2006 4A3 Turbomachinery I

4A3 Dr T Hynes

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1\. (a) Euler's Work equation $\Rightarrow \quad \Delta h_o = U(V_{\theta 2} - V_{\theta 3})$

For a repeating stage $\quad V_{x2} = V_{x3} = V_{x1} \quad \Rightarrow \quad \Delta h_o = U V_x (\tan\alpha_2 - \tan\alpha_1)$

$\alpha_3 = \alpha_1$

i.e. $\quad \psi = \dfrac{\Delta h_o}{U^2} = \varphi(\tan\alpha_2 - \tan\alpha_1) \quad \Rightarrow \quad \tan\alpha_2 = \tan\alpha_1 + \dfrac{\psi}{\varphi}$

$$\text{Reaction } \Lambda = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = 1 - \frac{\Delta h_{stator}}{\Delta h_{stage}} = 1 - \frac{\left(\frac{V_2^2}{2} - \frac{V_3^2}{2}\right)}{\Delta h_{o\,stage}}$$

($h_{o2} = h_{o3}$ & $\Delta h_{o\,stage} = \Delta h_{stage}$ for repeating stage)

$$= 1 - \frac{V_{\theta 2}^2 - V_{\theta 3}^2}{2\Delta h_{o\,stage}} \quad \text{since } V_{x2} = V_{x3}$$

$$= 1 - \frac{V_x^2}{2\Delta h_o}(\tan^2\alpha_2 - \tan^2\alpha_1) = 1 - \frac{\varphi^2}{2\psi}(\tan^2\alpha_2 - \tan^2\alpha_1)$$

$$\text{i.e. } \Lambda = 1 - \frac{\varphi^2}{2\psi}\left[\frac{\psi^2}{\varphi^2} + \frac{2\psi}{\varphi}\tan\alpha_1 + \tan^2\alpha_1 - \tan^2\alpha_1\right]$$

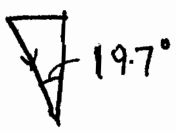
$$= 1 - \frac{\psi}{2} - \varphi\tan\alpha_1$$

$$\Rightarrow \quad \underline{\psi = 2(1 - \Lambda - \varphi\tan\alpha_1)}$$

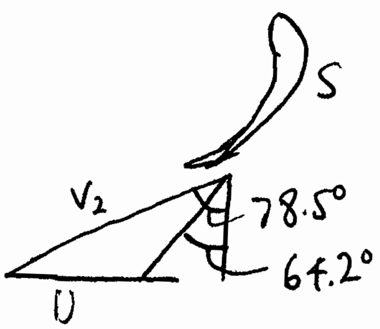
[30%]

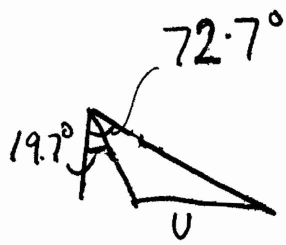
(b) For 20% reaction, $\varphi = .35$ & $\psi = 1.85$

$$\Rightarrow \tan\alpha_1 = \frac{1 - \Lambda - \psi/2}{\varphi} \quad \Rightarrow \quad \underline{\alpha_1 = -19.7^\circ}$$



$\tan\alpha_2 = 4.93 \quad \Rightarrow \quad \alpha_2 = 78.5^\circ$





[25%]

(c) $$\eta_{tt} = 1 - 0.04\,\frac{V_2^2 + W_3^2}{\Delta h_0} = 1 - 0.04\,\frac{\varphi^2(\tan^2\alpha_2 + \tan^2\alpha_3^{rel} + 2)}{4}$$

$$= 90.3\,\%$$

Stage Exit K.E. $= \frac{1}{2} V_3^2$

$$\therefore\ \eta_{ts} = 1 - 0.04\,\frac{V_2^2 + W_3^2}{\Delta h_0} - \frac{\frac{1}{2} V_3^2}{\Delta h_0} = .903 - .037 \rightarrow 86.6\%.$$

$$\left[\frac{\frac{1}{2} V_3^2}{\Delta h_0} = \frac{\varphi^2(\tan^2\alpha_3 + 1)}{24} = 0.037\right]$$

[20%]

(d) Low reaction stages are popular with steam turbine manufacturers because

(i) high stage loading obviously

but also (ii) low pressure drop across rotor $\Rightarrow$ easier tip sealing & less axial thrust

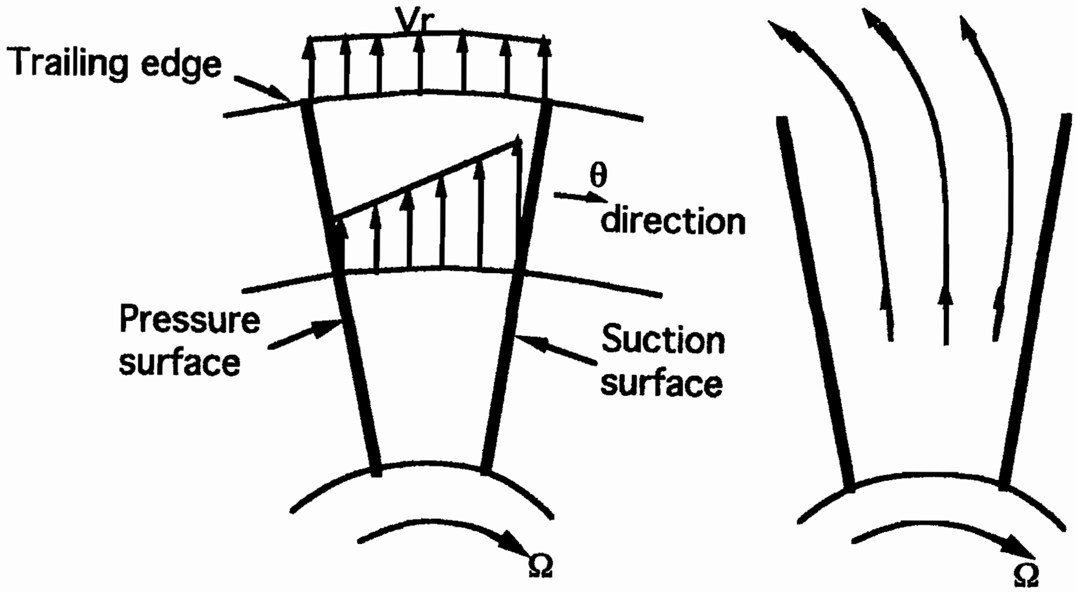
(iii) high camber blades $\Rightarrow$ stronger in bending

[This has to be balanced by greater turning in the blades leading to higher b.layer losses].

[25%]

2.

(a)



Flow moving radially outward is increasing its angular momentum. There is thus a cross-passage pressure gradient to balance this, with attendant velocity profile. [Sometimes referred to as Coriolis loading]. As the trailing edge is approached, this blade loading drops to zero (& velocity profile becomes uniform) and the angular momentum of the fluid remains constant for further radius change.

[20%]

(b) $$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma \eta_p}} = 1.401 \Rightarrow T_{02} = 420.4\ K$$

[15%]

(c) $c_p(T_{02} - T_{01}) = \Delta(UV_\theta) = U_2 V_{\theta 2}$ since $V_{\theta 1} = 0$

Slip factor $\sigma = \dfrac{V_\theta}{V_\theta^{ideal}}$ $\quad V_\theta^{ideal} = U + V_r \tan\beta \quad \beta = -40°$

$= U[1 - .3 \tan 40]$

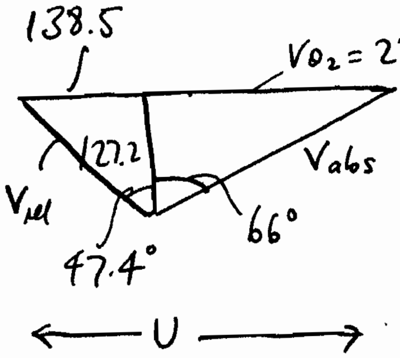
$$c_p \Delta T_0 = \sigma U^2 [1 - .3 \tan 40°]$$

$$\Rightarrow \quad U = \sqrt{\frac{1005 \times 120.4}{.9(1 - .3\tan 40°)}} = 424 \text{ m/s}$$

$$V_{\theta 2} = \sigma(1 - .3 \tan 40°)\, U = 285.5 \text{ m/s}$$

$$V_{r2} = 127.2 \text{ m/s}$$

$$\Rightarrow V_{abs} = 312.6 \text{ m/s}$$



$$V_{rel} = 188 \text{ m/s}$$

[25%]

(d) At exit $\dot{m} = \rho_2 V_{r2} A_2$ & $p_2 = \dfrac{p_{02}}{\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}} = 1.952 \times 10^5$ Pa

$$\left.\begin{aligned} T_2 &= T_{02} - \frac{V_{abs}^2}{2c_p} \\ &= 420.4 - \frac{312.6^2}{2 \times 1005} \end{aligned}\right\} \Rightarrow M_2 = \frac{V_{abs}}{\sqrt{\gamma R T_2}} = .81$$

$$= 372 \text{ K}$$

$$\therefore \rho_2 = \frac{1.952 \times 10^5}{287.1 \times 372} = 1.828 \text{ kg/m}^3$$

$$\therefore A_2 = 2.67 \times 10^{-3} \text{ m}^3 = 2\pi r_2 h \quad \text{giving} \quad \underline{h = 5.7 \text{ mm}}$$

At inlet $\dfrac{\dot{m}\sqrt{c_p T_{01}}}{p_{01} A_1} = f(.4) = .8056 \Rightarrow A_1 = 4.226 \times 10^{-3}$ m

$$= \pi r_t^2 [1 - h_R^2] \Rightarrow h_R = .4$$

[20%]

(e) Pressure rise across impeller greatly aided by the centrifugal pressure rise, which does not have the same deleterious effect on b-layers as the equivalent pressure rise would in an axial machine.

[20%]

3(a) $\eta_P = \dfrac{\text{work done against drag}}{\text{Change of KE of jet flow}}$

In level flight at constant speed, thrust = drag

$$\Rightarrow (\dot{m}_a + \dot{m}_f) V_j - \dot{m}_a V = \text{Drag}$$

$$\therefore \eta_P = \frac{V[(\dot{m}_a+\dot{m}_f)V_j - \dot{m}_a V]}{(\dot{m}_a+\dot{m}_f)\frac{V_j^2}{2} - \dot{m}_a\frac{V^2}{2}} = \frac{(1+F)V_j V - V^2}{\frac{1}{2}(1+F)V_j^2 - \frac{1}{2}V^2}$$

If $\dot{m}_f \ll \dot{m}_a \Rightarrow \eta_P = \dfrac{V_j V - V^2}{\frac{1}{2}V_j^2 - \frac{V^2}{2}} = \dfrac{2V}{V_j + V}$

[20%]

(b) $V = .85\sqrt{1.4 \times 287.1 \times 223.3} = 254.7$

$$p_{01} = 2.65\times10^4\left(1+.2(.85)^2\right)^{3.5} = 42500 \text{ Pa}$$

$$T_{01} = 223.3\left(1+.2(.85)^2\right) = 255.6 \text{ K}$$

$$\therefore p_{02} = 63750, \quad T_{02} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma\eta_P}} = 290.7 \text{ K}$$

Across nozzle p_{02} conserved & $p_2 = 2.65\times10^4$

$$\therefore M_J^2 = \frac{2}{\gamma-1}\left[\left(\frac{p_{02}}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = 1.425 \Rightarrow M_J = 1.194$$

So $T_2 = \dfrac{T_{02}}{1+\frac{\gamma-1}{2}M_J^2} = 226.2 \Rightarrow V_J = 360 \text{ m s}^{-1}$

$\therefore \eta_P = .829$

[35%]

(c) $\dot{m}_{fan} c_{p1}(T_{02} - T_{01}) = \dot{m}_{turbo} c_{p2}(T_{03} - T_{04})$

(i) $\therefore T_{03} - T_{04} = \dfrac{(1+10)\times 1005}{1250}(290.7 - 255.6) = 310.4$ °C

[10%]

$\Rightarrow T_{04} = 740$ K

(ii) Across turbine $\dfrac{p_{03}}{p_{04}} = \left(\dfrac{T_{03}}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}\eta_p} \Rightarrow p_{04} = 43.58\ \text{kPa}$

$$= 6.654$$

Across nozzle $\dfrac{T_J}{T_{04}} = \left(\dfrac{p_a}{p_{04}}\right)^{\frac{\gamma-1}{\gamma}} = .8915 \Rightarrow T_J = 660\ \text{K}$

$\therefore \quad V_{Jc} = 447.2$

$\therefore \quad \dfrac{V_{Jf}}{V_{Jc}} = .805$ [20%]

(iii)
$$\eta_p = \frac{V\left(\dot{m}_{jf} V_{jf} + \dot{m}_{jc} V_{jc} - (\dot{m}_f + \dot{m}_c) V\right)}{\dot{m}_{jf} \frac{V_{jf}^2}{2} + \dot{m}_{jc} \frac{V_{jc}^2}{2} - (\dot{m}_f + \dot{m}_c) \frac{V^2}{2}}$$

or, if fuel flow considered,

$$\eta_p = \frac{V\left(\dot{m}_{jf} V_{jf} + (\dot{m}_{jc} + \dot{m}_f) V_{jc} - (\dot{m}_f + \dot{m}_c) V\right)}{\dot{m}_{jf} \frac{V_{jf}^2}{2} + (\dot{m}_{jc} + \dot{m}_f) \frac{V_{jc}^2}{2} - (\dot{m}_f + \dot{m}_c) \frac{V^2}{2}}$$

[15%]

N.B. In this expression

$\dot{m}_f$ = fan jet flow

(total flow through the fan $= \dot{m}_f + \dot{m}_c$).