UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Monday 25 April 2005 2.30 to 4

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

*Answer not more than **three** questions*

All questions carry the same number of marks

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment
4D6 Data sheets (4 pages)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 A two storey building is modelled as the two-dimensional, rigid-jointed, linear elastic sway frame shown in Fig. 1. Its beams have a mass of 200 kg/m. The bending stiffness of the ground floor and first floor columns are $EI = 5000$ kNm^2 and $EI = 2500$ kNm^2 respectively.

(a) Determine which one of the following two mode shapes is the best estimate of the fundamental vibration mode:

$$\begin{bmatrix} 1 \\ 2/3 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 1/3 \end{bmatrix}$$

[30%]

(b) The building, initially at rest, is subject to a load pulse $F(t)$ applied to the 1st floor only. $F(t)$ is defined in Fig. 2. Calculate the maximum displacement of the top of the frame assuming vibration in the fundamental mode only. [40%]

(c) Obtain an improved estimate of the maximum dynamic displacement by considering the vibration response in both the first and second modes of the structure. [30%]

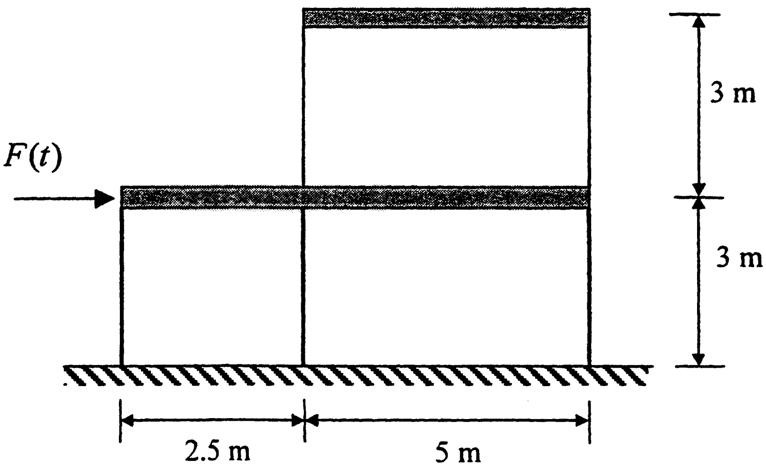

Fig. 1

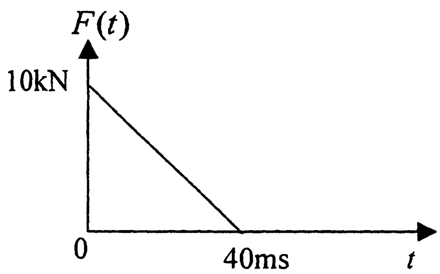

Fig. 2

2 (a) Describe lumped mass and distributed mass systems. Give one example of a suitable structure that you can analyse using these systems. [20%]

(b) A simply supported beam with flexural stiffness *EI* of 6×10^5 Nm^2 and the self weight of 400 kg/m is shown in Fig. 3. This beam is expected to undergo flexural vibrations with a mode shape as follows:

$$\bar{u}_n(x) = \sin\frac{n\pi x}{8}$$

where *n* indicates the mode of vibration. Determine the fundamental and second mode natural frequencies for this beam. [40%]

(c) An external time dependent load *F(t)* shown in Fig. 4 acts at 2 m from the right hand support as shown in Fig. 3. Determine the maximum deflection at point P that will occur due to this load;

i) using superposition of maximum amplitudes method;

ii) using SRSS method.

Comment on which of the above estimates would be more accurate. [40%]

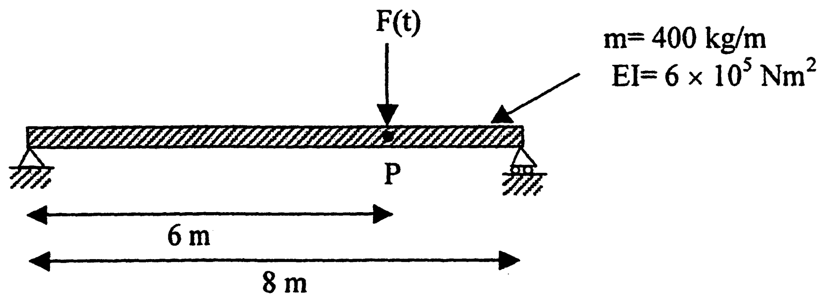

Fig. 3

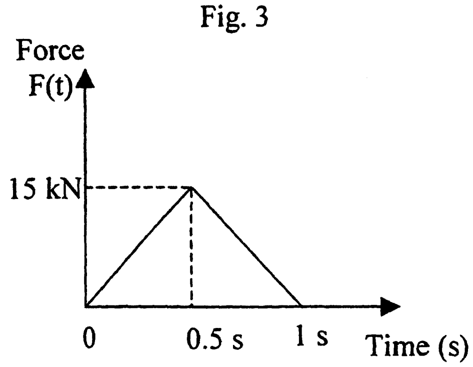

Fig. 4

3 (a) Explain why excess pore water pressures will be generated in loose, saturated sand when subjected to earthquake loading. [20%]

(b) A block foundation of dimensions 6 m × 6 m × 4 m shown in Fig. 5 is to be embedded to a depth of 4 m. This foundation is to be located in sandy soil that has a dry unit weight of 15 kN/m^3 and a void ratio of 0.67. The expected direction of ground shaking is shown in Fig. 5. The representative small-strain shear modulus of the sandy soil on the reference plane was determined to be 150 MN/m^2 and its Poisson's ratio may be taken as 0.3. Assuming that the block foundation is rigid, determine the horizontal, vertical and rotational stiffness due to the soil. [30%]

(c) The mass of the foundation including the participating soil around it was determined to be 691,200 kg. The mass moment of inertia of the block foundation and participating soil around it was determined to be 2,995,200 kgm^2 about the expected axis of rotation. Determine the natural frequencies of vibration for the horizontal, vertical and rotational degrees of freedom for the block foundation. [20%]

(d) A strong earthquake was experienced by the block foundation. Due to the large cyclic strains and excess pore pressure generation, the shear modulus of the sand has reduced to 3 MN/m^2. How does this affect the natural frequencies of vibration? Would you expect resonance of the block foundation under these conditions? [30%]

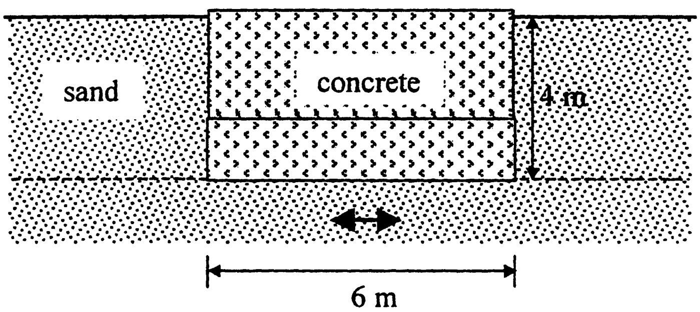

Fig. 5

4 (a) Explain what is meant by the terms mechanical admittance and aerodynamic admittance in the theory of buffeting of structures by the wind. [20%]

(b) A structural engineer has constructed a large finite element model of a suspension bridge and has extracted the modes shapes and the corresponding natural frequencies and modal masses. The engineer now wishes to model what happens to the bridge as a crowd of pedestrians walk across the bridge, but finds the idea of constructing a data file describing the time varying force on each finite element to be rather daunting.

i) Explain what is meant by the modal mass; [10%]

ii) Explain why it is important that the structural finite element model should have included in it nonlinear geometric effects; [10%]

iii) Briefly describe a procedure by which the engineer can estimate the dynamic response of the bridge which does not need the creation of an input file giving the time varying loading on each finite element; [20%]

iv) Briefly explain why random vibration theory can seriously underestimate the lateral vibrations of a pedestrian footbridge. [10%]

(c) The design for a light steel chimney is predicted to experience fatigue problems due to vortex-induced vibrations. An engineer argues that since oscillation amplitudes are approximately proportional to the reciprocal of the Scruton number, and the Scruton number is proportional to the mass per unit length, then increasing the mass but not the stiffness will improve matters. Explain how this may actually worsen the fatigue problems. (Hint: Strouhal Number $St = n_v D/U$, where n_v is the shedding frequency, D the diameter and U the wind velocity.) [30%]

END OF PAPER

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Engineering Tripos Part IIB/EIST Part II FOURTH YEAR

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Approximate SDOF model for a beam

for an assumed vibration mode $\bar{u}(x)$, the equivalent parameters are

$$M_{eq} = \int_0^L m\,\bar{u}^2 dx \qquad K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\,\bar{u} dx + \sum_i F_i\,\bar{u}_i$$

Frequency of mode $u(x,t) = U \sin\omega t\,\bar{u}(x)$ $\quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} \quad \omega = 2\pi f$

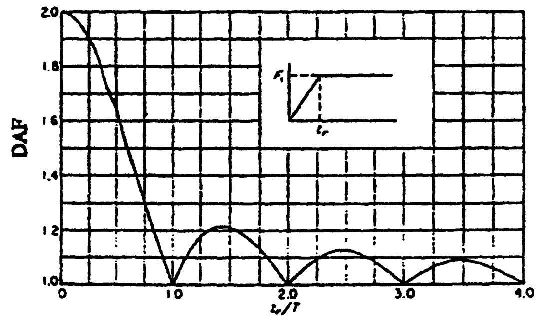
Modal analysis of simply-supported uniform beams

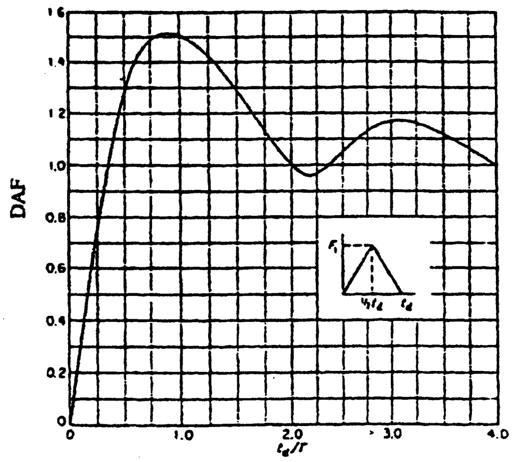
$$u_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

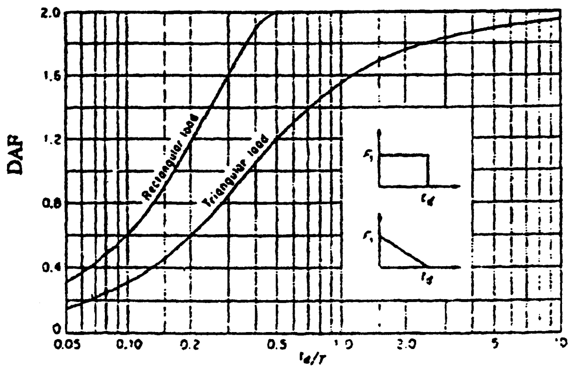
Ground motion participation factor

$$\Gamma = \frac{\int m\bar{u}dx}{\int m\bar{u}^2 dx}$$

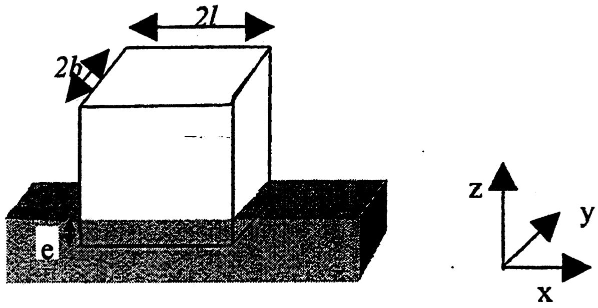
Dynamic amplification factors







Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions *2l* and *2b*, embedded to a depth *e* are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^3}{1-\nu}\left[3.2\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\frac{l}{b}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\left(\frac{l}{b}\right)^{4}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{tor} = Gb^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1+e}$$

where e is the void ratio, S_r is the degree of saturation, G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

Effective mean confining stress

$$p' = \sigma'_v \frac{(1+2K_o)}{3}$$

where σ'_v is the effective vertical stress, K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1-\nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{max} = 100 \frac{(3-e)^2}{(1+e)} (p')^{0.5}$$

where p' is the effective mean confining pressure in **MPa,** e is the void ratio and Gmax is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{max}} = \frac{1}{1+\gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a \,.\, e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{max}}{G_{max}}$$

where

$$\tau_{max} = \left[\left(\frac{1 + K_o}{2} \sigma'_v \sin \phi' \right)^2 - \left(\frac{1 - K_o}{2} \sigma'_v \right)^2 \right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity v_s as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

SPGM
January, 2004