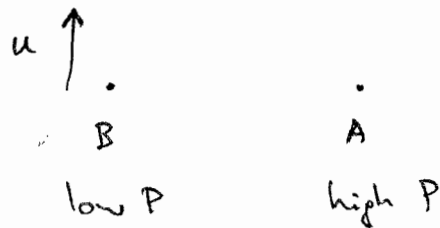


1(a) Geostrophic flow is the flow where there is a balance between Coriolis force and the pressure gradient. The velocity is normal to the pressure gradient. The governing equation for geostrophic flow  $\left(-\frac{1}{\rho} \nabla P = 2 \underline{\Omega} \times \underline{u}\right)$  (given in Data Card)

is derived by assuming that the viscous terms and inertial terms are zero in the N-S equations.

Geostrophic flow occurs far from the surface of the earth (outside the boundary layer), where the shear is small.

Pressure decreases going west (A to B)



Hence  $u$  is north.  
(southerly wind)

(b) Both anabatic and katabatic flows are caused by heat transfer from the surface of a slope to the air above. If the ground heats the air above it, the air rises up the slope  $\rightarrow$  anabatic. When the ground cools the air, the air runs down the slope  $\rightarrow$  katabatic.

Anabatic flows are unstable  $\Rightarrow$  turbulence increases

$\Rightarrow$  more mixing  $\Rightarrow$  flow gets thicker  $\Rightarrow$  slower.

In contrast, katabatic flows are stable  $\Rightarrow$  remain as a thin layer.

I would prefer an anabatic flow since the toxic gas will be more diluted due to the greater mixing.

(2)

1(c) (i) The Monin-Obukhov length is a measure of the height in the atmospheric boundary layer where buoyancy effects become important. Near the ground the velocity gradient is large and hence shear dominates. As we move higher, shear drops and buoyancy comes in.

(ii) Since M-O length is +ve, flow is stable

$$\text{Hence (from Data Card)} \quad \frac{dU}{dz} = \frac{u^*}{\kappa z} \left( 1 + 4.7 \frac{z}{L} \right) \quad (1)$$

The logarithmic profile (for neutral stability)

$$\text{gives that} \quad \frac{dU}{dz} = \frac{u^*}{\kappa z} \quad (2)$$

Hence using Eq (2) instead of Eq (1) at  $z=1\text{m}$  for  $L=10\text{m}$  will involve an error

$$\frac{u^*_{\text{Eq 2}}}{u^*_{\text{Eq 1}}} = \frac{\kappa z \frac{dU}{dz}}{\kappa z \frac{dU}{dz} \left( 1 + 4.7 \frac{L}{10} \right)} = 0.68$$

$\Rightarrow$  32% error

2 (a) Steady flow, no mean velocity, taking  $x_3$  as up gives

$$\frac{\partial}{\partial x_3} \left( \lambda \frac{\partial T}{\partial x_3} - \rho c_p \overline{u_3 \theta} \right) = 0$$

$$\Rightarrow \lambda \frac{\partial T}{\partial x_3} - \rho c_p \overline{u_3 \theta} = \text{constant}$$

At  $x_3 = 0$  (ground)  $\overline{u_3 \theta} = 0 \Rightarrow \lambda \frac{\partial T}{\partial x_3} = \text{heat flux at ground}$   
 $= -Q$

$$\Rightarrow \lambda \frac{\partial T}{\partial x_3} - \rho c_p \overline{u_3 \theta} = -Q \quad (1)$$

(b) The turbulent kinetic energy equation becomes

$$\frac{\overline{f_i u_i}}{\rho} = \varepsilon \Rightarrow \frac{\rho}{T} \overline{u_3 \theta} = \varepsilon \quad \text{in our case (2)}$$

Neglecting molecular conduction term, Eq (1)

gives that  $\rho c_p \overline{u_3 \theta} = Q \Rightarrow \underline{\underline{\varepsilon = \frac{\rho}{T} \frac{Q}{\rho c_p}}}$

(c) From  $\varepsilon = \frac{u'^3}{L}$ ,  $u'^3 = \frac{\rho Q L}{T \rho c_p}$   
 $\Rightarrow u' = \left( \frac{\rho Q L}{T \rho c_p} \right)^{1/3}$

(d) This problem now concerns the decay of turbulence assuming no production mechanism.

The turbulence kinetic energy equation becomes

$$\frac{d\left(\frac{3}{2}u'^2\right)}{dt} = -\frac{u'^3}{L}$$

$$\text{Let } u'^2 = \alpha \quad \Rightarrow \quad \frac{d\alpha}{dt} = -\frac{2}{3} \frac{\alpha^{3/2}}{L} \quad \Rightarrow \quad \alpha^{-3/2} d\alpha = -\frac{2}{3L} dt$$

$$\Rightarrow \alpha^{-1/2} = \frac{t}{3L} + C$$

$$\text{At } t=0, u'^2 = u_0'^2$$

$$\Rightarrow \frac{1}{u_0'} = \frac{t}{3L} + \frac{1}{u_0'}$$

$$\Rightarrow \frac{u_0'}{u'(t)} = 1 + \frac{t}{\frac{3L}{u_0'}}$$

$$\Rightarrow \frac{u'(t)}{u_0'} = \left(1 + \frac{t}{3T_0}\right)^{-1} \quad \text{where } T_0 = \frac{L}{u_0'} = \text{eddy timescale (or turnover time)}$$

$$\text{So, if } u'(t)/u_0' = 10\%, \quad \frac{t}{3T_0} = 9$$

i.e. after 27 initial eddy turnover times.

Using the result from (c),

$$u'/u_0' \text{ will be } 10\% \text{ at } t = 3 \cdot 9 \cdot L \left(\frac{8QL}{T_p C_p}\right)^{-1/3}$$

(a) Main constituents:

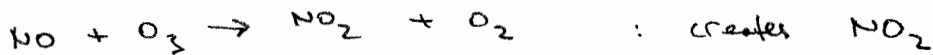
NO : primary pollutant, emitted by cars, industry, power generation

VOC : volatile organic compounds, emitted by refineries, industry

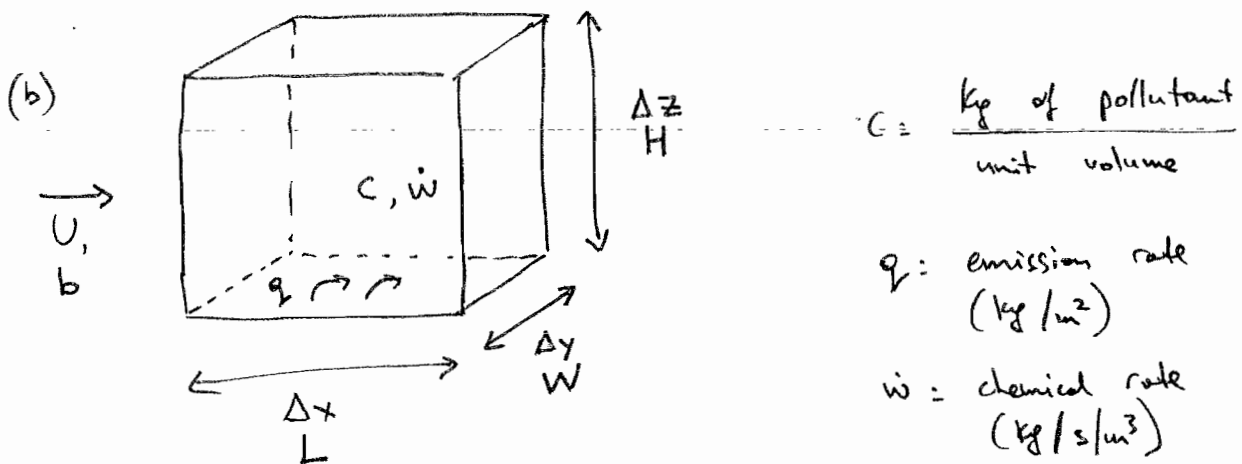
NO<sub>2</sub> : "brown gas", created from NO

O<sub>3</sub> : exists in low background concentrations, created from NO<sub>2</sub>

Main reactions:



NO also becomes acid (HNO<sub>2</sub>) ⇒ acid rain



Mass conservation:  $\frac{d}{dt} (LWH c) = b \cdot U \cdot WH - c \cdot U \cdot W \cdot H + q \cdot LW + \dot{w} WHL$

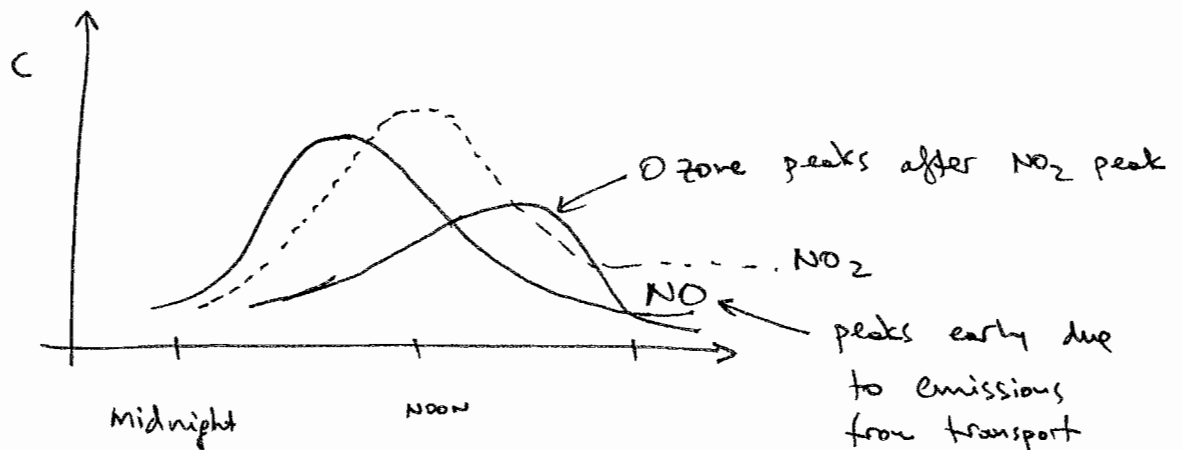
$\Rightarrow \frac{dc}{dt} = (b - c) \frac{U}{L} + \frac{q}{H} + \dot{w}$

(c) Steady state  $\Rightarrow \frac{dc}{dt} = 0$

$$\Rightarrow c = b + \frac{qL}{HU} + \frac{wL}{U}$$

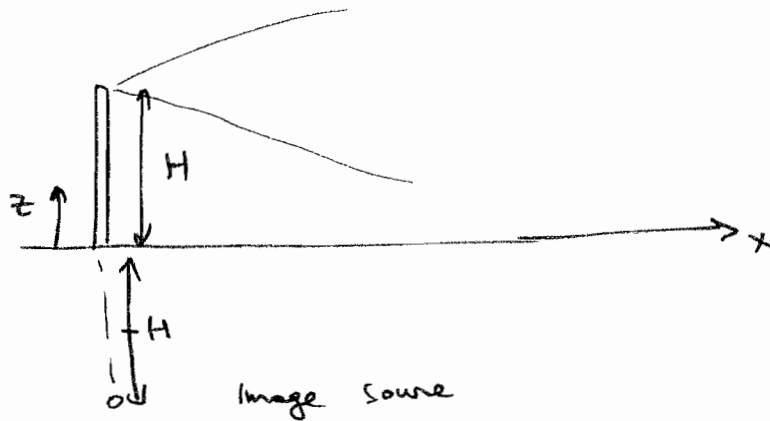
- Depending on reaction,  $w \rightarrow 0$  at night for photolysis reactions
- As day progresses,  $q$  increases & then decreases at night (due to economic activity)
- $H$  may decrease to very small values at night due to stability of atmosphere (mixing height)

$\Rightarrow$  depending on pollutant, we get a different picture of evolution during the day:



- An inert pollutant will probably accumulate at night (high  $c$  due to low  $H$ ). During the day,  $H \uparrow$  and  $U$  tends to be higher, hence  $c$  may decrease (if  $q$  does not increase!).

4 (a)



Maximum concentration on ground ( $z=0$ ) will occur at  $y=0$  (immediately downwind of the stack)

Gaussian plume equation with ground effect:

$$\phi(x, y, z) = \frac{Q}{2\pi U \sigma_y \sigma_z} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \left[ \exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right) \right]$$

At  $z=0$ ,  $y=0$ ,  $\sigma_y = \sigma_z = \alpha x$ , we get:

$$\phi(x, 0, 0) = \frac{Q}{\pi U} \frac{1}{\alpha^2 x^2} \exp\left(-\frac{H^2}{2\alpha^2 x^2}\right) \quad (1)$$

Maximum concentration will occur where  $\frac{d\phi}{dx} = 0$

$\Rightarrow$  differentiating Eq (1) w.r.t.  $x$  gives

$$\frac{Q}{\pi U} \left( \frac{2H^2}{2\alpha^2 x^3} \exp\left(-\frac{H^2}{2\alpha^2 x^2}\right) \frac{1}{\alpha^2 x^2} - \frac{2}{\alpha^2 x^3} \exp\left(-\frac{H^2}{2\alpha^2 x^2}\right) \right) = 0$$

$$\begin{aligned} (\Rightarrow) \quad \frac{2}{\alpha^2 x^3} &= \frac{1}{\alpha^2 x^2} \frac{H^2}{\alpha^2 x^2} \quad (\Rightarrow) \quad x = \frac{H}{\sqrt{2} \alpha} \\ &\quad \left( \text{OR when } \sigma = \underline{\underline{H/\sqrt{2}}} \right) \end{aligned}$$

(b) For stable to neutrally-stable atmospheres, the horizontal dispersion coefficient is usually higher because in the vertical direction the turbulent lengthscale is restricted by the surface. <sup>horizontal velocity fluctuation > vertical.</sup> For unstable atmospheres, a lot of turbulence is generated in the vertical direction (term  $\overline{f_3 u_3} / \rho$  in the turbulent kinetic energy equation) and hence diffusion in the vertical direction is greater than in horizontal.

Hence:  $\frac{\sigma_y}{\sigma_z} > 1$  for stable & neutral

$\frac{\sigma_y}{\sigma_z} < 1$  for unstable