1). By continuity the ~~Flow turning in a choked turbine cascade~~ exit flow angle of a choked turbine cascade is determined by

a).

$$\alpha_2 = \cos^{-1}\left(\frac{P_{01}^*}{P_{02}} \cdot \frac{O}{S} \cdot \frac{f(1)}{f(1.2)}\right)$$

$$= \cos^{-1}\left(\frac{1}{0.9} \cdot 0.35 \cdot \frac{1.281}{1.2432}\right)$$

$$= \cos^{-1}(0.4007) = \underline{66.38^\circ}$$

$\cos\alpha_2 = \frac{P_0^*}{P_{02}} \cdot \frac{O}{S} \cdot \frac{f(1)}{f(M_2)}$

with fixed O/S, it is expected that both P_0^*/P_{02} and $f(1)/f(M_2)$ will increase with increasing M_2 as it exceeds unity thus $\cos\alpha_2$ increases leading to $\alpha_2 \downarrow$

$\delta_2 = \kappa_2 - \alpha_2 \uparrow$.

[30%]

b).

$$\alpha_2 = \cos^{-1}\left(\frac{P_{01}^*}{P_{02}} \cdot \frac{O}{S} \cdot \frac{f(1)}{f(1.2)}\right)$$

$$= \cos^{-1}\left(\frac{1}{0.9} \cdot 0.35 \times 1.1 \times \frac{1.281}{1.2432}\right)$$

$$= \cos^{-1}(0.4408) = 63.85^\circ$$

$$\Delta\delta = 66.38^\circ - 63.85^\circ = \underline{2.53^\circ}$$

Increasing O/S by 10% while keeping exit pressure the same and throat choked means mass flow increases by 10% but tangential blade force is more or less the same. thus results in a reduction in the flow turning.

[20%]

c). For transonic turbine most turning takes place upstream of the throat and the flow is accelerating. downstream of the throat the blade turning is very small and flow would not over-accelerated.

For compressor, inlet flow is supersonic and any turning will cause flow to further accelerate, resulting high shock loss and seperation due to strong shock. The transonic compressor blade does not turn flow ~~absolute fra~~ relative to the blade but achives substantial flow turning in absolute frame of reference by slowing the flow down through shock waves. At high mach number, reversed camber is used in the fore part of the blade to pressure compress the flow, in order to reduce the Mach number upstream of the shock wave ~~to~~ reduce the shock loss as well as the risk of separation. [30%]

d). Prandtl-Meyer expansion:

$$\alpha_1 + \nu_1 = \alpha_E + \nu_E \quad , \quad 65^\circ + 8.99 = \alpha_E + 13.38$$

$$\underline{\alpha_E = 60.61^\circ}$$

Continuity:

$$f(M_1)\cos\alpha_1 = f(M_E)\left(\cos\alpha_E - \frac{t}{s}\right)$$

$$1.149 \cdot 0.4226 = 1.0573 \cdot \left(\cos 60.61^\circ - \frac{t}{s}\right)$$

$$\frac{t}{s} = -\frac{1.149 \cdot 0.4226}{1.0573} + 0.49075 = \underline{\underline{0.0315}}$$

The throat for transonic compressor blade is defined as along the expansion wave that intersects the leading edge of the neighbouring blade. As this expansion wave remains a simple wave without interacting with other waves, the flow properties along it must be constant so the assumption is rather accurate.

[25%]

2)

a).

$V_m \sin\phi \frac{\partial V_m}{\partial m}$ radial component of flow acceleration along streamline

$\frac{V_m^2}{r_m}\cos\phi$ radial component of meridional centripetal acceleration due to streamline curvature.

$-\frac{V_\theta^2}{r}$ ~~radial component~~ centripetal acceration which is due to tangential velocity and is always radial inwards

assumptions made: flow is steady, inviscid ~~and~~ axisymmetrical.

$\frac{1}{\rho}\frac{\partial p}{\partial r}$: ~~the~~ force pressure gradient in radial direction. [25%]

b). from meridional stream line curvature equation. for a cyclindrical flow path with negligible radial velocity.

$\phi \to 0$ thus $\sin\phi \to 0$, or when $dV_m/dm \to 0$.

$V_m \cdot \sin\phi \cdot \frac{dV_m}{dm}$ term can be ignored. and $V_m \to V_x$

for parallel & straight streamlines on meridional plane, $r_m \to \infty$ and $\frac{V_m^2}{r_m}\cos\phi$ ~~term~~ term can be ignored. this leads to simple radial equilibrium equation: $\frac{V_\theta^2}{r} = \frac{1}{\rho}\frac{dp}{dr}$

use $dh - T ds = \frac{dp}{\rho}$ and $h_0 = h + \frac{V^2}{2} = h + \frac{V_\theta^2}{2} + \frac{V_x^2}{2}$

$\frac{dh}{dr} - T\frac{ds}{dr} = \frac{dp}{\rho dr}$ $\quad \frac{dh_0}{dr} - \frac{d}{dr}\left(\frac{V_\theta^2}{2} + \frac{V_x^2}{2}\right) - T\frac{ds}{dr} = \frac{dp}{\rho dr} = \frac{V_\theta^2}{r}$

rearrange:

$$\underline{\frac{d}{dr}\frac{V_x^2}{2}} = \frac{dh_0}{dr} - \frac{T ds}{dr} - V_\theta\frac{dV_\theta}{dr} - \frac{V_\theta^2}{r} = \underline{\underline{\frac{dh_0}{dr} - T\frac{ds}{dr} - \frac{V_\theta}{r}\frac{d(rV_\theta)}{dr}}}$$

h_0, s, and rV_θ are conserved quantites in the gaps between the blade rows as the SRE equation applied. [25%]

c). Cylindrical flow path and $V_r \to 0$. SRE can be used. loss $\to 0$. $T\frac{ds}{dr}$ can be dropped.

$$\Rightarrow \frac{d}{dr}\left(\frac{V_x^2}{2}\right) = \frac{dh_0}{dr} - \frac{V_\theta}{r}\frac{d}{dr}(rV_\theta)$$

$\Rightarrow$ inlet flow uniform $\Rightarrow$ h_0 uniform at inlet.

$\because$ flow after stator uniform.

$$\frac{d}{dr}(rV_\theta) = 0. \Rightarrow rV_\theta = \text{const}. \Rightarrow \underline{\text{free vortex flow}}$$

exit of stage. uniform flow + $V_\theta = 0$.

$$\Rightarrow \frac{d}{dr}\frac{V_x^2}{2} = 0 + \frac{V_\theta}{r}\frac{d}{dr}(rV_\theta) = 0 \Rightarrow \frac{dh_0}{dr} = 0.\ h_{0_2} \text{ Uniform}.$$

as h_{01} is uniform. $\underline{\Delta h_0 \text{ i.e. work distribution uniform along span.}}$ [2%]

d). Very low reaction at hub will lead to high diffusion on the blade hub/suction surface corner, with enhanced secondary flow due to high loading. this potentially is to create high loss, and should be avoided.

at mid span. 50% reaction $\Rightarrow \dfrac{\Delta h\ \text{stator}}{\Delta h\ \text{stage}} = 0.5$.

$\Delta h\ \text{stator} = \frac{1}{2}V_{\theta m}^2$; $\Delta h\ \text{stage} = U_m \cdot \Delta V_{\theta m}$

$\Rightarrow U_m = V_{\theta m}$, also $\Delta h_0 = V_{\theta m}^2$

at hub. for free vortex flow. $V_{\theta_h} = V_{\theta m} \cdot \dfrac{r_m}{r_h}$.

$$\Delta h\ \text{stator} = \frac{1}{2}V_{\theta h}^2 = \frac{1}{2}V_{\theta m}^2 \cdot \left(\frac{r_m}{r_h}\right)^2$$

$$\Lambda_h = 1 - \frac{\Delta h\ \text{stator}}{\Delta h\ \text{stage}} = 1 - \frac{\frac{1}{2}V_{\theta m}^2 \cdot \left(\frac{r_m}{r_h}\right)^2}{V_{\theta m}^2} = 1 - \frac{1}{2}\left(\frac{r_m}{r_h}\right)^2$$

$$\frac{r_h}{r_t} = 0.6,\quad r_t = 2r_m - r_h, \Rightarrow \frac{r_m}{r_h} = \frac{4}{3}$$

$$\underline{\Lambda_h = 1 - \frac{1}{2}\cdot\frac{16}{9} = 1 - \frac{16}{18} \doteq 0.111}$$

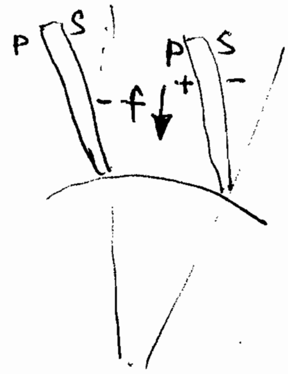
[30%]

3). a).

The reaction of a turbine stage is usually to increase from hub to tip. with the lowest at the hub and the highest at the tip. Together with the pitch variation from low to high from hub to tip. this means the tangential pressure gradient across the blade at the hub is higher than that at the tip. thus provides larger driving force for generating the secondary flow.

Blade lean (towards pressure surface) will reduce the pressure gradient in radial direction thus helps ~~to~~ increase the hub reaction that will weaken the secondary flow. also local forward sweep at the hub can shift loading to higher radial sections and reduce the leading edge loading at the hub, this will also help to weaken the secondary flow. [20%]

b). Reaction is determined by radial pressure gradient, which provide centripetal force for swirling flow. if the blade is leant, the blade force can provide part of the radial force for the radial equilibrium thus reliefs the static pressure gradient. As the pressure gradient is positive in radial direction. the blade lean should also provide radially inward force. thus the blade should be leant with pressure surface down as sketched.



[20%]

c). Euler equations are inviscid and very fast to be solved numerically compared to N-S equations.

It ~~was~~ is more accurate for turbine blade design because viscous effect is weaker in turbine blades than in compressor blades.

∵ 1). favourable pressure gradient

2). relative displace thickness compared with the blade ~~thick~~ thickness

However error will increase as loading increases, also at transonic speeds where shock wave interacts with the boundary layer.

For compressor blades, viscous effects cannot be ignored. if Euler ~~eq~~ solver is to be used. it needs to be coupled with a boundary solver ~~to~~ take the viscous effect into account. This also applies to the transonic turbine blades.

Also in reality AVDR is important so a two-D solve may not be sufficient and should be modified to "Q3D" to take streamtube variation into account. [2.8]

d). For internal flow with choked areas, the mass ~~conser~~ flow is very important as near sonic condition. small mass flow error will result in large error in other flow property such as velocity and pressure. in addition, ~~the~~ total work ~~and~~ is directly proportional ~~to~~ the mass flow rate. The conservations of the energy and momenta have direct impact on efficiency calculation which is very important for turbomachinery.

To ensure the conservations, it is common ~~to~~ cast the governing equations in conservative form, ~~which not only reduce~~ and to use finite volume integration methods instead of finite difference methods. Both will reduce the discretisation errors in conservative properties, especially where the primary flow variables are not contineous, such as acrossing shock waves.

[20%]

e). The characteristics theory points out that for subsonic flow - there is one characterics running upstream and four downstream for 3-D Euler equations. thus at the inlet we need four conditions to specify waves running downstream and at the ~~inlet~~ exit one is needed to fix the upstream running wave.

Usually, at the inlet, Stagnation pressure P_0 and stagnation temperature T_0, together with two flow angles in radial and tangential directions are specified, the other independant variable, usually either static pressure or density, is explolated from the flow down stream;

at exit. usually the static pressure is specified and all other variables are extrapolated from the flow upstream.

for a transonic fan, although the relative flow varies from subsonic to supersonic, the absolute inlet flow is still subsonic through the entire span, thus absolute subsonic inlet flow boundary conditions should be specified.

[2 [illegible]]