ENGINEERING TRIPOS PART IIB

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Monday 9 May 2005 2.30 to 4

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Module 4D7

CONCRETE AND MASONRY STRUCTURES

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Attachments: (i) Concrete and Masonry Structures Formula and Data sheet (4 pages)*
*(ii) The Cumulative Normal Distribution Function (1 page).*

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.**

(TURN OVER

1 (a) **Either:** Discuss briefly the effect of fire on reinforced concrete structures. Outline what you, as a structural engineer, would look for and do to assess the structural integrity of a fire-damaged reinforced concrete structure. Suggest possible non-destructive tests you might consider to assist in your evaluation.

**Or:** There have been a number of advances in concrete material technology such as the introduction of (i) High Strength or High Performance Concretes, (ii) Reactive Powder Concrete (RPC) and (iii) Fibre Reinforced Concrete (FRC). Briefly outline two of these developments and explain their potential advantages over conventional reinforced concrete structures and where they might be used in practice. [35%]

(b) A steel mill produces steel reinforcing bars which are found to have mean yield strength of 529 MPa and standard deviation of 42 MPa. The steel yield strength may be assumed normally distributed. The bars are all of 20 mm diameter and the variability in the bar diameter can be assumed negligible.

(i) Determine both the characteristic and design yield strengths for the bars, assuming $\gamma_m$ for steel of 1.15, and hence calculate the probability that a bar chosen at random will have less than the design yield strength. [15%]

(ii) A single reinforcing bar is to be designed to carry a permanent dead load with mean value of 65 kN and standard deviation of 20 kN. This is the only load expected to act on the bar and may be assumed to be normally distributed. What would be the value of the characteristic load and the design load, assuming $\gamma_f$ of 1.35? Is the bar adequately safe to carry this loading based on the Eurocode design criterion? [15%]

(iii) An alternative measure of safety is the reliability index. What is the reliability index $\beta$ and hence probability of failure in tension for any randomly selected bar subjected to this loading? What are the sensitivity factors for load and resistance for this example? [15%]

(iv) Discuss briefly the advantages and disadvantages of the reliability index and the Eurocode design criteria as measures of structural adequacy. In the above example, would you consider the bar "safe" to carry the given loading? Justify your decision. [20%]

2 (a) Eurocode 2 specifies that concrete structures should be designed and constructed to satisfy two fundamental principles: (i) Fitness for Purpose and (ii) avoidance of Disproportionate Collapse. Explain these principles and discuss carefully how you would ensure that a structure conforms to these requirements. [40%]

(b) Whole life costing (WLC) principles are to be used to decide between two options for installing a new expansion joint for an existing short-span concrete bridge.

Option 1 involves installing a stainless steel joint with capital cost of £40,000. This joint is expected to last 20 years before requiring replacement. Installation requires closure of the bridge for 5 days. Maintenance costs for this joint are estimated at £5,000 per annum throughout its life.

Option 2 would involve installing a new FRP (fibre reinforced plastic) joint that is lightweight and quicker to install requiring only two days' bridge closure, but is more expensive at £70,000 and has a shorter predicted life expectancy of 15 years. Maintenance costs for this joint are estimated at £3,000 per annum.

The traffic delay cost incurred for each full day of closure of the bridge during installation is £20,000. All costs given are in 2005 prices. Both the capital cost of the joints and the traffic delay costs may be assumed to be incurred at the start of the accounting years in which the joint is replaced. Traffic delay costs during maintenance may be neglected.

Assuming a discount rate of 6% per annum for discounting in annual steps, and using continuous discounting for the maintenance costs, determine whether the stainless steel or FRP expansion joint should be recommended on economic grounds if the required remaining design life for the bridge is 60 years. How would you allow for traffic delay costs during annual maintenance? Would it make any difference if the traffic delay costs were not borne by the bridge owner? [60%]

(TURN OVER

3 (a) Give reasons why it might be important to limit crack widths and deflections in reinforced concrete structures. Explain briefly why the bond properties of the reinforcement influence crack width, and why after a certain stage no new bending cracks develop. Outline briefly why engineers designing ordinary reinforced concrete structures are often able to avoid doing detailed calculations for the serviceability limit states of cracking and deflection. [35%]

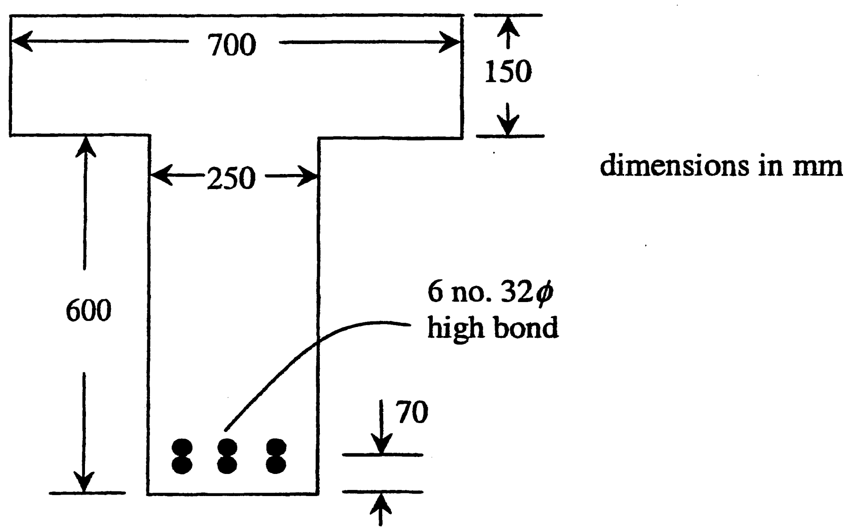



Fig. 1

(b) A reinforced concrete beam has the uniform cross-section shown in Fig. 1. The steel bars have elastic modulus $E_s$ which is ten times the effective modulus $E_{ce}$ of the concrete.

(i) Consider the beam bent about a horizontal axis in the figure, with all the materials in their elastic ranges except that due to cracking the concrete carries no tensile stress. Show that for zero axial force in the beam the neutral axis is approximately 106 mm below the bottom of the top flange. Evaluate the second moment of area $I_{cr}$ for the beam in this condition, transformed to equivalent concrete. [20%]

(ii) The beam is simply supported over a 12 m span, and carries at SLS a load of 200 kN uniformly distributed across the span. The unit weight of concrete is $25\ \mathrm{kN/m^3}$ including an allowance for the reinforcement, and $E_s$ is 205 GPa.

(cont.

Estimate the strain $\varepsilon_{sm}$ in the steel bars at midspan at SLS. Hence, considering the entire tension zone to calculate the steel proportion $\rho_r$, estimate the maximum crack width at SLS. [25%]

(iii) Estimate the maximum deflection at SLS, assuming that the cracked-elastic behaviour occurs throughout. Explain briefly why this is an over-estimate, and how to improve it. [20%]

4 (a) A concrete column of solid circular cross-section, radius $R$, is reinforced by a large number $n$ of longitudinal steel bars each of area $a$ equally spaced near the perimeter of the section on a circle of radius $0.9R$. The concrete has design cube strength $f_{cd}$, and takes uniform $0.6 f_{cd}$ on the compression zone at failure, which occurs with maximum strain $\varepsilon_{cm}$. The steel has design yield strength $f_{yd}$ in tension and compression and yield strain $\varepsilon_y$.

At a certain cross-section the column is subjected to compressive axial force $N$ combined with moment $M$ about an axis through the column centre. Without doing any detailed calculations, outline how to construct an interaction diagram of the values of $N$ and $M$ at failure, on appropriate dimensionless axes. Explain the significance of the quantities

(i) $naf_{yd} / \pi R^2 f_{cd}$;

(ii) $\varepsilon_y / \varepsilon_{cm}$.

How would you allow for any prestress in the steel? [40%]

(b) A column of rectangular cross-section, breadth $b$ and depth $h$, in concrete of similar properties to the above, is bent at a critical section about its major axis. It is to be designed to be just on the point of failure under axial force $N = 0.2 f_{cd} bh$ combined with moment $M = 0.13 f_{cd} bh^2$. Steel reinforcing bars may be provided near bottom and/or top faces, with cover $0.1h$ to bar centres.

Assuming that $\varepsilon_y / \varepsilon_{cm}$ is small, develop two alternative designs for the reinforced section, by considering the applied moment about appropriate steel positions, or otherwise

(i) with no steel near one face;

(ii) with steel provided such that the neutral axis at failure goes through the centre of the section.

Up to what value of $\varepsilon_y / \varepsilon_{cm}$ would these results still be valid? [45%]

An engineer claims that the second design is the optimum, i.e. uses the minimum possible total area of steel bars. Discuss briefly what other calculations might be done, or arguments brought to bear, to validate this claim. Why are such calculations not usually done in routine design practice? [15%]

5 (a) In what circumstances is it permissible to neglect torque in concrete beams being designed for the ultimate limit state? Explain briefly why beams carrying torque should be reinforced by a combination of closed stirrups and longitudinal bars. [15%]

A reinforced concrete beam with a $600 \times 400$ mm rectangular cross-section is reinforced by closed stirrups of 8 mm diameter at 100 mm spacing and cover 30 mm, plus 8 longitudinal bars of diameter 25 mm just inside the stirrups at each corner and the centre of each face. All the steel has design yield strength 250 MPa.

(i) Modelling the beam as a hollow box, and using the truss analogy, estimate the ultimate torsional strength of the beam, assuming that both sets of bars yield at failure. [25%]

(ii) Explain why it is reasonable to take the thickness of the box walls as twice the cover to the stirrup centres. Estimate the slope of the compression members of the truss at failure, and the minimum cube strength of concrete required to ensure that failure is indeed by yielding of the steel (take the effectiveness factor for concrete as 0.6). [30%]

(b) Explain briefly why a horizontal masonry beam between rigid immovable abutments would be expected to sustain very high vertical loads, even when constructed from a large number of stone elements with thin vertical joints in weak mortar. What factors would limit the magnitude of load that could in fact be carried? Explain briefly why the boundary conditions on longitudinal displacement at the supports are important for the behaviour of reinforced concrete beams. Discuss whether any apparent extra load-carrying capacity, related to restricted movement at the supports, could be relied upon in design practice and would be useful. [30%]

**END OF PAPER**

**Module 4D7 : Concrete and masonry structures**

## Formula and Data Sheet

The purpose of this sheet is to list certain relevant formulae (mostly from Eurocode 2) that are so complex that students may not remember them in full detail. Symbols used in the formulae have their usual meanings, and only minimal definitions are given here. The sheet also gives some typical numerical data.

### Material variability and partial safety factors

The word 'characteristic' usually refers to a 1 in 20 standard. At SLS, usually $\gamma_m = 1.0$ on all material strengths, $\gamma_f = 1.0$ on all loads.

At ULS, usually $\gamma_m$ is 1.15 for steel, 1.5 for concrete; and $\gamma_f$ is 1.4 for permanent loads, 1.6 for live loads (possibly reduced for combinations of rarely-occurring loads).

The difference between two normally-distributed variables is itself normally distributed, with mean equal to the difference of means, and variance the sum of the squares of the standard deviations.

### Cement paste

The density of cement particles is approx. 3.15 times that of water. On hydration, the solid products have volume approx. 1.54 times that of the hydrated cement, with a fixed gel porosity approx. 0.6 times the hydrated cement volume. This gives capillary porosity about

$\left[3.15\frac{W}{C} - 1.14h\right] \Big/ \left[1 + 3.15\frac{W}{C}\right]$ for hydration degree h : and gel/space ratio (gel volume / gel + capillaries) $2.14h \Big/ \left[h + 3.15\, W/C + a\right]$

### Mechanical properties of concrete

Cracking strain typically $150 \times 10^{-6}$, strain at peak stress in uniaxial compression typically 0.002. Lateral confinement typically adds about 4 times the confining stress to the unconfined uniaxial strength, as well as improving ductility. In plane stress, the peak strength under biaxial compression is typically 20% greater than the uniaxial strength.

### Durability considerations

Present value of some future good : $S_i / (1 + r)^i$ for stepped, or $S_i / \exp(r_c t_i)$ for continuous discounting.

Water penetration : cumulative volume uniaxial inflow / unit area is sorptivity times square root of time. On sharp-wet-front theory penetration depth is $\left\{2k\left(H + h_c\right)/\Delta n\right\}^{1/2} t^{1/2}$.

Uniaxial diffusion into homogeneous material : $\dfrac{\partial c}{\partial t} = D\dfrac{\partial^2 c}{\partial x^2}$

solution $\quad c = c_o\left(1 - \mathrm{erf}(z)\right),\ z = x/2\sqrt{Dt}$

Table of erf (z) :

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| z | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | |
| erf (z) | 0 | 0.11 | 0.22 | 0.33 | 0.43 | 0.52 | 0.60 | 0.68 | |
| | | | | | | | | | |
| z | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | $\infty$ |
| erf(z) | 0.74 | 0.80 | 0.84 | 0.88 | 0.91 | 0.93 | 0.95 | 0.97 | 1.00 |

Passivation for pH > 12 and $Cl^-$ < 0.4% by weight cement.

Corrosion unlikely for corrosion current < 0.2 $\mu A/cm^2$, resistivity > 100 k $\Omega$ cm, half-cell potential > –200 mV (but probable for < –350 mV).

**SLS : cracking**

Steel $A_s > k_c \; k \; f_{ct.ef} \; A_{ct}/f_{yk}$ in tension zone, to produce multiple cracks.

Then, limitation to about 0.3 mm under quasi-permanent loads depending on exposure.

Maximum (characteristic) width $w_k = \beta . s_{rm} . \varepsilon_{sm}$ ($\beta$ usually 1.7)

where spacing $s_{rm} = 50\text{mm} + 0.25 k_1 k_2 \phi/\rho_r$

with $k_1$ 0.8 for high bond, 1.6 for plain bars; $k_2$ 1.0 for tension
0.5 for bending.

**SLS : deflection**

Interpolated curvature $\kappa = (1-\xi)\,\kappa_{un} + \xi\,\kappa_{cr}$

$$\text{where} \quad \xi = 1 - \beta_1 \beta_2 \left(\frac{\sigma_{sr}}{\sigma_s}\right)^2$$

$\beta_1$ is 1.0 for high bond, 0.5 for plain bars

$\beta_2$ is 1.0 for short-term, 0.5 for sustained load

$\sigma_{sr}$ is steel stress, for cracked section, but using loads which first cause cracking at the section considered. $\sigma_s$ is current steel stress, calculated for cracked section.

**ULS : moment and axial force**

It is usual to assume failure at a cross-section to occur when the extreme-fibre compressive strain in the concrete reaches a limiting value, often $\varepsilon_{cm} = 0.0035$. The yield strain of steel $\varepsilon_y$ of course depends on strength, as roughly $f_y/E$.

Initial calculations often use uniform stress of 0.6 $f_{cd}$ on the compression zone at failure.

With these assumptions, for a singly-reinforced under-reinforced rectangular beam

$$M_u = A_s f_y d(1 - 0.5x/d); \quad x/d = \frac{A_s f_y}{0.6 f_{cd} bd};$$

over-reinforcement for $x/d > 0.5$.

For Tee beams, effective flange width b in compression is of order

$b_w + l_o/5 \leq b_{actual}$, where $l_o$ is span between zero-moment points.

For long columns, extra deflection prior to material failure is of order

$e_2 = \frac{l_o^2}{\pi^2} \kappa_m$ where $\kappa_m$ is curvature at mid-height at failure and $l_o$ is effective length. Eurocode multiplies by further factor K, which is 1 for

$\frac{l_o}{r} > 35$, and $\frac{l_o}{20r} - 0.75$ for $15 \leq l_o/r \leq 35$,

r being radius of gyration of gross concrete section.

**Shear in reinforced concrete**

For unreinforced webs at ULS, shear strength in Code is

$$V_{Rd1} = b_w d\left\{\tau_{Rd}\ k\left(1.2 + 40\rho_1\right) + 0.15\ N/A_c\right\}$$

where $\rho_1$ is $A_s/bd$ for tension steel, $\tau_{Rd}$ is tabulated function of $f_{cd}$, and $k = 1.6 - d$ (metres) $\geq 1$ (and is 1 for more than 50% steel curtailment).

In 'standard' design method, for $V_{sd} > V_{Rd1}$

$V_{Rd} = V_{Rd1} + V_{Rd3} < V_{Rd2}$ (tabulated in Eurocode)

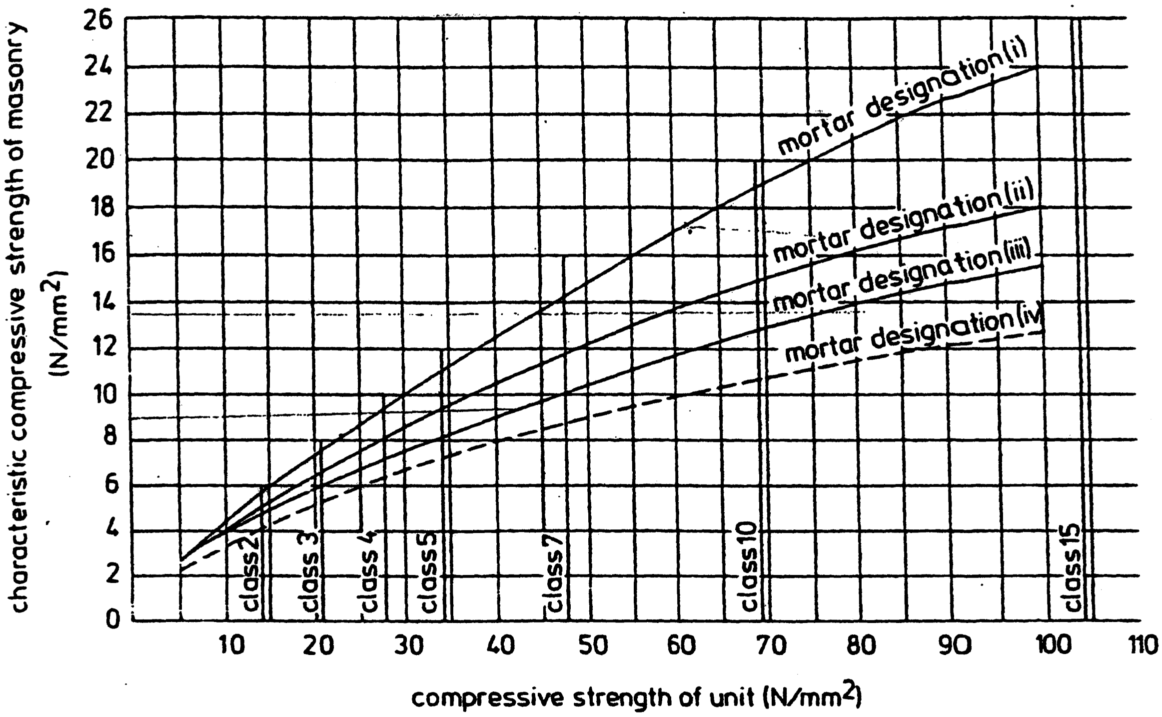
Stirrup term $V_{Rd3}$ follows from truss analogy with 45° struts and "web" depth 90% of effective;

$$V_{Rd3} = A_{sw}\ f_{wyd}\ (0.9d)/s.$$

**Torsion at ULS**

Based on truss analogy with variable strut angle, for a thin-walled box section; shear flow

$$q = f_{yd}\left\{\left(A_w/s\right)\left(\Sigma A_\ell/u\right)\right\}^{1/2}\ ;\ \sigma \leq\ \nu\ f_{cd}\,.$$




interpolation for classes of loadbearing bricks not shown on the graph may be used for average crushing strengths intermediate between those given on the graph, as described in clause 10 of BS 3921: 1985 and clause 7 of BS 187: 1978.

**Figure 5.6(a)** Characteristic compressive strength, $f_k$, of brick masonry (see Table 5.4)

CTM
October 1997
Revised CRM
February 2003

## THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}\,dx \quad \text{FOR} \quad 0{\cdot}00 \le u \le 4{\cdot}99.$$

| $u$ | ·00 | ·01 | ·02 | ·03 | ·04 | ·05 | ·06 | ·07 | ·08 | ·09 |
|---|---|---|---|---|---|---|---|---|---|---|
| ·0 | ·5000 | ·5040 | ·5080 | ·5120 | ·5160 | ·5199 | ·5239 | ·5279 | ·5319 | ·5359 |
| ·1 | ·5398 | ·5438 | ·5478 | ·5517 | ·5557 | ·5596 | ·5636 | ·5675 | ·5714 | ·5753 |
| ·2 | ·5793 | ·5832 | ·5871 | ·5910 | ·5948 | ·5987 | ·6026 | ·6064 | ·6103 | ·6141 |
| ·3 | ·6179 | ·6217 | ·6255 | ·6293 | ·6331 | ·6368 | ·6406 | ·6443 | ·6480 | ·6517 |
| ·4 | ·6554 | ·6591 | ·6628 | ·6664 | ·6700 | ·6736 | ·6772 | ·6808 | ·6844 | ·6879 |
| ·5 | ·6915 | ·6950 | ·6985 | ·7019 | ·7054 | ·7088 | ·7123 | ·7157 | ·7190 | ·7224 |
| ·6 | ·7257 | ·7291 | ·7324 | ·7357 | ·7389 | ·7422 | ·7454 | ·7486 | ·7517 | ·7549 |
| ·7 | ·7580 | ·7611 | ·7642 | ·7673 | ·7703 | ·7734 | ·7764 | ·7794 | ·7823 | ·7852 |
| ·8 | ·7881 | ·7910 | ·7939 | ·7967 | ·7995 | ·8023 | ·8051 | ·8078 | ·8106 | ·8133 |
| ·9 | ·8159 | ·8186 | ·8212 | ·8238 | ·8264 | ·8289 | ·8315 | ·8340 | ·8365 | ·8389 |
| 1·0 | ·8413 | ·8438 | ·8461 | ·8485 | ·8508 | ·8531 | ·8554 | ·8577 | ·8599 | ·8621 |
| 1·1 | ·8643 | ·8665 | ·8686 | ·8708 | ·8729 | ·8749 | ·8770 | ·8790 | ·8810 | ·8830 |
| 1·2 | ·8849 | ·8869 | ·8888 | ·8907 | ·8925 | ·8944 | ·8962 | ·8980 | ·8997 | ·90147 |
| 1·3 | ·90320 | ·90490 | ·90658 | ·90824 | ·90988 | ·91149 | ·91309 | ·91466 | ·91621 | ·91774 |
| 1·4 | ·91924 | ·92073 | ·92220 | ·92364 | ·92507 | ·92647 | ·92785 | ·92922 | ·93056 | ·93189 |
| 1·5 | ·93319 | ·93448 | ·93574 | ·93699 | ·93822 | ·93943 | ·94062 | ·94179 | ·94295 | ·94408 |
| 1·6 | ·94520 | ·94630 | ·94738 | ·94845 | ·94950 | ·95053 | ·95154 | ·95254 | ·95352 | ·95449 |
| 1·7 | ·95543 | ·95637 | ·95728 | ·95818 | ·95907 | ·95994 | ·96080 | ·96164 | ·96246 | ·96327 |
| 1·8 | ·96407 | ·96485 | ·96562 | ·96638 | ·96712 | ·96784 | ·96856 | ·96926 | ·96995 | ·97062 |
| 1·9 | ·97128 | ·97193 | ·97257 | ·97320 | ·97381 | ·97441 | ·97500 | ·97558 | ·97615 | ·97670 |
| 2·0 | ·97725 | ·97778 | ·97831 | ·97882 | ·97932 | ·97982 | ·98030 | ·98077 | ·98124 | ·98169 |
| 2·1 | ·98214 | ·98257 | ·98300 | ·98341 | ·98382 | ·98422 | ·98461 | ·98500 | ·98537 | ·98574 |
| 2·2 | ·98610 | ·98645 | ·98679 | ·98713 | ·98745 | ·98778 | ·98809 | ·98840 | ·98870 | ·98899 |
| 2·3 | ·98928 | ·98956 | ·98983 | $\cdot 9^2$ 0097 | $\cdot 9^2$ 0358 | $\cdot 9^2$ 0613 | $\cdot 9^2$ 0863 | $\cdot 9^2$ 1106 | $\cdot 9^2$ 1344 | $\cdot 9^2$ 1576 |
| 2·4 | $\cdot 9^2$ 1802 | $\cdot 9^2$ 2024 | $\cdot 9^2$ 2240 | $\cdot 9^2$ 2451 | $\cdot 9^2$ 2656 | $\cdot 9^2$ 2857 | $\cdot 9^2$ 3053 | $\cdot 9^2$ 3244 | $\cdot 9^2$ 3431 | $\cdot 9^2$ 3613 |
| 2·5 | $\cdot 9^2$ 3790 | $\cdot 9^2$ 3963 | $\cdot 9^2$ 4132 | $\cdot 9^2$ 4297 | $\cdot 9^2$ 4457 | $\cdot 9^2$ 4614 | $\cdot 9^2$ 4766 | $\cdot 9^2$ 4915 | $\cdot 9^2$ 5060 | $\cdot 9^2$ 5201 |
| 2·6 | $\cdot 9^2$ 5339 | $\cdot 9^2$ 5473 | $\cdot 9^2$ 5604 | $\cdot 9^2$ 5731 | $\cdot 9^2$ 5855 | $\cdot 9^2$ 5975 | $\cdot 9^2$ 6093 | $\cdot 9^2$ 6207 | $\cdot 9^2$ 6319 | $\cdot 9^2$ 6427 |
| 2·7 | $\cdot 9^2$ 6533 | $\cdot 9^2$ 6636 | $\cdot 9^2$ 6736 | $\cdot 9^2$ 6833 | $\cdot 9^2$ 6928 | $\cdot 9^2$ 7020 | $\cdot 9^2$ 7110 | $\cdot 9^2$ 7197 | $\cdot 9^2$ 7282 | $\cdot 9^2$ 7365 |
| 2·8 | $\cdot 9^2$ 7445 | $\cdot 9^2$ 7523 | $\cdot 9^2$ 7599 | $\cdot 9^2$ 7673 | $\cdot 9^2$ 7744 | $\cdot 9^2$ 7814 | $\cdot 9^2$ 7882 | $\cdot 9^2$ 7948 | $\cdot 9^2$ 8012 | $\cdot 9^2$ 8074 |
| 2·9 | $\cdot 9^2$ 8134 | $\cdot 9^2$ 8193 | $\cdot 9^2$ 8250 | $\cdot 9^2$ 8305 | $\cdot 9^2$ 8359 | $\cdot 9^2$ 8411 | $\cdot 9^2$ 8462 | $\cdot 9^2$ 8511 | $\cdot 9^2$ 8559 | $\cdot 9^2$ 8605 |
| 3·0 | $\cdot 9^2$ 8650 | $\cdot 9^2$ 8694 | $\cdot 9^2$ 8736 | $\cdot 9^2$ 8777 | $\cdot 9^2$ 8817 | $\cdot 9^2$ 8856 | $\cdot 9^2$ 8893 | $\cdot 9^2$ 8930 | $\cdot 9^2$ 8965 | $\cdot 9^2$ 8999 |
| 3·1 | $\cdot 9^3$ 0324 | $\cdot 9^3$ 0646 | $\cdot 9^3$ 0957 | $\cdot 9^3$ 1260 | $\cdot 9^3$ 1553 | $\cdot 9^3$ 1836 | $\cdot 9^3$ 2112 | $\cdot 9^3$ 2378 | $\cdot 9^3$ 2636 | $\cdot 9^3$ 2886 |
| 3·2 | $\cdot 9^3$ 3129 | $\cdot 9^3$ 3363 | $\cdot 9^3$ 3590 | $\cdot 9^3$ 3810 | $\cdot 9^3$ 4024 | $\cdot 9^3$ 4230 | $\cdot 9^3$ 4429 | $\cdot 9^3$ 4623 | $\cdot 9^3$ 4810 | $\cdot 9^3$ 4991 |
| 3·3 | $\cdot 9^3$ 5166 | $\cdot 9^3$ 5335 | $\cdot 9^3$ 5499 | $\cdot 9^3$ 5658 | $\cdot 9^3$ 5811 | $\cdot 9^3$ 5959 | $\cdot 9^3$ 6103 | $\cdot 9^3$ 6242 | $\cdot 9^3$ 6376 | $\cdot 9^3$ 6505 |
| 3·4 | $\cdot 9^3$ 6631 | $\cdot 9^3$ 6752 | $\cdot 9^3$ 6869 | $\cdot 9^3$ 6982 | $\cdot 9^3$ 7091 | $\cdot 9^3$ 7197 | $\cdot 9^3$ 7299 | $\cdot 9^3$ 7398 | $\cdot 9^3$ 7493 | $\cdot 9^3$ 7585 |
| 3·5 | $\cdot 9^3$ 7674 | $\cdot 9^3$ 7759 | $\cdot 9^3$ 7842 | $\cdot 9^3$ 7922 | $\cdot 9^3$ 7999 | $\cdot 9^3$ 8074 | $\cdot 9^3$ 8146 | $\cdot 9^3$ 8215 | $\cdot 9^3$ 8282 | $\cdot 9^3$ 8347 |
| 3·6 | $\cdot 9^3$ 8409 | $\cdot 9^3$ 8469 | $\cdot 9^3$ 8527 | $\cdot 9^3$ 8583 | $\cdot 9^3$ 8637 | $\cdot 9^3$ 8689 | $\cdot 9^3$ 8739 | $\cdot 9^3$ 8787 | $\cdot 9^3$ 8834 | $\cdot 9^3$ 8879 |
| 3·7 | $\cdot 9^3$ 8922 | $\cdot 9^3$ 8964 | $\cdot 9^4$ 0039 | $\cdot 9^4$ 0426 | $\cdot 9^4$ 0799 | $\cdot 9^4$ 1158 | $\cdot 9^4$ 1504 | $\cdot 9^4$ 1838 | $\cdot 9^4$ 2159 | $\cdot 9^4$ 2468 |
| 3·8 | $\cdot 9^4$ 2765 | $\cdot 9^4$ 3052 | $\cdot 9^4$ 3327 | $\cdot 9^4$ 3593 | $\cdot 9^4$ 3848 | $\cdot 9^4$ 4094 | $\cdot 9^4$ 4331 | $\cdot 9^4$ 4558 | $\cdot 9^4$ 4777 | $\cdot 9^4$ 4988 |
| 3·9 | $\cdot 9^4$ 5190 | $\cdot 9^4$ 5385 | $\cdot 9^4$ 5573 | $\cdot 9^4$ 5753 | $\cdot 9^4$ 5926 | $\cdot 9^4$ 6092 | $\cdot 9^4$ 6253 | $\cdot 9^4$ 6406 | $\cdot 9^4$ 6554 | $\cdot 9^4$ 6696 |
| 4·0 | $\cdot 9^4$ 6833 | $\cdot 9^4$ 6964 | $\cdot 9^4$ 7090 | $\cdot 9^4$ 7211 | $\cdot 9^4$ 7327 | $\cdot 9^4$ 7439 | $\cdot 9^4$ 7546 | $\cdot 9^4$ 7649 | $\cdot 9^4$ 7748 | $\cdot 9^4$ 7843 |
| 4·1 | $\cdot 9^4$ 7934 | $\cdot 9^4$ 8022 | $\cdot 9^4$ 8106 | $\cdot 9^4$ 8186 | $\cdot 9^4$ 8263 | $\cdot 9^4$ 8338 | $\cdot 9^4$ 8409 | $\cdot 9^4$ 8477 | $\cdot 9^4$ 8542 | $\cdot 9^4$ 8605 |
| 4·2 | $\cdot 9^4$ 8665 | $\cdot 9^4$ 8723 | $\cdot 9^4$ 8778 | $\cdot 9^4$ 8832 | $\cdot 9^4$ 8882 | $\cdot 9^4$ 8931 | $\cdot 9^4$ 8978 | $\cdot 9^5$ 0226 | $\cdot 9^5$ 0655 | $\cdot 9^5$ 1066 |
| 4·3 | $\cdot 9^5$ 1460 | $\cdot 9^5$ 1837 | $\cdot 9^5$ 2199 | $\cdot 9^5$ 2545 | $\cdot 9^5$ 2876 | $\cdot 9^5$ 3193 | $\cdot 9^5$ 3497 | $\cdot 9^5$ 3788 | $\cdot 9^5$ 4066 | $\cdot 9^5$ 4332 |
| 4·4 | $\cdot 9^5$ 4587 | $\cdot 9^5$ 4831 | $\cdot 9^5$ 5065 | $\cdot 9^5$ 5288 | $\cdot 9^5$ 5502 | $\cdot 9^5$ 5706 | $\cdot 9^5$ 5902 | $\cdot 9^5$ 6089 | $\cdot 9^5$ 6268 | $\cdot 9^5$ 6439 |
| 4·5 | $\cdot 9^5$ 6602 | $\cdot 9^5$ 6759 | $\cdot 9^5$ 6908 | $\cdot 9^5$ 7051 | $\cdot 9^5$ 7187 | $\cdot 9^5$ 7318 | $\cdot 9^5$ 7442 | $\cdot 9^5$ 7561 | $\cdot 9^5$ 7675 | $\cdot 9^5$ 7784 |
| 4·6 | $\cdot 9^5$ 7888 | $\cdot 9^5$ 7987 | $\cdot 9^5$ 8081 | $\cdot 9^5$ 8172 | $\cdot 9^5$ 8258 | $\cdot 9^5$ 8340 | $\cdot 9^5$ 8419 | $\cdot 9^5$ 8494 | $\cdot 9^5$ 8566 | $\cdot 9^5$ 8634 |
| 4·7 | $\cdot 9^5$ 8699 | $\cdot 9^5$ 8761 | $\cdot 9^5$ 8821 | $\cdot 9^5$ 8877 | $\cdot 9^5$ 8931 | $\cdot 9^5$ 8983 | $\cdot 9^6$ 0320 | $\cdot 9^6$ 0789 | $\cdot 9^6$ 1235 | $\cdot 9^6$ 1661 |
| 4·8 | $\cdot 9^6$ 2067 | $\cdot 9^6$ 2453 | $\cdot 9^6$ 2822 | $\cdot 9^6$ 3173 | $\cdot 9^6$ 3508 | $\cdot 9^6$ 3827 | $\cdot 9^6$ 4131 | $\cdot 9^6$ 4420 | $\cdot 9^6$ 4696 | $\cdot 9^6$ 4958 |
| 4·9 | $\cdot 9^6$ 5208 | $\cdot 9^6$ 5446 | $\cdot 9^6$ 5673 | $\cdot 9^6$ 5889 | $\cdot 9^6$ 6094 | $\cdot 9^6$ 6289 | $\cdot 9^6$ 6475 | $\cdot 9^6$ 6652 | $\cdot 9^6$ 6821 | $\cdot 9^6$ 6981 |

Example: $\Phi(3{\cdot}57) = \cdot 9^3\,8215 = 0{\cdot}9998215$.

**4D7 2005 Answers**

1. (b) (i) 460 MPa 400 MPa 0.00107
   (ii) 97.9 kN 132.2 kN
   (iii) 4.22 $1.22 \times 10^{-5}$

2. (b) including traffic delay costs, steel about £281k, FRP about £233k

3. (b) (i) $1.241 \times 10^{10}$ mm$^4$
   (ii) $691 \times 10^{-6}$ 0.155 mm
   (iii) 24.5 mm

4. (b) (i) $A_t = 0$, $A_b f_{yd} = 0.141\ f_{cd}bh$
   (ii) $A_t f_{yd} = 0.0188\ f_{cd}bh$ (in compression), $A_b f_{yd} = 0.1188\ f_{cd}bh$

5. (a) (i) 94.4 kNm
   (ii) $25.2^{\circ}$ 17 MPa