UNIVERSITY OF CAMBRIDGE

ENGINEERING TRIPOS PART IIB

Monday 25 April 2005 2.30 to 4

Module 4F6

SIGNAL DETECTION AND ESTIMATION

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The **approximate** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

1 a) Describe how Cox's axioms lead to a derivation of Bayes rule in terms of probabilities as degrees of belief. [30%]

b) Explain the advantages and disadvantages of using Bayesian methods. [30%]

c) A container has 64 coins in it and one is known to be double headed. A coin is picked from the container, flipped and found to land with a head upwards. The coin is flipped a further two times and each time it lands with a head facing upwards. For these three independent flips of the coin what are the probabilities that the coin is the double headed coin ? [40%]

2 a) Outline the differences between *Maximum Likelihood*, *Maximum A-Posteriori* and *Minimum Variance* estimation. [20%]

Consider the following binary communications problem where we assume that the signals, Y, corresponding to the two hypotheses H_1 and H_2 are $+1$ and -1 respectively. You are required to estimate the value of the signal based upon a single observation

$$Z = Y + V$$

where V is assumed to be normally distributed, $N(0, \sigma^2)$.

The prior probabilities for the two hypotheses are assumed to be the same, i.e.,

$$p(y) = 0.5\delta(y-1) + 0.5\delta(y+1)$$

b) Derive an expression for $p(z)$ and hence for the posterior density $p(y|z)$. [20%]

c) Show that the maximum a-posteriori estimate is given by

$$\hat{y}_{MAP} = sgn(z)$$

where $sgn(z) = +1$ if $z \geq 0$ and $sgn(z) = -1$ if $z < 0$. [30%]

d) Show that the minimum mean square estimate is given by

$$y_{ms} = \tanh(\frac{z}{\sigma^2})$$

[30%]

(TURN OVER

3 a) Define the terms *unbiased*, *efficient*, *consistent* and *point* estimators. [20%]

b) Show that for any *unbiased* estimator the variance satisfies the Cramer-Rao lower bound. [40%]

c) Consider the estimation of the mean, θ_1, and variance, θ_2, for a sequence of N statistically independent Gaussian random variables.

Derive the *Fisher Information Matrix* and hence show that

$$E(\hat{\theta}_1 - \theta_1)^2 \geq \frac{\theta_2}{N}$$

$$E(\hat{\theta}_2 - \theta_2)^2 \geq \frac{2\theta_2^2}{N}$$

[40%]

4 a) Describe, in detail, the *Neyman-Pearson* decision rule applied to detection theory and discuss the advantages and disadvantages of this decision rule over the *Likelihood ratio* criterion. [40%]

b) A radar transmits M successive pulses. The return pulses at the receiver are denoted by $s_1(t), s_2(t), \ldots\ldots, s_M(t)$.

Additive noise samples are denoted by $n_1(t), n_2(t), \ldots\ldots, n_M(t)$.

These noise waveforms are sample functions of white Gaussian noise with spectral density $N_0/2$. It is assumed that the signals $s_i(t)$ are completely known and that each exists over an interval T seconds long. The received signals are $r_i(t) = s_i(t) + n_i(t)$.

A decision based on these M received signals is to be made to determine if a target is present.

Using the likelihood ratio criterion, show that the hypothesis, H_1, that the signal is present should be based upon the inequality,

$$\sum_{i=1}^{M} \int_0^T r_i(t)s_i(t)dt \geq \frac{1}{2}N_0 \ln \lambda_0 + \frac{1}{2}\sum_{i=1}^{M} E_i$$

and state what the terms λ_0 and E_i represent. [60%]

END OF PAPER