4B11 2006 CRIB

Q1.

a) Show that the far field diffraction pattern of the aperture, of amplitude A, shown in Figure 1 is given by the following expression. State any assumptions made.

$$F(u,v) = Aa^2\mathrm{sinc}(\pi au)\mathrm{sinc}(\pi av)$$

b) Use a simple graphical technique to derive the far field diffraction pattern of a binary amplitude (A [0,1]) grating. State any assumptions made and estimate the efficiency of light diffracted into the first order.

c) Using the same technique as in part (b), show how the far field diffraction pattern for a binary amplitude grating can be used to derive the far field diffraction pattern for a binary phase (A [-1, +1]) grating. Estimate the efficiency diffracted into the first order.

d) Explain what features of the binary phase grating or hologram effect the shape of each spatial frequency element or spot within the far field diffraction pattern. Give two reasons why this is an important factor when using binary phase gratings to steer light into waveguide structures in a telecommunications optical switch.

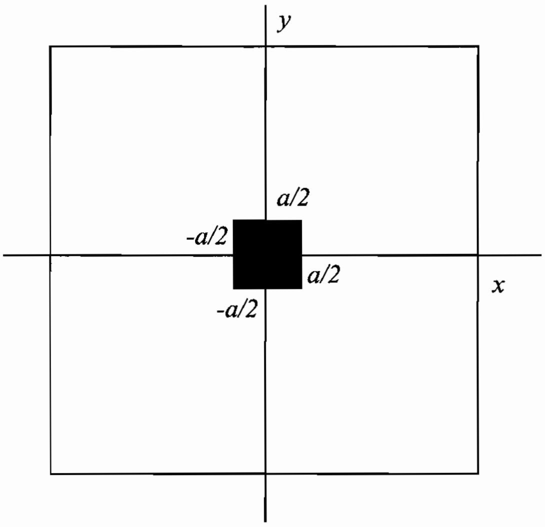


Figure 1. (*The white area represents amplitude zero*)

Crib. 1

a) Far field diffraction pattern = Fourier transform of the aperture.

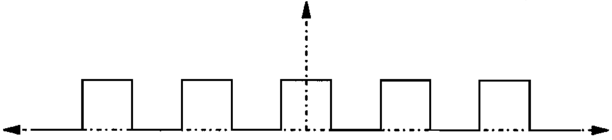
$$F(u,v)=\iint_{\pm\infty} f(x,y)e^{2\pi j(ux+vy)}dxdy = A\int_{-a/2}^{a/2} e^{2\pi j(ux)}dx\int_{-a/2}^{a/2} e^{2\pi j(vy)}dy$$

$$=A\left[\frac{e^{2\pi j u}}{2\pi j u}\right]_{-a/2}^{a/2}\left[\frac{e^{2\pi j v}}{2\pi j v}\right]_{-a/2}^{a/2} = \frac{A}{2\pi j u}\left(e^{a\pi j u}-e^{-a\pi j u}\right)\frac{1}{2\pi j v}\left(e^{a\pi j v}-e^{-a\pi j v}\right)$$

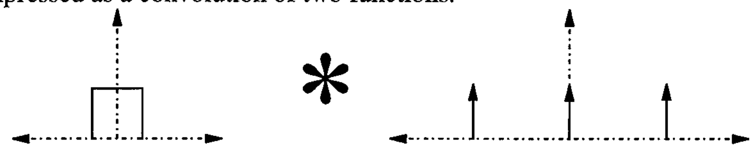
$$=\frac{A}{\pi u}\sin\pi u\frac{1}{\pi v}\sin\pi v = Aa^2\mathrm{sinc}(\pi a u)\mathrm{sinc}(\pi a v)$$

Assumptions: far field is coaxial from the propagating axis. The far field is far enough from the aperture to be in the Faunhopher region. Aperture is illuminated by a coherent, monochromatic plane wave source. Aperture is constant and much smaller than the distance to the far field.

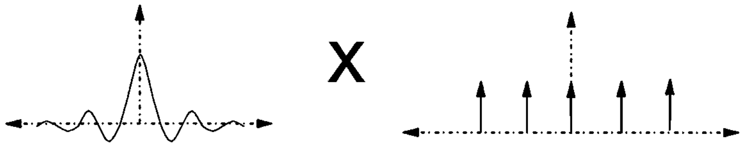
b) If the 1D amplitude grating is viewed from the end (as a 1-D function).



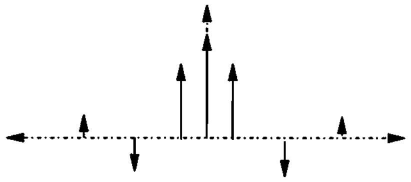
Which can be expressed as a convolution of two functions.



After the Fourier transform.



Gives the final result.

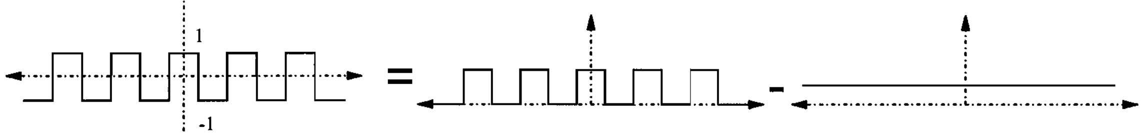


The train of delta functions has a sinc envelope and every second delta function is suppressed by the zeros of the sinc. The 2-D far field region of the 2-D grating shown above will be the same structure as the 1-D case (a sinc enveloped train of delta functions) superimposed onto the y axis of the 2-D far field plane.

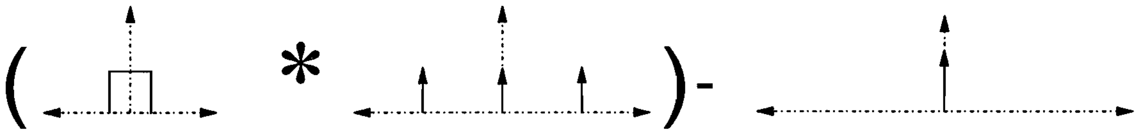
Assumptions: No deadspace, 50:50 mark space ratio. Uniform coherent, monochromatic illumination.

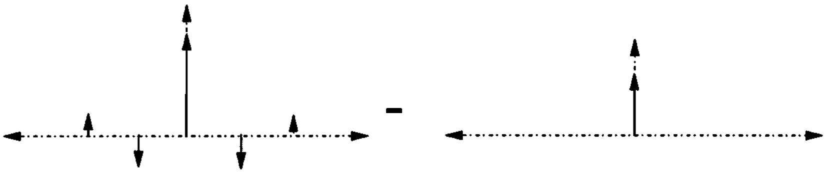
Estimated efficiency = 50% (amplitude)*0.41 = 20%

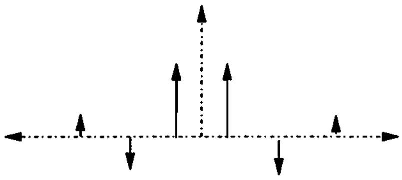
c) A 1-D grating $A \in [+1,-1]$ can be made by subtracting DC from a 1-D grating, $A \in [0,1]$.



Hence in the Fourier domain we have:







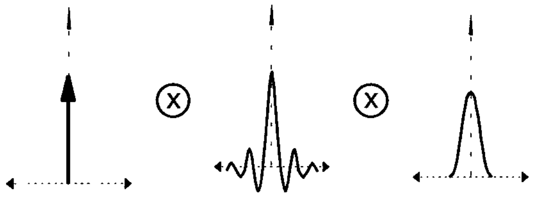
Hence the far field diffraction pattern is the same as for part b, but with no zero order and the efficiency is now 41%.

d) In the examples so far, we have assumed that the illumination of the hologram is uniform and that the hologram and lens extend to infinity. This is not the case in the real world, as there are a finite number of hologram pixels creating an aperture over the hologram and the light used to illuminate it will not be uniformly distributed. In all the examples we are assuming that the illumination source is a collimated monochromatic laser which generates high quality parallel wavefronts with a wider diameter than the hologram or the lenses. Such a source will usually have a intensity distribution which can be expressed as a Gaussian beam profile or function. The most useful property of the Gaussian function is that its FT is another Gaussian with a different scale factor (i.e. size). If a lens is illuminated with a collimated Gaussian source, then the focal plane will also contain a Gaussian profile which is scaled by the focal length. The actual structure of the replay field of a hologram is even more complex, as the Gaussian beam profile of the source is also apertured by the hologram and lens leading to sinc like sidelobes on the FT'ed Gaussian profile.

The entire illumination system can be modeled as a sequence of multiplied functions. The input illumination distribution $g(x,y)$ times the hologram aperture $d(x,y)$ times the total aperture of the FT lens (if it has a smaller diameter than the hologram) $p(x,y)$. Hence effect of the FT on these functions results in a convolution of their transforms.

$$F(u,v) = G(u,v) \otimes D(u,v) \otimes P(u,v)$$

The ideal hologram replay field $H(u,v)$ is designed as an array of delta functions in desired positions. The lens aperture $p(x,y)$ is a large circular hole, so the FT $P(u,v)$ will be a first order Bessel function (like a circular sinc function). The hologram aperture is a large square of size $N\Delta$ and its FT, $H(u,v)$ will be a sharp sinc function. The effect of the FT of the illumination $G(u,v)$ is to add a Gaussian profile. Hence, the profile of the spots in the hologram replay field will not be delta functions, they will be delta functions convolved with a Bessel function convolved with a sinc function convolved with a Gaussian function.



In a telecoms switch using waveguides or fibres, the launch condition states that the mode launched must be stable in order to propagate, hence we must match the desired mode of the waveguide. This is very difficult if the spot from the hologram is distorted due to apodisation. This mismatch appears as a loss in the switch as well as back reflections. The sidelobes of the spots is also a problem as in telecoms, crosstalk below 40dB is required and this means that the sidelobes must be less to 40dB which is difficult to maintain.

Q2

a) Define the terms retardance and birefringence when referred to a liquid crystal material. What features of the liquid crystal material leads to these properties.

b) Explain what is meant by the term surface stabilised ferroelectric liquid crystal. What is the predominant feature of this class of materials and what are the switching characteristics you would expect to see in a device made with this material?

c) A generalised Jones matrix for a waveplate at an angle ψ to the vertical axis and retardance Γ is given below. Show how a surface stabilised ferroelectric liquid crystal (FLC) pixel, with a switching angle of θ, can be configured to perform binary phase modulation. Show that it is independent of ψ, Γ and θ.

$$W = \begin{pmatrix} e^{-j\Gamma/2}\cos^2\psi + e^{j\Gamma/2}\sin^2\psi & -j\sin\frac{\Gamma}{2}\sin(2\psi) \\ -j\sin\frac{\Gamma}{2}\sin(2\psi) & e^{j\Gamma/2}\cos^2\psi + e^{-j\Gamma/2}\sin^2\psi \end{pmatrix}$$

d) Explain why a normal planar aligned nematic liquid crystal material is inherently polarisation dependant. If this also the case for a surface stabilised FLC material?

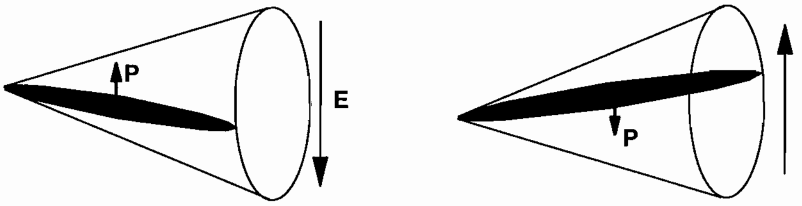
Q2 crib

a) Any material which passes light will have some effect on the that light as it passes through it. This normally takes the form of slowing down the wave and we define the refractive index n of the material as the ratio of the speed of light in a vacuum to the speed of light within the material. Most materials (such as water or window glass) exhibit *isotropic* behavior which means that n is constant for any direction that the light travels through the material. Some materials, however are anisotropic which means that n varies with the direction of propagation. If the crystal has structure such as hexagonal or triagonal, different directions of light will see very different crystalline structures. The effect of this anisotropy is called birefringence and is property which is exploited in retarders. In a birefringent material, each eigenwave sees a different refractive index and will propagate at a different speed. This leads to a phase retardation between the two eigenwaves which is dependent on the thickness of the birefringent material and the wavelength of the light. The preferred directions of propagation within the crystal are defined as the extraordinary (or fast) axis and the ordinary (or slow) axis. An eigenwave that passes in the same direction as the extraordinary axis sees a refractive index n_e and the eigenwave that passes along the ordinary axis sees n_o. For light of wavelength λ passing through a birefringent crystal of thickness t, we define the retardation Γ as.

$$\text{Retardation}\ \Gamma = \frac{2\pi\ t}{\lambda}(n_e - n_o)\ \text{Birefringence}\ \Delta n = (n_e - n_o)$$

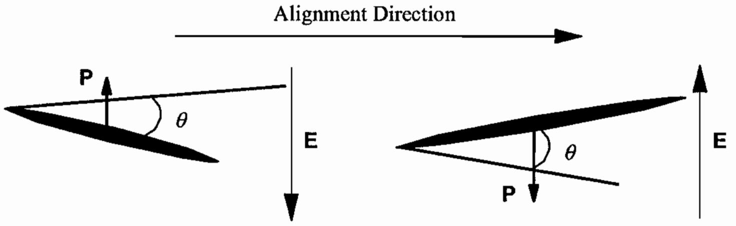
The ordered or crystalline properties arise from the shape of the molecules and their interactions as they tumble about and is purely a statistical average of their motion. Common molecular shapes for liquid crystals are rod-like (calimatic) and disc-like (discotic). It is the interaction between the molecular order and polarised light, which creates a large electro-optical effect such as those seen in LC displays.

b) One of the most useful smectic mesophases is the smectic C (SmC) phase as the molecules are highly ordered and form layers with the molecules tilted within each layer. When an electric field is applied across the molecules there is little interaction as the field is almost perpendicular to the main axis of the LC molecules. The smectic C structure can be improved by adding chirality to the molecular structure which adds an extra dipole perpendicular to the molecular axis of the LC material. This is often is referred to as chiral smectic C or SmC*.



The addition of this dipole due to the chirality of the molecules means that the molecules are in an ordered structure but they are free to move. An applied field across the molecules is now parallel to the extra dipole and the interaction will exert a force on the molecules causing them to rotate about their molecular axis. The dipole **P** of the molecule, which is perpendicular to its length is often referred to as the Spontaneous Polarisation and means that the overall bulk of the LC will have a net potential, hence these materials are called ferroelectric liquid crystals (FLCs). When an electric field **E** is applied to the cell, there is an interaction between the **E** and **P**, which forces the molecule to move around the cone to a point of equilibrium. If the field is changed, the molecules move again. The SmC* phase in thick cells is not ferroelectric because in the equilibrium state the **P** dipoles of the molecules interact with each other forming a helix along the axis of the cell which results in no overall retardance.

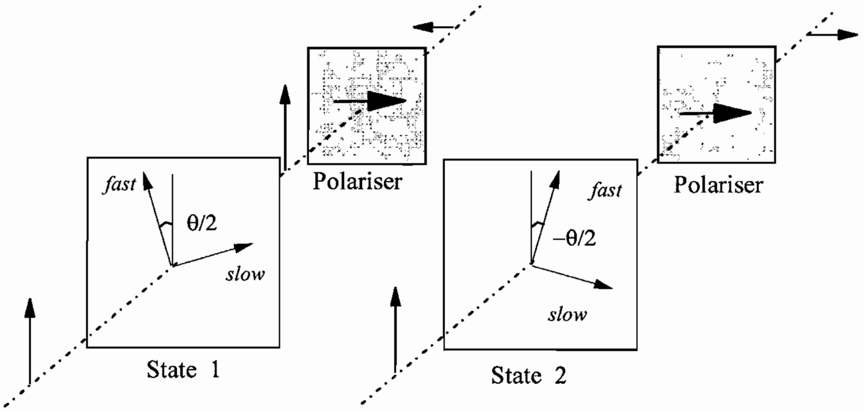
If the FLC is restricted to a cell thickness of 2-5μm then the helix of along the cell is suppressed and the molecules are bounded into two stable states either side of the director cone. The angle between these two states is defined as the switching angle θ. This is referred to as a surface stabilised FLC geometry an creates a high degree of ferroelectricity and creates a large birefringent electro-optical effect. The penalty for doing this is that the molecules are only stable in the two states an therefore the modulation will only be binary. The up side to this binary modulation is that it can be very fast (~10μsec) and that the stability can lead to the molecules remaining in the two states in what is known as bistable switching.



c) The FLC crystal acts like a switchable waveplate whose fast and slow axes can be in two possible states separated by the angle θ and whose retardation depends on the thickness and birefringence of the FLC. A pixel with retardation Γ at an angle θ can be represented using Jones matrix notation as:

$$\begin{pmatrix} e^{-j\Gamma/2}\cos^2\theta + e^{j\Gamma/2}\sin^2\theta & -j\sin\frac{\Gamma}{2}\sin(2\theta) \\ -j\sin\frac{\Gamma}{2}\sin(2\theta) & e^{j\Gamma/2}\cos^2\theta + e^{-j\Gamma/2}\sin^2\theta \end{pmatrix}$$

If the light is polarised so that its direction bisects the switching angle and an analyser (polarisor) is placed after the pixel at 90° to the input light, then phase modulation is possible. If we start with vertically polarised light, then the FLC pixel fast axis positions must bisect the vertical axis and will be oriented at angles of $\theta/2$ and $-\theta/2$ respectively for each state. Once again we can use Jones matrices to express the system.



State 1.

$$\begin{pmatrix}V_x'\\ V_y'\end{pmatrix}=\begin{pmatrix}1&0\\ 0&0\end{pmatrix}\begin{pmatrix}e^{-j\Gamma/2}\cos^2\frac{\theta}{2}+e^{j\Gamma/2}\sin^2\frac{\theta}{2} & -j\sin\frac{\Gamma}{2}\sin(\theta)\\ -j\sin\frac{\Gamma}{2}\sin(\theta) & e^{j\Gamma/2}\cos^2\frac{\theta}{2}+e^{-j\Gamma/2}\sin^2\frac{\theta}{2}\end{pmatrix}\begin{pmatrix}0\\ V_y\end{pmatrix}$$

$$=\begin{pmatrix}-V_y j\sin\frac{\Gamma}{2}\sin(\theta)\\ 0\end{pmatrix}$$

State 2

$$\begin{pmatrix}V_x'\\ V_y'\end{pmatrix}=\begin{pmatrix}1&0\\ 0&0\end{pmatrix}\begin{pmatrix}e^{-j\Gamma/2}\cos^2\frac{\theta}{2}+e^{j\Gamma/2}\sin^2\frac{\theta}{2} & j\sin\frac{\Gamma}{2}\sin(\theta)\\ j\sin\frac{\Gamma}{2}\sin(\theta) & e^{j\Gamma/2}\cos^2\frac{\theta}{2}+e^{-j\Gamma/2}\sin^2\frac{\theta}{2}\end{pmatrix}\begin{pmatrix}0\\ V_y\end{pmatrix}$$

$$=\begin{pmatrix}V_y j\sin\frac{\Gamma}{2}\sin(\theta)\\ 0\end{pmatrix}$$

From these two expression we can see that the difference between the two states is just the minus sign, which means that the light has been modulated by 180° (π phase modulation). Moreover, the phase modulation is independent of the switching angle θ and the retardation Γ. These parameters only effect the loss in transmission through the pixel which can be gained by squaring the above expressions.

$$T=V_y^2\sin^2(\theta)\sin^2\left(\frac{\Gamma}{2}\right)$$

Hence maximum transmission (and therefore minimum loss) occurs when $\Gamma=\pi$ and $\theta=\pi/2$.

d) A nematic LC material which is planar aligned (parallel to the glass surfaces) will always be polarisation sensitive. As the electric field is applied, the indices dipole in the nematic causes the molecules to rotate out of the plane of the cell (ie perpendicular to the glass plates) until it is in the homeotropic state. When viewed y the optical signal passing through the LC, the electric field parallel to the molecules sees the change in birefringence with applied voltage, however the optical field perpendicular to the LC molecules sees no change in birefringence it is polarisation dependant.

For an FLC, the switching is in the plane of the cell (ie parallel to the electrodes), hence all polarisations will see a change in birefringence as the molecules rotate. This means that the FLC is in fact polarisation insensitive, however only for binary modulation. The case for multi-level phase is more subjective for a FLC material and requires a perfect 90 degree switching material.

Q3.

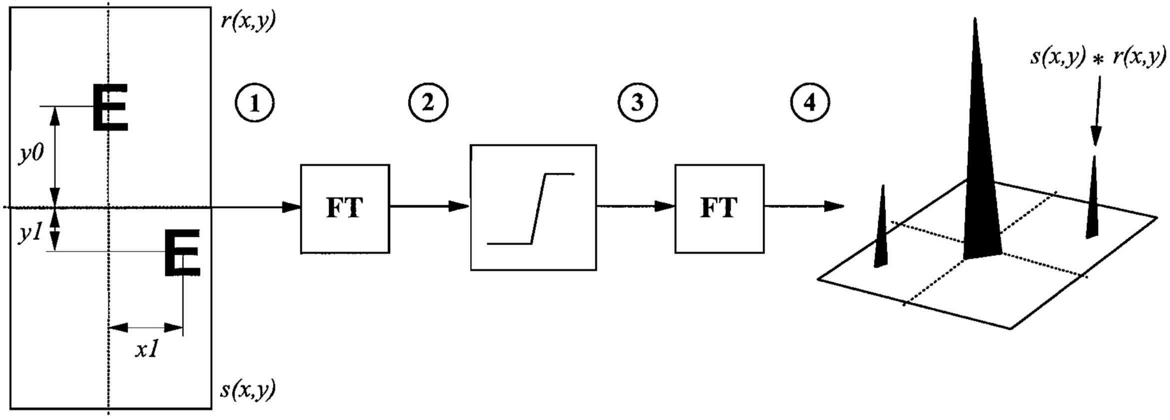
a) Sketch the general functional layout of a joint transform correlator (JTC). Explain what each component is for and how it is implemented optically. Using this functional layout, show how the principle of correlation occurs assuming a square law non-linearity.

b) Why is the structure of the non-linearity so important to the operation of the JTC? Describe two ways in which this non-linearity can be implemented using liquid crystal technology.

c) What is the main limitation of this form of JTC? How can the symmetry of the optical system in part (a) be exploited to simplify the overall JTC architecture? What are the technological implications of this change?

d) Explain briefly how a JTC could be used to implement a complex image processing task such as motion estimation of objects moving between video frames.

Q3 crib
a)



1) Input device – liquid crystal SLM, 2. Fourier transform lens, 2) optical non-linearity (SLM or OASLM), 4) Fourier transform lens, 5) Output detector (CCD)

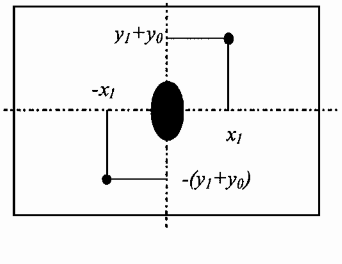
In plane 1, the input *s(x,y)* and reference *r(x,y)* are displayed side by side in an optical system and then transformed by a single lens into plane 2.

$$S(u,v)e^{-j2\pi(x_1u-y_1v)}+R(u,v)e^{-j2\pi y_0 v}$$

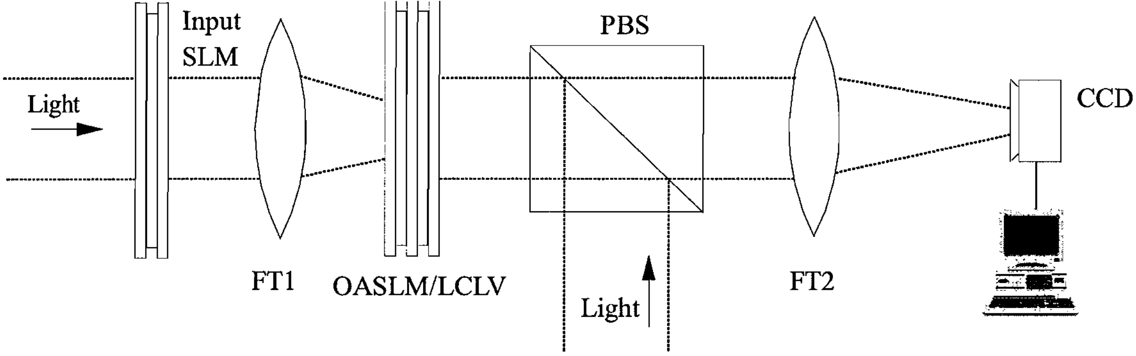
The nonlinearity between planes 2 and 3 creates the correlation and in its simplest form can be modeled by a square law detector such as photodiode or CCD camera which takes the magnitude squared of the light falling upon it.

$$S^2(u,v)+R^2(u,v)+S(u,v)R(u,v)e^{-j2\pi\left(x_1u-\left(y_0+y_1\right)v\right)}+S(u,v)R(u,v)e^{-j2\pi\left(-x_1u+\left(y_0+y_1\right)v\right)}$$

The final plane 4 is after the second FT, with the central DC terms proportional to FT [R^2 + S^2] and the two symmetrical correlation peaks spaced by (x_1,y_1+y_0) and $(-x_1,-(y_1+y_0))$.

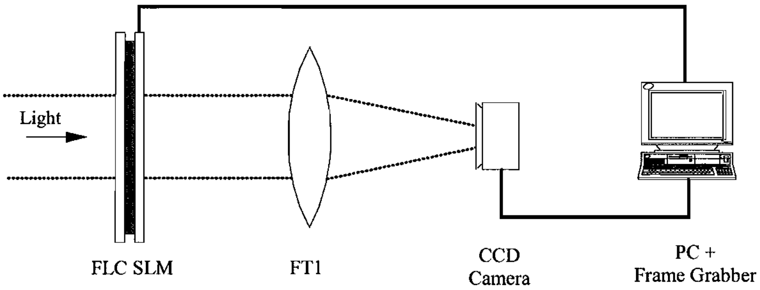


b) The JTC works on the basis of a non-linearity working on the spectrum of the input objects to create the product of the two Fourier transforms. This was modeled as a square law detector, but this gives undesirable broad peaks. A much better correlation peak is obtained when the degree of non-linearity is increased as high as possible. A simple square root function on the spectrum gives good narrow peaks, but the best performance is when the spectrum is thresholded. Hence the JTC was originally built using an optically addressed FLC SLM.



The system above shows how an actual JTC can be built based on a FLC OASLM acting as the non-linearity. This gives great performance as the photoconductor of the OASLM acts at a square law detector and the FLC (in surface stabilised mode) acts as a binary thresholding function. Once the read and write light for the OASLM have been set, it acts as a non-linearity at speeds in excess of 1kHz. The full JTC is too big and bulky as it required two lasers and a lot of optical components. In the case of the 1/f JTC, the non-linearity is performed using a combination of a CCD, computer and FLC SLM. This gives a lot more flexibility to the system and the function of the correlator.

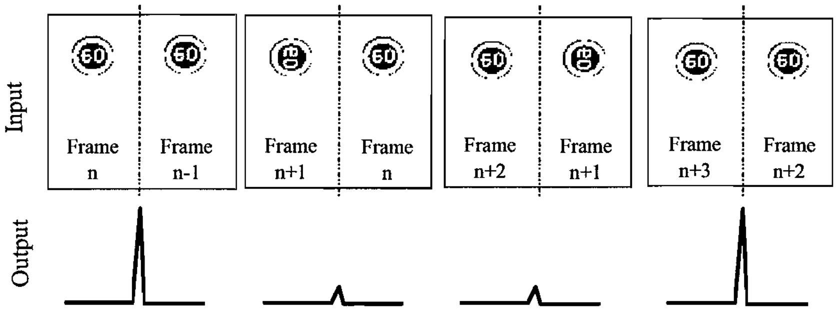
c) A better layout is to exploit the symmetry about the OALSM or non-linearity. If the optical system is split at this point, then the JTC just becomes two Fourier transforms and in fact can be done with a single laser, SLM and camera by doing two passes through the Fourier transform lens. This is known as the 1/f JTC.



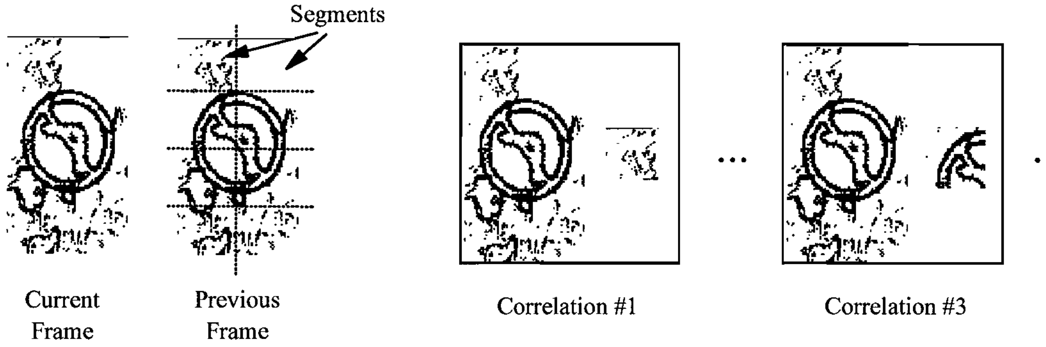
The input and reference images are displayed side by side on a FLC SLM as in a full JTC. The SLM is illuminated by a collimated laser beam and the images are Fourier transformed by a single lens in its focal plane. This spectrum is then imaged onto a CCD camera. The spectrum is then non-linearly processed before being displayed onto the SLM again to form the correlation information. The 1/f JTC is a two-pass system, using the same lens to perform the second Fourier transform.

If the spectrum was directly Fourier transformed, the result would be the two symmetrical correlation peaks characteristic of the JTC along with a huge zero order located in the centre of the output plane. The quality of the correlation peaks and the zero order can be improved by non-linearly processing the spectrum that also suits the available FLC SLM technologies. Some of the best results have been reported by using binary thresholds on the spectrum to improve the correlation peaks. A binarised spectrum produces good sharp correlation peaks and reduced zero order. If the binarised spectrum is converted to binary phase modulation [-1,+1], then the zero order can be reduced to around the height of the correlation peaks.

d) One elegant scenario is when a correlator is used to compare sequential frames in a video stream. In such an application, the current frame is the unknown and the previous frame is the 'reference'. Events that occur from frame to frame can now easily be tracked. The current frame and previous frame are synchronised with the progress of objects through the system. If the sequence does not change, then the output correlations remain from frame to frame, however, when a change occurs (in the example a rotated roadsign), then the correlation between frames is interrupted. Moreover, the cycle of distortion can be detected by looking at the sequence of disturbances about the first detected defect.



It is also possible to use the optical comparator for motion estimation and tracking. In this case, the current and previous frames are compared to see if there has been any motion between frames. The previous frame is broken into a number of smaller segments, which may or may not be overlapping as shown in Figure 10. Each separate segment is then correlated with the whole current frame at a rate much higher then the video stream rate.



Q4.

a) Explain, with the aid of sketches, what is mean by the terms fan out and fan in when applied to a shutter based optical switch.

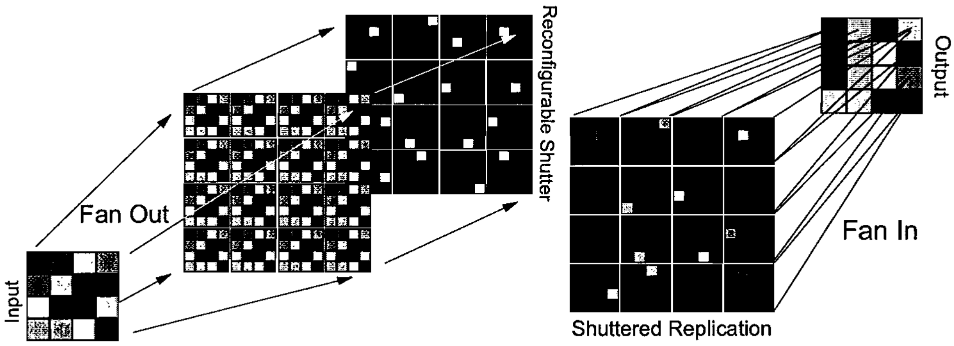
b) Sketch the basic structure of an *n* input to *n* output optical switch based on shutters or shadow logic. Define each optical component in the system and define the relationship between the number of input ports and the number of shutters required.

c) Discuss what are the main two limits to the scalability of such a shutter based optical switch. How important is the shutter switching speed? Name two suitable technologies which could be used for this shutter. What are their limitations?

d) If this shutter based switch were to be used as a packet switch, then it must be capable of allowing multiple input channels to be switched to a single output channel. How could the structure of part (b) be modified to allow this? What are the repercussions on the shutter technology?

Q4 crib.

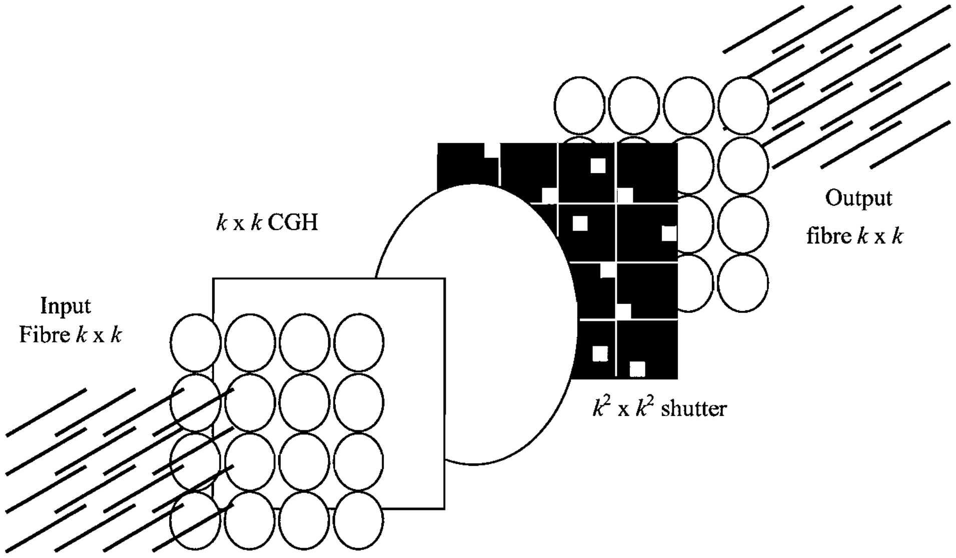
a)



The basic idea is to <u>fan out</u> (or replicate) the input light source array, such that if there are *n* inputs (in a *k* x *k* array, $k = n^2$) to the switch, then they would be <u>replicated</u> as an array of the input array *k* x *k* times. This can be done by the illumination of a *k* x *k* spot CGH or Damman grating to replicate the source inputs. The use of the CGH to replicate the inputs comes from the CGH property that the spots in the replay field are the FT of input illumination. In past examples this has been Gaussian, but it could equally be an input structure or an array of input sources.

The fanned out inputs are replicated as an *k* x *k* array of the input *k* x *k* array, all of which are incident on the shadow logic SLM or shutter. This device operates as a shutter array to block or pass the desired light from the inputs. For each replicated input, there will be one open shutter which selects the input source to be routed to a particular output, hence there have to be k^2 x k^2 shutters on the SLM. The position of the shutter selects the input source and the replication position selects the output to be routed to. The final stage is the most difficult as it <u>fans in</u> each replicated input array to a particular output position. The output plane is formed by the overlaying of all the replicated input sources.

b) The 1 to *n* switch can be expanded for *n* inputs (in an *k* x *k* array) to make a *n* by *n* optical crossbar. The requirements of the shutter SLM are now more critical. The input array of *k* x *k* fibres ($n = k^2$) has been fanned out to a *k* x *k* array of the *k* x *k* inputs. The shutter SLM requires a pixel for each replicated input so it must have a resolution of k^2 x k^2 so for a 100x100 input switch we would require a 10000x10000 pixel SLM!



c) The power appearing at the output will be the same for a given input fibre power (assuming all the inputs have the same power).

$$P_{out} = \frac{P_{in}}{n}$$

Hence the loss of the switch scales wit n, which is not a very desirable trait of a shutter based switch. More importantly, the crosstalk at each output will increase as the crosstalk from each shuttered input (of contrast ratio *B*) not selected will add when the replicated images are fanned into the output.

$$C = \frac{B}{n-1}$$

The expression of the crosstalk shows how the shadow routed crossbar can only operate for small values of *n*. If *n* is too large for a given *B*, then the crosstalk signal down the other channels will be larger than the actual routed signal leading to incorrect signals. The fan in to the output also becomes very critical, as each of the shuttered replications has to be overlapped onto a single output plane. This becomes an extremely difficult optical task when *n* is large.

The shutter speed is a very complex issue as many think the switch should reconfigure at the packet rate, however simulations show that a reconfiguration speed of the order of 1usec is perfectly adequate for most applications, especially those using IP packets. There is little point in reconfiguring any faster than the time it takes to process the packet header and addresses. In the 1usec shutter technologies there are two main choices; Electroclinic LCs, which are fast, but difficult to align and often very inefficient (and therefore lossy), and PLZT which is fast, but expensive and difficult to fabricate in large arrays. PLZT also need high addressing voltages.

d) The key point here is that in the full definition of a packet switch, any input can send data to any output in any combination, hence it could be possible that all inputs might want to send packets to a single common output port. In the extreme case, if there are n inputs, then there should be n3 paths to allow all possible combinations. This is clearly impractical, hence a modest number of paths are added known as 'speed up' in the packet switch.

Hence if there are extra paths, then there must be extra shutters and extra fan out and fan in optics to allow these extra paths. Simulations have shown that a modest speed up of 20% more paths will keep the packet loss below 1e-8 which is acceptable for most applications.