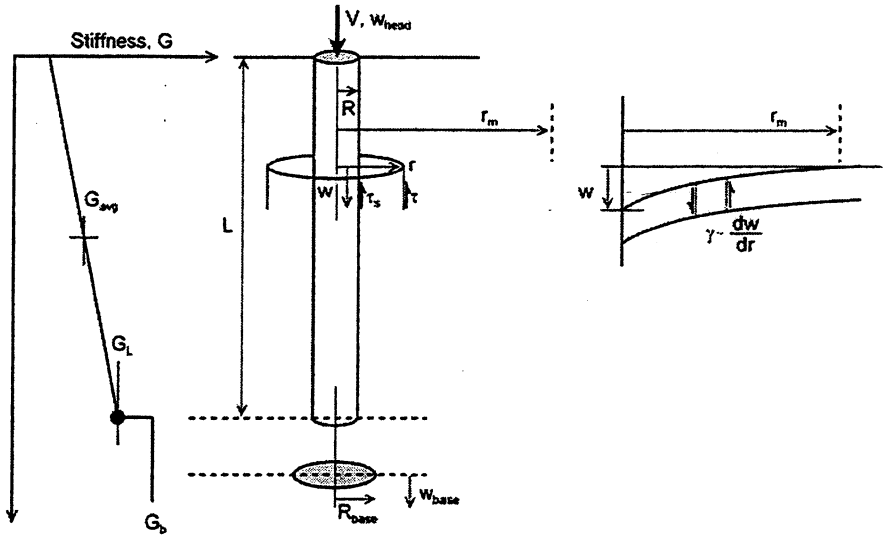
Compatibility:

$$\gamma \approx \frac{dw}{dr}$$

Elasticity:

$$\frac{\tau}{\gamma} = G$$

Integrate to magical radius, $r_m$, for shaft stiffness, $\tau_s/w$.




Nomenclature for settlement analysis of single piles

**Combined response of base (rigid punch) and shaft:**

$$\frac{V}{w_{head}} = \frac{Q_b}{w_{base}} + \frac{Q_s}{w} \qquad \frac{V}{w_{head}} = \frac{4R_{base}G_{base}}{1-\nu} + \frac{2\pi L G_{avg}}{\zeta}$$

$$\frac{V}{w_{head}DG_L} = \frac{2}{1-\nu}\frac{G_{base}}{G_L}\frac{D_{base}}{D} + \frac{2\pi}{\zeta}\frac{G_{avg}}{G_L}\frac{L}{D} \qquad \frac{V}{w_{head}DG_L} = \frac{2}{1-\nu}\frac{\eta}{\xi} + \frac{2\pi}{\zeta}\rho\frac{L}{D}$$

These expressions are simplified using dimensionless variables:

Base enlargement ratio, eta $\eta = R_{base}/R = D_{base}/D$ Slenderness ratio $L/D$

Stiffness gradient ratio, rho $\rho = G_{avg}/G_L$ Base stiffness ratio, xi $\xi = G_L/G_{base}$

It is often assumed that the dimensionless zone of influence, $\zeta = \ln(r_m/R) = 4$.

More precise relationships, checked against numerical analysis are:

$$\zeta = \ln\left\{\{0.5 + (5\rho(1-\nu) - 0.5)\xi\}\frac{L}{D}\right\} \qquad \text{for } \xi=1: \qquad \zeta = \ln\left\{5\rho(1-\nu)\frac{L}{D}\right\}$$

## 5.2 Settlement of a compressible pile

$$\frac{V}{w_{head}DG_L} = \frac{\dfrac{2\eta}{(1-\nu)\xi} + \rho\dfrac{2\pi}{\zeta}\dfrac{\tanh\mu L}{\mu L}\dfrac{L}{D}}{1 + \dfrac{1}{\pi\lambda}\dfrac{8\eta}{(1-\nu)\xi}\dfrac{\tanh\mu L}{\mu L}\dfrac{L}{D}}$$

where $\mu = \dfrac{\sqrt{8/\zeta\lambda}}{D}$ Pile compressibility

$\lambda = E_p/G_L$ Pile-soil stiffness ratio