ENGINEERING TRIPOS PART IIB

Wednesday 4 May 2005 2.30 to 4

Module 4F2

ROBUST MULTIVARIABLE CONTROL

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 (a) State the Small Gain Theorem, defining any terms used. [10%]

(b) Consider the family of plants:

$$G = \left\{ G_0(s)[1 + \Delta(s)W(s)G_0(s)]^{-1}, \quad \|\Delta\|_\infty < \epsilon \right\},$$

where W and Δ are taken to be stable transfer function matrices.

Take a controller K in a standard feedback configuration (see Fig. 1). With the aid of a diagram find a necessary and sufficient condition that depends only on W, K and G_0 such that K stabilises all systems in G. [30%]

(c) In addition to robust stability, assume we have as well a performance criterion that the closed loop transfer function $T_{d\to y}$ should satisfy $\|T_{d\to y}\|_\infty \leq \alpha$ for all Δ satisfying $\|\Delta\|_\infty < \epsilon$. Formulate this as a structured singular value test. [20%]

(d) Let

$$G = \left\{ \frac{10}{s+2+\delta}, \quad |\delta| < 1, \quad \delta \in \mathbb{R} \right\}$$

and a constant controller $k \in \mathbb{R}$. Find the range of k for *nominal* and *robust* stability. [20%]

(e) For G as in part (d), design a controller K that rejects step inputs from d *for all* the models in G (i.e., $y(t) \to 0$ as $t \to \infty$ when $d(t) = c$, $t \geq 0$ *for all* the models in G). [20%]

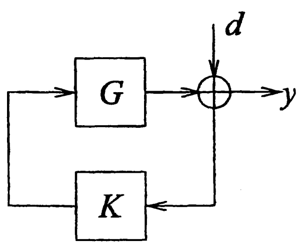

Fig. 1

2 The value function $V(x,k)$ for the dynamic game with

$$x_{k+1} = f(x_k, u_k, w_k),$$

where $x \in \mathbb{R}^n$, $w \in W \subseteq \mathbb{R}^p$, $u \in U \subseteq \mathbb{R}^m$, and cost

$$J(x_0, u(\cdot), w(\cdot)) = \sum_{k=0}^{h-1} c(x_k, u_k, w_k) + J_h(x_h)$$

satisfies the equation

$$V(x,k) = \min_{u \in U} \max_{w \in W} \Big(c(x,u,w) + V(f(x,u,w), k+1) \Big).$$

(a) What do w , u and $V(x,k)$ represent? Give a definition of $V(x,k)$. [30%]

(b) Consider the particular game with $x \in \mathbb{R}$, $u \in \mathbb{R}$, $w \in \mathbb{R}$, $h = 3$ and

$$\begin{aligned} f(x,u,w) &= x + u + w \\ c(x,u,w) &= u^2 - w^2 \\ J_3(x) &= x^2 \end{aligned}$$

Find the optimal cost starting from the initial state $x_0 = 5$. [70%]

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3 Consider the stable linear system $G(s)$ with state-space realization

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

(a) Define the $\mathcal{H}_\infty$ norm of this system. [10%]

(b) What does $\|G(s)\|_\infty < \gamma$ imply about the relationship between $u(t)$ and $y(t)$? [20%]

(c) Assume there exists an $X = X^T > 0$ satisfying

$$A^T X + XA + C^T C + \frac{1}{\gamma^2} XBB^T X = 0.$$

By considering

$$V(t) = x(t)^T Xx(t) + \int_0^t y(\tau)^T y(\tau)\, d\tau - \gamma^2 \int_0^t u(\tau)^T u(\tau)\, d\tau,$$

show that $\|G(s)\|_\infty < \gamma$. [40%]

(d) Let $A = -1$, $B = 1$, $C = 1$. Find the smallest value of γ for which the Riccati equation in (c) has a positive solution. Find $G(s)$, and check that this value equals $\|G(s)\|_\infty$. [30%]

END OF PAPER