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Part II B Module 4A14 Silent Aircraft Initiative

Solutions 2006

Qu1 a) The unsteady force is a dipole source.

The compactness ratio = $\dfrac{\text{source distance from origin} \times \text{frequency (radians/s)}}{\text{speed of sound}}$

$$= \frac{0.4 \times 2\pi 45}{340} = 0.3,$$

treat as compact and neglect variation in retarded time over the wheel. Then

$$p'(\underset{\sim}{x},t) = \frac{\underset{\sim}{x} \cdot \dot{\underset{\sim}{F}}(t - |\underset{\sim}{x}|/c_0)}{4\pi|\underset{\sim}{x}|^2 c_0}$$

where $\underset{\sim}{F}$ is the force exerted on the air by the wheels.

Take direction of motion to be in 1-direction, $\dot{\underset{\sim}{F}}(t) = (0, \dot{F}_2, \dot{F}_3)$ since for a wheel the fluctuating force is in a plane perpendicular to direction of motion.

$$p'(\underset{\sim}{x},t) = \frac{x_2 \dot{F}_2 + x_3 \dot{F}_3}{4\pi|\underset{\sim}{x}|^2 c_0}$$

$$\overline{p'^2} = \left(\frac{1}{4\pi|\underset{\sim}{x}|^2 c_0}\right)^2 \overline{\left(x_2^2 \dot{F}_2^2 + x_3^2 \dot{F}_3^2\right)}$$

where we have assumed that the two components $\dot{F}_2$ and $\dot{F}_3$ are uncorrelated.

Since $\overline{\dot{F}_2^2} = \overline{\dot{F}_3^2}$ i.e. the mean square value of the force has the same value in any direction normal to the flight path (see lecture notes)

$$= \frac{1}{2}\overline{(\dot{f}_{rms})^2}$$

Hence $\overline{p'^2} = \left(\dfrac{1}{4\pi|\underset{\sim}{x}|c_0}\right)^2 \sin^2\theta \; \dfrac{1}{2}\dot{f}_{rms}^2$ where θ is the angle from the direction of motion

$x_1 = |\underset{\sim}{x}|\cos\theta$

$(x_2^2 + x_3^2)^{1/2} = |\underset{\sim}{x}|\sin\theta$

Assuming that $\dot{f}_{rms} = 2\pi \times 45 \, f_{rms}$ since the characteristic frequency is 45Hz,

we obtain

$$\overline{p'^2} = \left(\frac{1}{4\pi|\underset{\sim}{x}|c_0}\right)^2 \frac{1}{2}(2\pi \times 45)^2 \sin^2\theta \left(k\,\frac{1}{2}\rho_0 V^2 D^2\right)^2$$

$$= \frac{1}{32|\underset{\sim}{x}|^2}\left(\frac{k\,45\,\rho_0 V^2 D^2}{c_0}\right)^2 \sin^2\theta$$

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Qu1 cont.)

The radiated sound power follows from integrating the flux of acoustic energy through a large sphere of radius R

$$\text{Sound power} = \int_{S_\infty} p' \underset{\sim}{v}' \cdot d\underset{\sim}{S} = \int_{\theta=0}^{\pi} \frac{\overline{p'^2}}{\rho_0 c_0} 2\pi R^2 \sin\theta \, d\theta$$

since the sound pressure is symmetric about the direction of flight

$$= \frac{2\pi}{\rho_0 c_0} \frac{1}{32} \left(\frac{k\ 45 \rho_0 V^2 D^2}{c_0} \right)^2 \int_{\theta=0}^{\pi} \sin^2\theta \sin\theta \, d\theta$$

$$= \frac{\pi}{16} \frac{\rho_0}{c_0^3} \left(k\ 45 V^2 D^2 \right)^2 \frac{4}{3}$$

$$\text{since} \int_{\theta=0}^{\pi} \sin^2\theta \sin\theta \, d\theta = \int_{\theta=0}^{\pi} (1-\cos^2\theta) \sin\theta \, d\theta$$

$$= \int_{1}^{-1} -(1-u^2) \, du \qquad \begin{aligned} \cos\theta &= u \\ -\sin\theta d\theta &= du \end{aligned}$$

$$= \left[\left(1 - \tfrac{1}{3} u^2 \right) \right]_{-1}^{1} = \frac{4}{3}$$

$= 0.86\ \text{W}$ with $\rho_0 = 1.2\ \text{kg/m}^3$, $c_0 = 340\ \text{m/s}$

$$= 10 \log_{10} \left(\frac{0.86}{1 \times 10^{-12}} \right) \text{dB} = \underline{\underline{119\ \text{dB}}}$$

b) Would get additional noise from unsteady forces on the struts (mid-frequency range) and from the small details of hydraulic hoses, etc. (high freq) Could also get cavity noise from unsteady air storage in the undercarriage well.

c) The noise can be reduced by streamlining the struts, enclosing the hoses and by either streamlining the wheel assembly or by going to an arrangement of just two larger wheels. However, because of the V^6 dependence of the sound power on velocity the largest influence on noise is a reduction in approach velocity. Steeper glide slope increases the distance above the ground and can also help. Any cavity noise would be an effective sound sound and should be eliminated, eg by closing the undercarriage doors and sealing well.

Question 2

Start from the formula on datasheet

$$p' = \frac{1}{4\pi} \sum \frac{\partial^2}{\partial x_i \partial x_j} \int \frac{T_{ij}(\underline{y}, t - |\underline{x}-\underline{y}|/c_0)}{|\underline{x}-\underline{y}|}$$

In far field replace $|\underline{x}-\underline{y}|$ by $|\underline{x}|$ in denominator

$$p' \sim \frac{1}{4\pi} \sum \frac{\partial^2}{\partial x_i \partial x_j} \frac{1}{|\underline{x}|} \int T_{ij}(\underline{y}, t - |\underline{x}-\underline{y}|/c_0)$$ {5%

Now note that derivatives of $1/|\underline{x}|$ are small in far field, so neglect.

$$p' \sim \frac{1}{4\pi|\underline{x}|} \sum \frac{\partial^2}{\partial x_i \partial x_j} \int T_{ij}(\underline{y}, t - |\underline{x}-\underline{y}|/c_0)$$ {5%

Now suppose lengthscale and velocity scale of turbulence are L and u_0, so timescale is L/u_0. Variation of retarded time over source is L/c_0, so require

$$L/u_0 \gg L/c_0 \Rightarrow M = \frac{u_0}{c_0} \ll 1$$ {5%}

If so $$p' \sim \frac{1}{4\pi|\underline{x}|} \sum \frac{\partial^2}{\partial x_i \partial x_j} \int T_{ij}(\underline{y}, t - |\underline{x}|/c_0)$$ {5%

in far field $$\frac{\partial}{\partial x_i} \sim -\frac{1}{c_0} \hat{x}_i \frac{\partial}{\partial t}$$

$$\therefore p' \sim \frac{1}{4\pi c_0^2 |\underline{x}|} \hat{x}_i \hat{x}_j \frac{\partial^2}{\partial t^2} \int T_{ij}(\underline{y}, t - \frac{|\underline{x}|}{c_0})$$ {5%

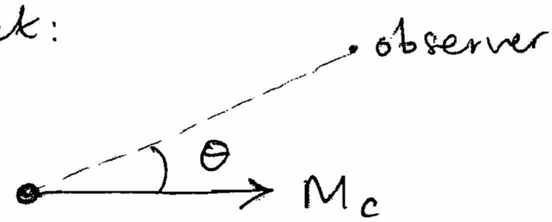
Qu2 cont.) Now, $\frac{\partial}{\partial t} = O\left(\frac{L}{u_0}\right)^{-1}$, from timescale of turbulence, $T_{ij} = O(\rho_0 u_0^2)$ and this is integrated over volume $O(L^3)$. {10%}

$$O(p') = \frac{1}{c_0^2 |\underline{x}|} \left(\frac{u_0}{L}\right)^2 \cdot \rho_0 u_0^2 \cdot L^3 \qquad (*)$$

$$= \frac{L}{|\underline{x}|} \rho_0 c_0^2 \; m^4. \qquad \{10\%\}$$

$$\therefore \text{intensity} \propto (p')^2 \propto m^8 \qquad = [45\%$$

Doppler effect:



frequency of received signal is

$$\frac{\omega}{1 - M_c \cos\theta} \qquad \{10\%\}$$

The radiating volume is $l^2 l_r$

$$l_r = \frac{l}{1 - M_c \cos\theta} \qquad \{10\%$$

$\therefore$ in $(*)$ above

$$O(p') = \frac{1}{c_0^2 |\underline{x}|} \left(\frac{u_0}{L}\right)^2 \frac{1}{(1-M_c\cos\theta)^2} \rho_0 u_0^2 \frac{L^2 \cdot L}{(1-M_c\cos\theta)}$$

$$O(p') \propto \frac{m^4}{(1-M_c\cos\theta)^3} \qquad \{5\%$$

Qu 2 cont.)

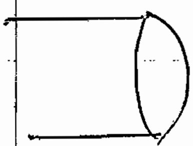
For ensemble of eddies, number radiating {5%

changes by $1 - M_c \cos\theta$

$\therefore$ total intensity $\propto \left[\dfrac{m^4}{(1-M_c\cos\theta)^3} \right]^2 (1 - M_c \cos\theta)$ {5%]

$$= \frac{m^8}{(1-M_c\cos\theta)^5}$$

= [35%]



nozzle area A

total sound intensity

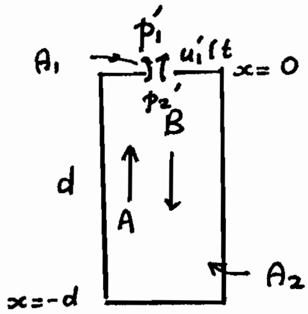
$\propto m^8 A$ {10%}

Thrust $= \rho_0 m^2 A$ = constant {10%}

$\Rightarrow$ sound intensity $\propto (A)^{-4} A = A^{-3}$ = [20%]

at fixed thrust

Qu 3.



$$p'(x,t) = \left(Ae^{-i\omega x/c_0} + Be^{i\omega x/c_0}\right)e^{i\omega t}$$

$$u' = \frac{1}{\rho_0 c_0}\left(Ae^{-i\omega x/c_0} - Be^{i\omega x/c_0}\right)e^{i\omega t}$$

$u' = 0$ at $x=-d$, $\quad Ae^{i\omega d/c_0} = Be^{-i\omega d/c_0} = C$ say

Then at $x_2 = 0^-$ $\quad p_2'(x,t) = e^{i\omega t}C\left[e^{-i\omega d/c_0} + e^{i\omega d/c_0}\right] = 2C\cos\left(\frac{\omega d}{c_0}\right)e^{i\omega t}$

$$u'(0,t) = \frac{e^{i\omega t}C}{\rho_0 c_0}\left[e^{-i\omega d/c_0} - e^{i\omega d/c_0}\right] = \frac{-2iC}{\rho_0 c_0}\sin\left(\frac{\omega d}{c_0}\right)e^{i\omega t}$$

$$u_1' = \frac{A_2}{A_1}u'(0,t) = -\frac{2iC}{\rho_0 c_0}\sin\left(\frac{\omega d}{c_0}\right)e^{i\omega t}\frac{A_2}{A_1}$$

Hence $\quad p_2' = -\frac{A_1}{A_2}\cot\left(\frac{\omega d}{c_0}\right)\frac{\rho_0 c_0 u_1'}{i} = i\rho_0 c_0\frac{A_1}{A_2}\cot\left(\frac{\omega d}{c_0}\right)u_1'$

Apply condition across the neck $\quad p_2' - p_1' = \rho_0 c_0 k u_1'$

$$i\rho_0 c_0\frac{A_1}{A_2}\cot\left(\frac{\omega d}{c_0}\right)u_1' - p_1' = \rho_0 c_0 k u_1'$$

$$\underline{\underline{p_1' = \left(i\frac{A_1}{A_2}\cot\left(\frac{\omega d}{c_0}\right) - k\right)\rho_0 c_0 u_1'}}$$

(ii) To optimally absorb sound we want $|u_1'|$ to be as large as possible for a particular p_1'. In general

$$|u_1'| = \frac{1}{\rho_0 c_0}\frac{|p_1'|}{\left|i\frac{A_1}{A_2}\cot\left(\frac{\omega d}{c_0}\right) - k\right|} = \frac{|p_1'|}{\rho_0 c_0\left(\frac{A_1^2}{A_2^2}\cot^2\left(\frac{\omega d}{c_0}\right) + k^2\right)^{1/2}}$$

To make $|u_1'|$ large, we want $\cot^2\left(\omega d/c_0\right)$ as small as possible, i.e.

$$\cos\left(\frac{\omega d}{c_0}\right) = 0.$$

The shortest depth that achieves this is $\frac{\omega d}{c_0} = \pi/2$ i.e $d = 1/4$ wavelength.
For a temperature of 600K, $c_0 \simeq 491$ m/s

$d = c_0/\omega \quad$ for 1kHz $= 0.078$m

i.e. <u>depth of 78 mm</u>

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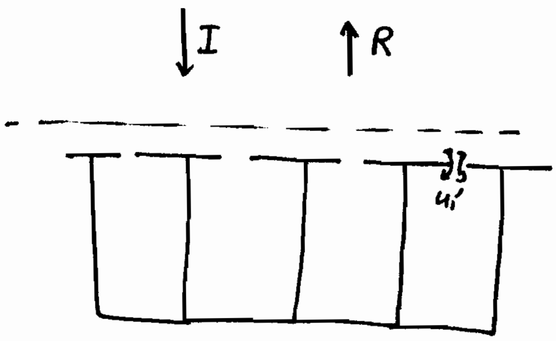
Qu3 cont.)

(iii) Now consider an incident plane wave



The area averaged velocity at $x=0^+$ is $A_1/A_2\, u_1'(t)$ where A_1/A_2 is the porous

$$p_1'(t) = (I+R)\, e^{i\omega t}$$

$$\frac{A_1}{A_2}\, u_1'(t) = \frac{1}{\rho_0 c_0}(I-R)\, e^{i\omega t}$$

On the resonance condition $p_1'(t) = |u_1'|\rho_0 c_0 k$

Hence $$(I+R) = \frac{A_2}{A_1}(I-R)\frac{\rho_0 c_0 k}{\rho_0 c_0}$$

i.e. $$I\left(1-\frac{A_2}{A_1}k\right) = R\left(-1-\frac{A_2 k}{A_1}\right)$$

$$R = -I\,\frac{(A_1 - A_2 k)}{A_1 + A_2 k}$$

Proportion of incident energy absorbed $= 1 - \dfrac{|R|^2}{|I|^2} = 1 - \left(\dfrac{A_1 - A_2 k}{A_1 + A_2 k}\right)^2$

$$= \frac{4 A_1 A_2 k}{(A_1 + A_2 k)^2}$$

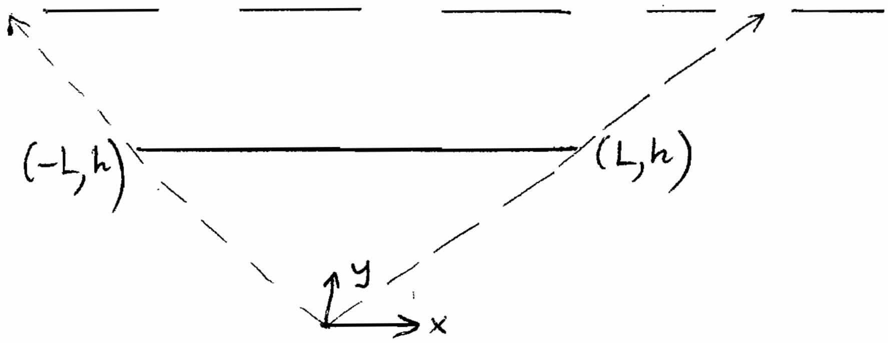
$$= \frac{4\, A_1/A_2\, k}{\left(k + A_1/A_2\right)^2}$$

For $k = 0.1$, $A_1/A_2 = 0.05$, proportion of incident energy $= \dfrac{4 \times 0.1 \times 0.05}{(0.1 + 0.05)^2}$

$$= 0.89$$

Question 4

(i)



The rays from (0,0) to edges make an angle

$$\tan^{-1} L/h$$

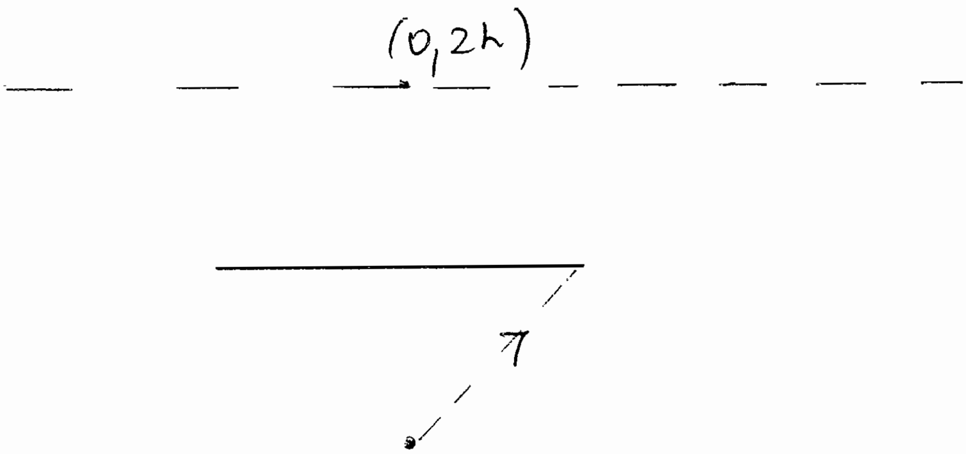
with vertical. Only sound propagating in $y>0$ can reach $y = 2h$

$\therefore$ fraction of acoustic energy crossing $y = 2h$ is $\dfrac{2\left[\frac{\pi}{2} - \tan^{-1}(L/h)\right]}{2\pi}$

$$= \frac{\tan^{-1}(h/L)}{\pi}$$

= [10%

(ii)



The field from the source in the absence of the plate is of the form

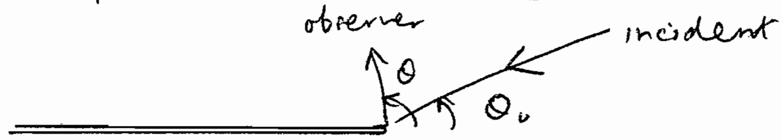
$$\frac{P_0}{\sqrt{r}} \exp(-ik_0 r)$$

for some constant p_0

$\therefore$ field reaching $(0, 2h)$ without plate is

$$\frac{p_0}{\sqrt{2h}} \exp(-2ik_0 h) \qquad (1)$$ {10%

notation for edge scattering



$$\theta_0 = -\pi + \tan^{-1}(h/L)$$ {10%
$$\theta = \pi - \tan^{-1}(h/L).$$

The field incident on the edge is

$$\frac{p_0}{(h^2+L^2)^{1/4}} e^{-ik_0 h} \times \text{plane wave}$$

Hence, diffracted field at $(0, 2h)$ is

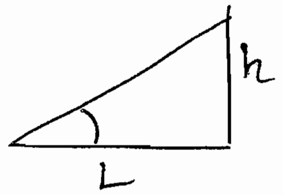
$$\underbrace{\frac{p_0}{(h^2+L^2)^{1/4}} e^{ik_0 h}}_{p_{inc}} \cdot \left(\frac{2}{\pi k_0 \sqrt{h^2+L^2}}\right)^{1/2} \frac{\sin\theta_0/2 \sin\theta/2}{\cos\theta + \cos\theta_0}$$ {10%

$$\times \exp\left(-\frac{i\pi}{4} - ik_0\sqrt{h^2+L^2}\right)$$

The total diffracted field at $(0, 2h)$ is double this (2 edges) {10%

$\therefore$ $\dfrac{\text{amplitude of diffracted field}}{\text{" " direct "}}$

$$= \frac{\dfrac{2p_0}{(h^2+L^2)^{1/4}} \left(\dfrac{2}{\pi k_0 \sqrt{h^2+L^2}}\right)^{1/2} \left|\dfrac{\sin\frac{\theta_0}{2} \sin\theta/2}{\cos\theta + \cos\theta_0}\right|}{\dfrac{p_0}{\sqrt{2h}}}$$



$$\cos\theta = \cos(\pi - \tan^{-1} h/L)$$
$$= -\cos(\tan^{-1} h/L) = \frac{-L}{\sqrt{h^2+L^2}}$$ {5%

$$\cos\theta_0 = \cos\theta.$$

$$1 - 2\sin^2 \theta_0/2 = \cos\theta_0 \quad \text{, and } |\sin\theta_0/2| = |\sin\theta/2|$$ {5%

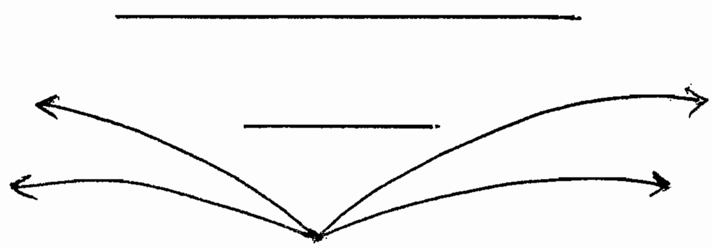
$$\therefore \text{ ratio } \rightarrow \frac{4\,h^{1/2}}{(h^2+L^2)^{1/2} \cdot k_0^{1/2}\pi^{1/2}} \frac{\sin^2\theta_0/2}{2|\cos\theta_0|}$$

$$= \frac{2h^{1/2}}{(h^2+L^2)^{1/2} k_0^{1/2}\pi^{1/2}} \frac{\frac{1}{2}\left(1 + \frac{L}{\sqrt{h^2+L^2}}\right)}{\frac{L}{\sqrt{h^2+L^2}}}$$

{10%}

$$= \frac{\sqrt{h}}{\sqrt{\pi}\cdot\sqrt{h^2+L^2}\sqrt{k_0}} \left(\frac{L + \sqrt{h^2+L^2}}{L}\right)$$ = [60%

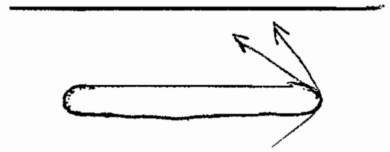
(iii) (a)



= [15%]

rays curve downwards, smaller fraction of energy reaches $y = 2h$

(b)



shadow is deeper, creeping rays shed into shadow zone, lower pressure @ $(0, 2h)$ = [15%]