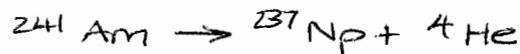


Q1.

(a)(i) The decay reaction is



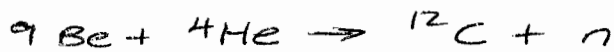
$$\begin{aligned}\Delta u &= 241.05682 - 237.04817 - 4 \cdot 0.00260 \\ &= 6.05 \times 10^{-3} \text{ u}\end{aligned}$$

$$1 \text{ u} \equiv 931.5 \text{ MeV}$$

$$\therefore \text{energy of } \alpha \text{ particle is } \underline{\underline{5.636 \text{ MeV}}}$$

[10%]

(ii) The source reaction is



$$\begin{aligned}\Delta u &= 9 \cdot 01219 + 4 \cdot 00260 - 12 \cdot 00000 - 1 \cdot 00867 \\ &= 6.12 \times 10^{-3} \text{ u} \\ &= 5.701 \text{ MeV}\end{aligned}$$

Assuming neutron receives all this plus all the energy of the α particle, maximum neutron

$$\text{energy} = \underline{\underline{11.337 \text{ MeV}}}$$

[15%]

(b) Number of ${}^{241}\text{Am}$ atoms present

$$\begin{aligned}N &= N_A \frac{m}{M} = 6.022 \times 10^{23} \times \frac{0.01}{0.241} \\ &= 2.50 \times 10^{22}\end{aligned}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{432 \times 365 \times 24 \times 3600} = 5.088 \times 10^{-11} \text{ s}^{-1}$$

$$\begin{aligned}\therefore {}^{241}\text{Am} \text{ activity} &= \lambda N = 2.50 \times 10^{22} \times 5.088 \times 10^{-11} \\ &= 1.272 \times 10^{12} \text{ Bq}\end{aligned}$$

$$\begin{aligned}\therefore \text{Neutron source rate} &= 0.5 \times 1.272 \times 10^{12} \\ S &= 6.36 \times 10^{11} \text{ n s}^{-1}\end{aligned}$$

$$\therefore \text{Flux at 1 m} = \frac{S}{4\pi} = \underline{\underline{5.06 \times 10^{10} \text{ n m}^{-2} \text{ s}^{-1}}}$$

[30%]

(c) For one-dimensional diffusion

$$\frac{d^2\phi}{dx^2} - \frac{\phi}{L^2} = 0$$

Q1 (cont.)

By inspection the solution is

$$\phi = A e^{x/L} + B e^{-x/L}$$

In a non-multiplying medium ϕ physically cannot increase indefinitely $\Rightarrow A = 0$

$$\therefore \phi = B e^{-x/L}$$

$$\text{BC } \phi = \phi_0 \text{ at } x = 0 \Rightarrow B = \phi_0$$

$$\therefore \phi = 0.01 \phi_0 \text{ when } e^{-x/L} = 0.01$$

$$\therefore \frac{x}{L} = \ln 100$$

$$\therefore \text{If } L = 0.12 \text{ m}$$

$$\underline{\underline{x = 0.55 \text{ m}}}$$

A lead shield more than 0.5 m thick is going to be enormously heavy. Lead is not a good shield against neutrons, though it is very effective against other forms of radiation. [25%]

- (d) To make an effective neutron shield you require
- (i) a moderator to slow the neutrons to energies where they are more readily captured (water, graphite or a polymer, eg. polythene, are possible choices depending on the application)
 - (ii) a material with a high neutron capture cross-section for thermal neutrons (eg. cadmium, boron, indium) - the same materials used in control rods.
 - (iii) something to shield against the γ 's arising from the neutron capture reactions - lead would be suitable for this.

[20%]

Q2.

- (a) I = Iodine-135 population
 X = Xenon-135 population
 f_i = I-135 yield fraction from fission
 Σ_f = macroscopic fission cross-section
 ϕ = neutron flux
 λ_i = I-135 decay constant
 λ_x = X-135 decay constant
 f_x = X-135 yield fraction from fission
 σ = X-135 microscopic capture cross-section
 t = time

[15%]

- (b) In steady state $\frac{dI}{dt} = 0$, so the steady-state I-135 population

$$I_0 = \frac{f_i \Sigma_f \phi}{\lambda_i}$$

and the X-135 population

$$\begin{aligned} X_0 &= \frac{f_x \Sigma_f \phi + \lambda_i I_0}{\lambda_x + \sigma \phi} \\ &= \frac{(f_i + f_x) \Sigma_f \phi}{\lambda_x + \sigma \phi} \end{aligned}$$

The poison effect of X-135 is given by

$$\begin{aligned} \rho_x &= -\frac{\sigma X_0}{\nu \Sigma_f} = -\frac{\sigma (f_i + f_x) \phi}{\nu (\lambda_x + \sigma \phi)} \\ &= \frac{-2.75 \times 10^{-22} (0.061 + 0.003) 15 \times 10^{16}}{2.4 (2.027 \times 10^{-5} + 2.75 \times 10^{-22} \times 15 \times 10^{16})} \\ &= \underline{\underline{-0.01788}} \end{aligned}$$

[20%]

- (c) At shutdown $\phi = 0$

$$\therefore \frac{dI}{dt} = -\lambda_i I$$

$$\therefore I = I_0 e^{-\lambda_i t} \text{ by inspection}$$

Q2 (cont.)

$$\frac{dX}{dt} + \lambda_x X = \lambda_i I = \lambda_i I_0 e^{-\lambda_i t}$$

PI $X_p = A e^{-\lambda_i t}$ by inspection

$$\therefore A = \frac{\lambda_i I_0}{\lambda_x - \lambda_i}$$

CF $X_c = B e^{-\lambda_x t}$ by inspection

$$\therefore \text{GS } X = \frac{\lambda_i I_0}{\lambda_x - \lambda_i} e^{-\lambda_i t} + B e^{-\lambda_x t}$$

BC $X = X_0$ at $t = 0 \Rightarrow B = X_0 - \frac{\lambda_i I_0}{\lambda_x - \lambda_i}$

$$\therefore X = \frac{\lambda_i I_0}{\lambda_x - \lambda_i} e^{-\lambda_i t} + \left(X_0 - \frac{\lambda_i I_0}{\lambda_x - \lambda_i} \right) e^{-\lambda_x t}$$

Substituting for I_0 and X_0

$$X = \frac{\delta_i \Sigma_f \phi}{\lambda_x - \lambda_i} e^{-\lambda_i t} + \left(\frac{(\delta_i + \delta_x) \Sigma_f \phi}{\lambda_x + \sigma \phi} - \frac{\delta_i \Sigma_f \phi}{\lambda_x - \lambda_i} \right) e^{-\lambda_x t}$$

$$\rho_x = - \frac{\sigma X}{v \Sigma_f}$$

$$\therefore \rho = - \left\{ \frac{\sigma \delta_i \phi}{v(\lambda_x - \lambda_i)} e^{-\lambda_i t} + \left[\frac{\sigma(\delta_i + \delta_x) \phi}{v(\lambda_x + \sigma \phi)} - \frac{\sigma \delta_i \phi}{v(\lambda_x - \lambda_i)} \right] e^{-\lambda_x t} \right\}$$

Substituting values

$$\rho = - \left\{ -0.12378 e^{-\lambda_i t} + 0.14166 e^{-\lambda_x t} \right\}$$

At $t = 2$ hours = 7200 s

$$\rho = -0.02178$$

At $t = 3$ hours = 10800 s

$$\rho = -0.02306$$

With 25% additional reactivity the poison effect that can be overcome is $1.25 \times (-0.01788) = -0.02235$
← from (b)

\therefore Restart is possible after 2 hours but not after 3

[65%]

Q3.

- (a) Magnox and CANDU reactors are refuelled on-line. AGRs were designed to be refuelled on-line but in practice are not, certainly not at full power. PWRs and BWRs are refuelled off-line as are Fast Breeders.

On-line refuelling increases availability (no shutdowns for refuelling) and also increases fuel utilisation (burnup). The main disadvantage is the increased risk of accidents. [20%]

- (b) Using the partial reactivity model and taking the reactivity to vary as $\rho_0(1 - \tau/T_1)$, the end-of-cycle condition for the first cycle is
- $$\rho_0(1 - \tau_1/T_1) = 0 \quad \text{as all batches are the same}$$

$$\therefore \underline{\tau_1 = T_1} \quad \text{the length of cycle 1}$$

The end-of-cycle condition for the second cycle is

$$\frac{1}{M} \rho_0(1 - \tau_2/T_1) + \frac{M-1}{M} \rho_0(1 - \frac{\tau_1 + \tau_2}{T_1}) = 0$$

fresh batch ↗
↖ retained batches

$$\therefore 1 - \frac{\tau_2}{T_1} + (M-1) \left(\underbrace{1 - \frac{\tau_1}{T_1}}_{\text{as } \tau_1 = T_1} - \frac{\tau_2}{T_1} \right) = 0$$

$$\therefore M \frac{\tau_2}{T_1} = 1 \quad \Rightarrow \quad \underline{\underline{\tau_2 = \frac{T_1}{M} = \frac{T_1}{M}}}$$

[20%]

- (c) The end-of-cycle condition for a general cycle is (taking $M = 3$)

Q3 (cont.)

$$\frac{1}{3}\rho_0 \left(1 - \frac{\tau_n}{T_1}\right) + \frac{1}{3}\rho_0 \left(1 - \frac{\tau_n + \tau_{n-1}}{T_1}\right) + \frac{1}{3}\rho_0 \left(1 - \frac{\tau_n + \tau_{n-1} + \tau_{n-2}}{T_1}\right) = 0$$

↑
freshhest batch
↑
oldest batch

$$\therefore T_1 - \tau_n + T_1 - \tau_n - \tau_{n-1} + T_1 - \tau_n - \tau_{n-1} - \tau_{n-2} = 0$$

$$\therefore 3T_1 - 3\tau_n - 2\tau_{n-1} - \tau_{n-2} = 0$$

$$\therefore \underline{\underline{\tau_n = T_1 - \frac{2}{3}\tau_{n-1} - \frac{1}{3}\tau_{n-2}}}}$$

$$\tau_1 = 18 \text{ months (given) } (= T_1)$$

$$\tau_2 = \tau_1/3 = 6 \text{ months (using result from (b))}$$

$$\therefore \tau_3 = T_1 - \frac{2}{3}\tau_2 - \frac{1}{3}\tau_1 = 18 - \frac{2}{3} \times 6 - \frac{1}{3} \times 18 = \underline{\underline{8 \text{ months}}}$$

$$\tau_4 = T_1 - \frac{2}{3}\tau_3 - \frac{1}{3}\tau_2 = 18 - \frac{2}{3} \times 8 - \frac{1}{3} \times 6 = \underline{\underline{10\frac{2}{3} \text{ months}}}$$

$$\tau_5 = T_1 - \frac{2}{3}\tau_4 - \frac{1}{3}\tau_3 = 18 - \frac{2}{3} \times 10\frac{2}{3} - \frac{1}{3} \times 8 = \underline{\underline{8\frac{2}{9} \text{ months}}}$$

From 4A1 datasheet the equilibrium cycle length is given by $\frac{\tau_{\infty}}{T_1} = \frac{2}{M+1}$

$$\therefore \tau_{\infty} = \frac{1}{2}T_1 \text{ for } M=3 = \underline{\underline{9 \text{ months}}}$$

[30%]

(d) Fuel utilisation $B_M = M\tau_{\infty} = \frac{2M}{M+1}T_1$

$$\text{For } M=3 \quad B_3 = 1.5T_1$$

For on-line refuelling M = the number of fuel channels which is typically $\gg 100$

$$\text{For large } M \quad B_M \rightarrow B_{\infty} = 2T_1$$

$$\therefore \frac{B_3}{B_{\infty}} = \frac{1.5T_1}{2T_1} = \underline{\underline{0.75}} \quad [15\%]$$

Q3 (cont.)

(e) Equilibrium operation in terms of cycle length can be achieved immediately by loading the core initially with M different batches the reactivities of which match the reactivities of the batches at the start of a cycle in equilibrium operation, i.e. one once burnt, one twice burnt etc. and one fresh. Different reactivities are achieved by using different enrichments. [15%]

Q4

(a)

Reprocessing starts with the shearing of the fuel pins to release the spent fuel which is then dissolved in hot concentrated nitric acid. Virtually all the spent fuel is dissolved in this process. The solution is then clarified by centrifugation and passed to the first extraction column. Here it is contacted with a solvent tri-butyl phosphate dissolved in odourless kerosene (TBP/OK) and the plutonium and uranium pass into the solvent phase leaving the fission products in the aqueous phase. The two phases separate and the fission products go to evaporation, vitrification and long term storage. The organic phase containing the Pu and U is then treated with a reducing agent which reduces the valency of the Pu. This renders the Pu soluble in dilute nitric acid so the organic stream is now mixed with dilute nitric acid in a second contactor and the Pu is separated off in the aqueous stream leaving the U still in the organic stream. The U is finally recovered from the organic stream by backwashing with very slightly acidified water.

Both Pu and U are further purified using TBP/OK washing and back washing, the Pu and U are usually finally recovered as oxides.

Waste arises at all stages of the process. The remains of the fuel pins are encapsulated in cement, the dissolver off gas is scrubbed to remove nitrous fumes, iodine and C14. Some krypton is discharged to the atmosphere. The fission product stream usually referred to as highly active liquor (HAL) is the most dangerous of all the waste arising, but its volume is very small. It is evaporated to reduce volume then encapsulated in a glass matrix (vitrification) and placed in stainless steel containers for long-term storage.

Wastes also arise from the purification of the U and Pu products; these are referred to as medium active liquors (MAL). They are also evaporated and mixed with the HAL for vitrification. Low active waste streams arising from a number of sources are subject to chemical treatment before discharge to the environment. There are also a number of low level solid waste arising which go to the controlled landfill at Drigg. [30%]

(b)

Reprocessing is essential for the old style magnox fuel because it is not suitable for long-term storage. PWR and AGR fuel can be stored safely, but there are still political problems finding suitable sites. The decisions by the UK and France to reprocess the more modern fuels were taken at a time of great energy shortage and the plan at that time was to recover the Pu to fuel fast reactors and also use the recovered U as a breeding blanket. The only real use for the recovered Pu at the moment is in the form of mixed oxide fuel (MOX) and with increasing energy prices reprocessing could again become attractive. The main down side of reprocessing is that, although it separates the various wastes, it does result in an overall increase in volume and there are some discharges to the environment. There is also the proliferation risk and the question of what to do with the recovered Pu, though in practice it is of little use for weapons as it is largely Pu240 and Pu241 whereas weapons grade material is largely Pu239. Thus, whilst reprocessing is not really financially viable at the moment, it is quite likely that it will be in the not-too-distant future. [20%]

(c)

$$F = P + W$$

$$X_F F = x_P P + x_W W$$

$$\begin{aligned} \Rightarrow F &= 100 + W \\ \Rightarrow 0.007F &= 3.5 + 0.003W \\ \Rightarrow 0.003F &= 0.3 + 0.003W \\ \Rightarrow 0.004F &= 3.2 \\ \Rightarrow \mathbf{F} &= \mathbf{800 \text{ tonnes}} \end{aligned}$$

$$\Rightarrow W = 700 \text{ tonnes}$$

$$\begin{aligned} \text{SWU} &= P(-\ln x_P) + W(-\ln x_W) - F(-\ln x_F) \\ &= 100(-\ln 0.035) + 700(-\ln 0.003) - 800(-\ln 0.007) \\ &\quad 335.2 \quad + \quad 4066.4 \quad - \quad 3969.5 \\ &= \mathbf{432.1 \text{ tonnes SWU}} \end{aligned}$$

[20%]

(d)

Recovered uranium = 96 tonnes
 Reprocessing loss 1% = 0.96 tonnes
 Available uranium = 95.04 tonnes

Mass balance on recovered uranium

$$\begin{aligned} 95.04 &= P_r + W_r \\ 0.008 \times 95.04 &= 0.035P_r + 0.003W_r \end{aligned}$$

$$\begin{aligned} \Rightarrow 0.7603 &= 0.035P_r + 0.003W_r \\ \Rightarrow 0.2851 &= 0.003P_r + 0.003W_r \\ \Rightarrow 0.4752 &= 0.032P_r \\ \Rightarrow \text{Recovered product at 3.5\% U235} &= 14.85 \text{ tonnes} \\ \Rightarrow \text{Recovered tails} &= 80.19 \text{ tonnes} \end{aligned}$$

$$\Rightarrow \text{Product from fresh feed} = 100 - 14.85 = 85.15 \text{ tonnes}$$

$$\Rightarrow \text{Amount of fresh feed} = 800 \times 0.8515 = 681.2 \text{ tonnes}$$

$$\Rightarrow \mathbf{\text{Saving} = 118.8 \text{ tonnes}}$$

SWU for reprocessed feed is given by:

$$\begin{aligned} &14.85(-\ln 0.035) + 80.19(-\ln 0.003) - 95.04(-\ln 0.008) \\ &= 49.78 \quad + \quad 465.84 \quad - \quad 458.88 \\ &= 56.74 \text{ tonnes SWU} \end{aligned}$$

$$\text{SWU for fresh feed} = 0.8515 \times 432.1 = 367.9 \text{ tonnes SWU}$$

$$\Rightarrow \text{Total SWU with reprocessing} = 367.9 + 56.7 = 424.6 \text{ tonnes SWU}$$

$$\Rightarrow \mathbf{\text{Saving} = 432.1 - 424.6 = 7.5 \text{ tonnes SWU}}$$

[20%]

(e)

The recovered plutonium could be used in a mixed oxide fuel or perhaps in a fast breeder reactor. It is not useable in nuclear weapons (see (b)).

[10%]