ENGINEERING TRIPOS PART IIB

Wednesday 27 April 2005 9 to 10.30

Module 4F9

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

This paper consists of three sections.

Answer not more than **one** *question from each section.*

Answers to questions in each section should be tied together and handed in separately.

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Supplementary page containing 7 copies of Fig. 2 (Question 3).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A *Medical Image Acquisition*

*Answer not more than **one** question from this section*

1 (a) The criterion function of the maximum likelihood expectation maximisation (ML-EM) algorithm for PET reconstruction is

$$L(Q|\Lambda) = \sum_i \left(q_i \ln \left(\sum_j c_{ij}\lambda_j \right) - \sum_j c_{ij}\lambda_j \right) .$$

Explain the meanings of the terms. Give typical values for the ranges of i and j. [20%]

(b) Describe three ways of reducing Poisson noise in ML-EM reconstructions. [20%]

(c) Describe *Compton scattering* and *photoelectric absorption*. Explain their relevance in the context of X-ray computed tomography. [20%]

(d) An X-ray phantom consists of seven homogeneous rods. The cross-section of each rod is a unit width regular hexagon, as shown in Fig. 1. The phantom is exposed to X-rays from different directions. Let $Q = -\ln(I/I_0)$, where I_0 is the X-ray intensity incident on the phantom and I is the transmitted intensity. Q is recorded for many paths through the phantom. The values recorded for a small number of these paths are shown in Fig. 1.

(i) Describe the additive algebraic reconstruction technique (AART) and the multiplicative algebraic reconstruction technique (MART), as applied to X-ray computed tomography. [20%]

(ii) Show the start of the process by which MART can be used to calculate the X-ray linear attenuation coefficients of the rods in Fig. 1. Start by assuming that all the rods have an attenuation coefficient of 6 and use a relaxation factor of 1. Perform enough of the calculation to use each Q value once. Explain your working. [20%]

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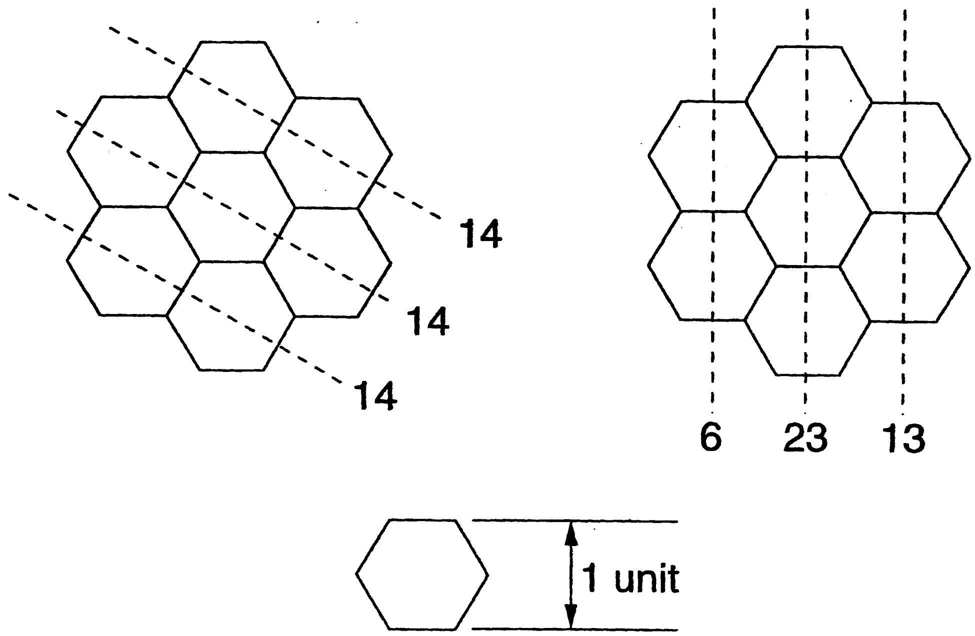

Fig. 1

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2 (a) Explain the meanings of the terms *speckle* and *specular reflection* in the context of medical ultrasound imaging. [20%]

(b) Explain how and why the performance of an ultrasound probe with a transmit frequency of 3.5 MHz differs from that of a probe with a transmit frequency of 10 MHz. [20%]

(c) Describe the signal processing necessary to convert a single line of radio frequency ultrasound data into one line of a B-scan image. [20%]

(d) Describe the continuous wave and pulsed wave Doppler ultrasound techniques and explain how they differ. [20%]

(e) Derive the linearised one-dimensional inviscid force equation for a small volume dV of fluid with density ρ, moving with variable velocity u under the influence of a variable pressure p. [20%]

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SECTION B *Curves, Surfaces and Interpolation*

Answer not more than one question from this section

3 (a) The data in Fig. 2 is a small section from a medical image. A clinician wants to work out the perimeter of a small feature comprising all data with values of 10 and above. The intention is to threshold the image at 10, then use contour following to extract the perimeter of the object.

(i) Describe two ambiguities which affect the contour following process in general, and the possible options to resolve them. [20%]

(ii) The contour following process is started by the clinician clicking on a pixel within the feature. Following this, the algorithm:

- Looks right until an edge is found
- Follows the edge, keeping to the pixel boundaries
- Stops when the first edge is found again

Sketch the result of this process, for initial clicks at the locations X and Y marked on Fig. 2, using both a 4-connected and an 8-connected contour following algorithm (copies of Fig. 2 are available on a separate sheet for you to hand in with your solution — use the top four copies for your four sketches). What is the perimeter estimate in each case? [30%]

(iii) How could this algorithm be improved to give more consistent results? [10%]

(b) In order to display the small feature more clearly, a magnified image is created with 100 times the resolution in each dimension, using either nearest-neighbour, bi-linear, or B-spline approximation.

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(i) Describe each approach, and discuss their relative advantages and disadvantages. [20%]

(ii) Thresholding and contour following is performed on the magnified image, using each of the approximation schemes mentioned above. Sketch the form of the result in each case (use the bottom three copies of Fig. 2 provided on the separate sheet). [20%]

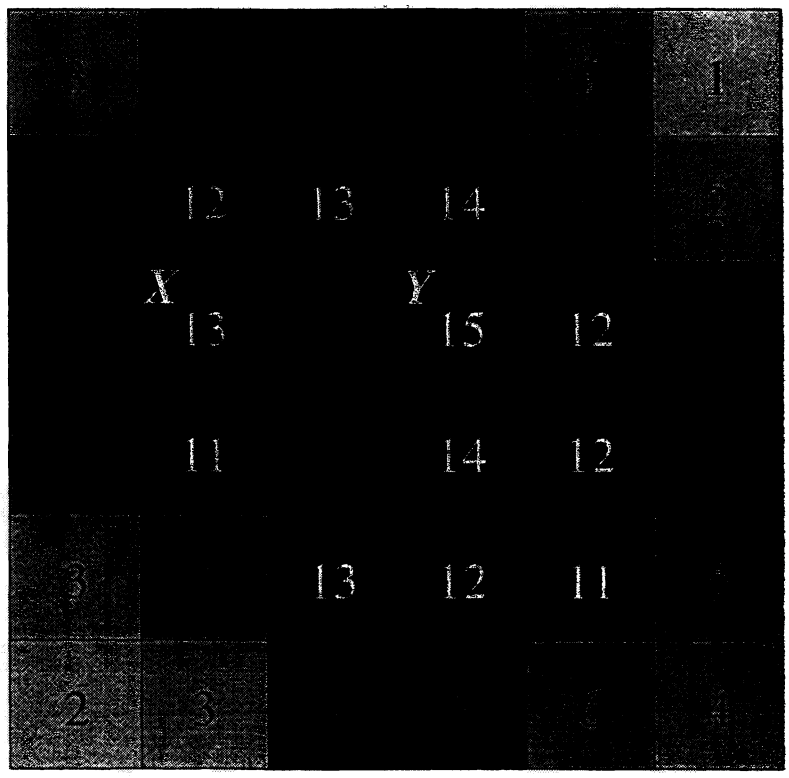

Fig. 2

4 (a) State four sources of error when using laser stripe scanning to digitise the surface of an object, and indicate what can be done, if anything, to reduce the error in each case. [20%]

(b) The *freehand* scanner of Fig. 3 is to be used to scan the surface of Fig. 4. The camera is mounted at 17° to the direction of laser light, such that the focal point is in line with the laser source, and at a distance of 0.3 m from it. The camera has focal length 5 cm, and the image extends 0.5 cm from the centre of the image plane.

(i) What is the range of distances Z from the laser source, measured along the direction of laser light, at which a surface can be correctly scanned? [20%]

(ii) Assuming the scanner is held in the optimal location and orientation, what is the maximum depth into the surface hole which can be correctly scanned? [20%]

(iii) The camera image has 128 pixels spanning the distance x in Fig. 3. Assuming the operator moves the scanner to achieve the best possible resolution at all times, what is the variation in depth resolution between the top of the hole and the depth found in part (ii)? [20%]

(c) Outline the processing steps required to generate a polygon mesh from the digitised surface data. The mesh should be suitable for real time display and manipulation. [20%]

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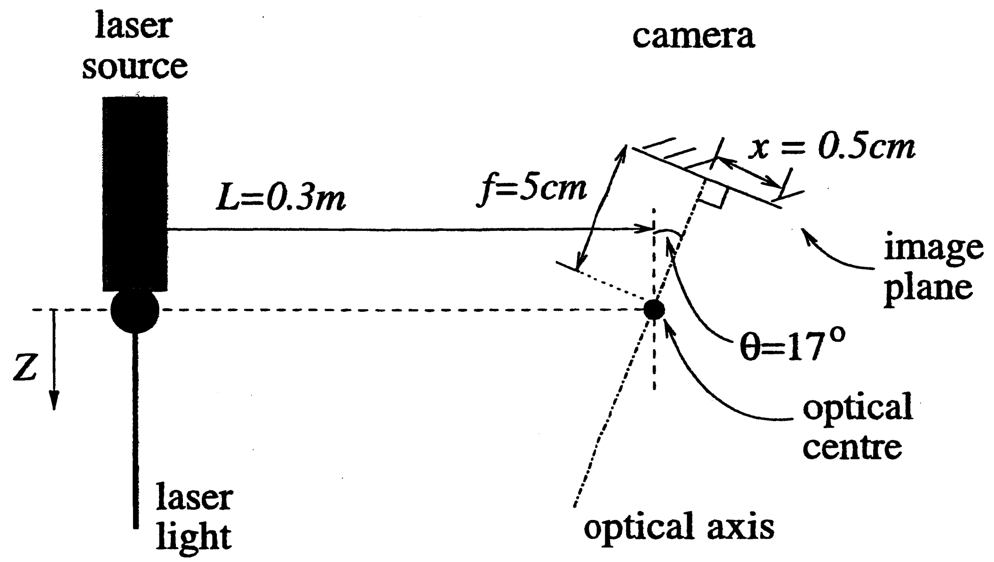

Fig. 3

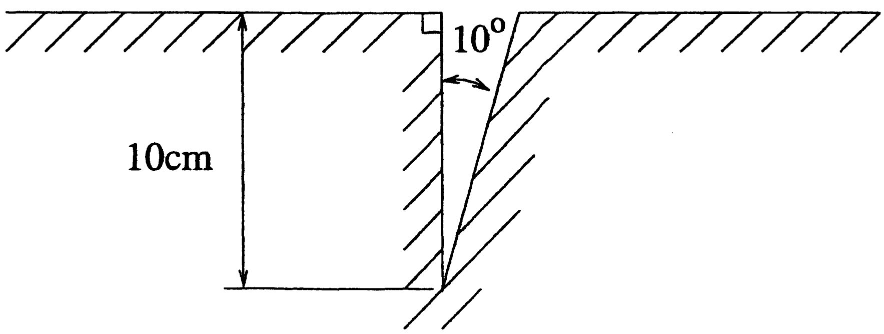

Fig. 4

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SECTION C *3D Graphical Rendering*

*Answer not more than **one** question from this section*

5 (a) Briefly describe what is meant by *volume rendering.* [20%]

(b) Figure 5 shows a volume rendering of a $n \times n \times n$ voxel array. Intensity and opacity attributes are accumulated along each ray according to

$$\begin{aligned} I_\lambda^{\text{out}} &= I_\lambda^{\text{in}} + c_\lambda \alpha \left(1 - \alpha^{\text{in}}\right) , \\ \alpha^{\text{out}} &= \alpha^{\text{in}} + \alpha \left(1 - \alpha^{\text{in}}\right) . \end{aligned}$$

(i) Explain the meanings of the various terms. [10%]

(ii) What are the initial values of I_λ^{in} and α^{in}? What happens if a voxel has opacity $\alpha = 0$ or $\alpha = 1$? [15%]

(iii) Derive an expression for α which ensures that each voxel makes an equal contribution to the rendering (which is then just a simple average of the colours accumulated along each ray). Express your answer in terms of i, the index of the voxel along the ray, as shown in Fig. 5. [25%]

(c) Figure 6 shows an alternative rendering, in which consecutive voxel planes are shifted to the left by a fraction f of a voxel. $f = \frac{1}{4}$ in the example in Fig. 6.

(i) Show that this is equivalent to rendering the voxel array rotated through an angle θ about an axis normal to the page. Derive an expression for f in terms of θ. [20%]

(ii) Note that the rays no longer pass through the voxel centres. How does this affect the rendering algorithm? [10%]

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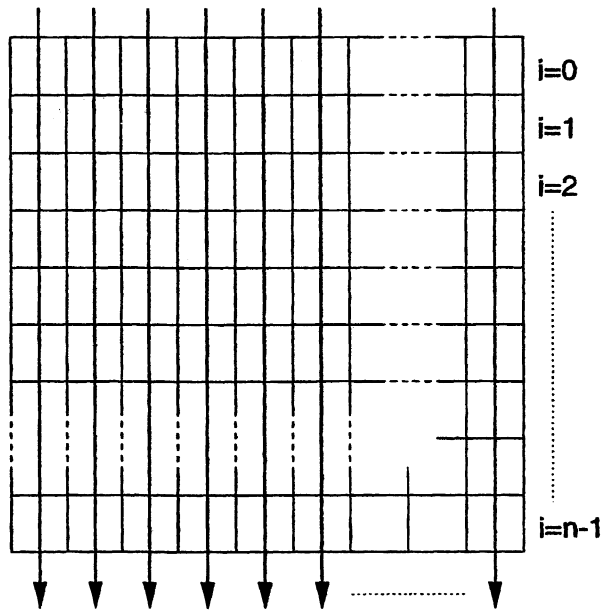

Fig. 5

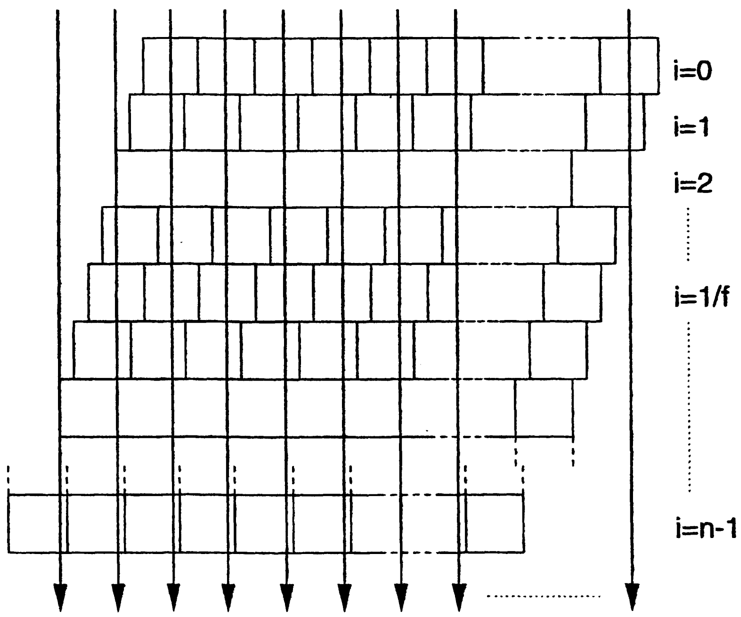

Fig. 6

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6 (a) Explain the purpose of a *view volume* in 3D surface rendering. [20%]

(b) Consider the perspective projection view volume in Fig. 7. When expressed in 3D screen coordinates (x_s, y_s, z_s), points within the view volume satisfy

$$-1 \leq x_s \leq 1\,, \quad -1 \leq y_s \leq 1\,, \quad 0 \leq z_s \leq 1\,.$$

Derive equations relating the 3D screen coordinates to the view coordinates. [20%]

(c) Now consider rendering a scene comprising just the two infinite planes $x_v = 1$ and $y_v = 1$, the former coloured red and the latter blue. The view volume has parameters $x_{\max} = y_{\max} = 1$, $n = d$ and $f = kd$, where k is large.

(i) Derive the equation of the line of intersection of the planes in 3D screen coordinates. [20%]

(ii) Hence sketch the appearance of the planes in the rendered image. Assume a very fine pixel array and perfect hidden surface removal. [20%]

(iii) Repeat (ii) for a crude, 2-bit hardware z-buffer. State any assumptions you make. [20%]

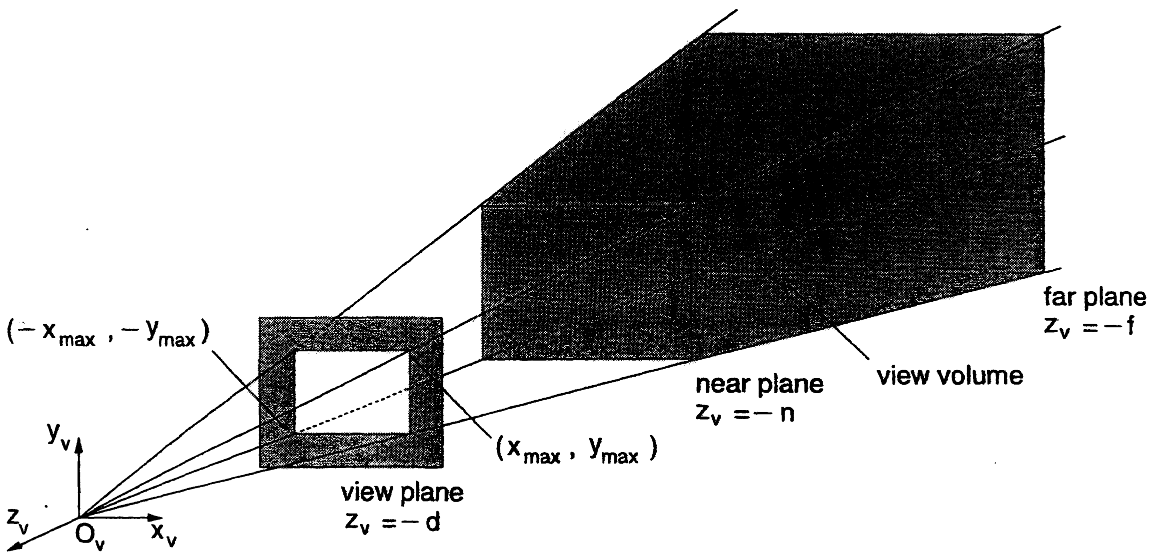

Fig. 7

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Engineering Tripos Part IIB

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To be handed in with your solution to Question 3

3	8	9	7	3	1
8	12	13	14	8	2
9	X 13	9	Y 15	12	7
8	11	8	14	12	8
3	7	13	12	11	5
2	3	9	9	6	4

4-connected start at X

3	8	9	7	3	1
8	12	13	14	8	2
9	X 13	9	Y 15	12	7
8	11	8	14	12	8
3	7	13	12	11	5
2	3	9	9	6	4

8-connected start at X

3	8	9	7	3	1
8	12	13	14	8	2
9	X 13	9	Y 15	12	7
8	11	8	14	12	8
3	7	13	12	11	5
2	3	9	9	6	4

4-connected start at Y

3	8	9	7	3	1
8	12	13	14	8	2
9	X 13	9	Y 15	12	7
8	11	8	14	12	8
3	7	13	12	11	5
2	3	9	9	6	4

8-connected start at Y

3	8	9	7	3	1
8	12	13	14	8	2
9	X 13	9	Y 15	12	7
8	11	8	14	12	8
3	7	13	12	11	5
2	3	9	9	6	4

nearest neighbour

3	8	9	7	3	1
8	12	13	14	8	2
9	X 13	9	Y 15	12	7
8	11	8	14	12	8
3	7	13	12	11	5
2	3	9	9	6	4

bi-linear

3	8	9	7	3	1
8	12	13	14	8	2
9	X 13	9	Y 15	12	7
8	11	8	14	12	8
3	7	13	12	11	5
2	3	9	9	6	4

B-spline