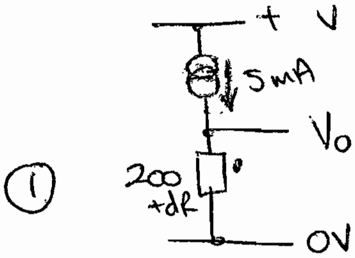


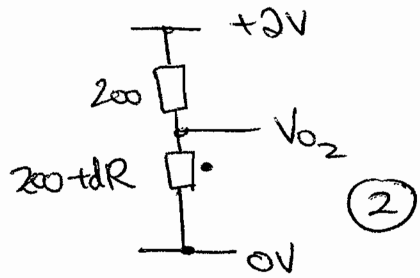
4B13 CRIB – 2006

1 (a) $R = Ae^{\beta'/T}$ with $\beta' = 3200$

$= 200$ @ $T = (273+20)\,K$

$\therefore A = 3.61 \times 10^{-3}$ and $\dfrac{dR}{dT} = -\beta'/T^2 \, R$





$$\frac{dR}{R} = -\beta'/T^2 \; dT = \frac{-3200}{293^2} \cdot dT = -0.0373 \; dT$$

$$\therefore dR = -7.45 \; \Omega/K \times 20 = -149 \; \Omega \quad \text{if linear}$$

for case ①, $V_{o_1} = (200 + dR) \times 0.005$ and $\dfrac{dV_{o_1}}{V_{o_1}} \simeq dR \times 0.005 = -0.745$

For case ②, $V_{o_2} = \dfrac{200+dR}{(400+dR)} \times 2$ and $\dfrac{dV_{o_2}}{V_{o_2}} \simeq \dfrac{dR}{2R} = -0.373$

Actual dR, from $(R+dR) = Ae^{\beta'/273+40}$ with $A = 3.61 \times 10^{-3}$, $R = 200$

$\Rightarrow dR = -100.6 \; \Omega$ from $(R+dR) = 99.4 \; \Omega$

$\therefore V_{o_1} = 0.497 \; V$ & $V_{o_2} = 0.664 \, V$

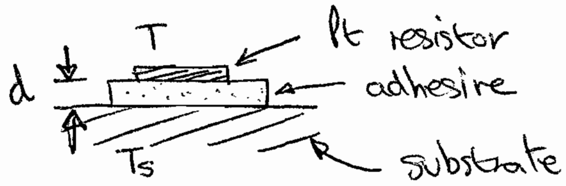
Hence non-linearity $= \left| \dfrac{\text{output non-linear} - \text{output linear}}{\text{linear output change}} \right| \times 100\%$

Non-lin. case ① $= \left| \dfrac{0.497 - (1-0.745)}{0.745} \right| \times 100\% = 32\%$

Non-lin. case ② $= \left| \dfrac{0.664 - (1-0.373)}{0.373} \right| \times 100\% = 9.9\%$

The simple potential divider is actually more linear, but gives a smaller output change with temperature

[30%]

1(b) (i) 

Heat flux to Pt $= \dfrac{kA(T_s - T)}{d} = m\,c_p\, dT/dt$

where $d = 10^{-4}$ m $\qquad A = 20\times10^{-6}$ m^2

$c_p = 1{\cdot}1\times10^{3}$ J/kg/K $\qquad m = 10^{-4}$ kg

$k = 0{\cdot}3$ W/K/m

$\therefore \dfrac{dT}{dt} = -\dfrac{kA\,T}{d\,m\,c_p} + C = -T/\tau + C$

This has a soln. of $T = X e^{-t/\tau} + Y$ where $\tau = \dfrac{m c_p d}{kA}$

and $C = T_s/\tau$

(X and Y are set by the boundary conds. ie: initial and final temperatures)

Hence, the time constant $\tau = \dfrac{10^{-4}.\ 1{\cdot}1\times10^{3}.\ 10^{-4}}{0{\cdot}3\ .\ 20\times10^{-6}} = 1{\cdot}83$ s

So, the 10%–90% rise-time $\simeq 2{\cdot}2\tau = 4{\cdot}0$ s and the -3dB frequency $= \dfrac{1}{2\pi\tau} = 0{\cdot}087$ Hz

(ii) with 10mA current, power dissipation $= 0{\cdot}01$ W

Hence, steady heat flow to substrate $= 0{\cdot}01\text{ W} = \dfrac{kA\,\Delta T}{d}$

where ΔT is the measurement error

$\Delta T = \dfrac{0{\cdot}01\ .\ 10^{-4}}{0{\cdot}3\ .\ 20\times10^{-6}} = 0{\cdot}17\ ^\circ\text{C}$ [50%]

(c) Stephan's Law $\qquad W = \epsilon\,\sigma_{SB}\,T^4$

If the iron is at 160°C $= 433$K with $\epsilon = 0{\cdot}5$, $\dfrac{W}{\sigma_{SB}} = 1{\cdot}76\times1$

$\epsilon = 0{\cdot}8$, $\dfrac{W}{\sigma_{SB}} = 2{\cdot}81\times$

But $\epsilon = 0{\cdot}95$ is assumed $\Rightarrow 1{\cdot}76\times10^{10} = 0{\cdot}95\,T_{0\cdot5}^4 \Rightarrow T_{0\cdot5} = 96^\circ$

or $2{\cdot}81\times10^{10} = 0{\cdot}95\,T_{0\cdot8}^4 \Rightarrow T_{0\cdot8} = 142^\circ$

$\therefore$ under-reads by 18–64°C

[20%]

2 (a) Advantages of using a group of references:–

(i) When these are chosen to be close to the quantity being measured, only small differences have to be determined so almost eliminating the uncertainties of the meter measuring the differences.

(ii) A group of references allow their periodic calibration to be done more simply. The one which is affected least by travel is sent for calibration – its value is traced to the others on its return.

(iii) Noise in any item, both unknown or from the references themselves can be attributed to its source unambiguously.

(iv) ~~Any~~ deficiencies in the references is diminished by the number of references. As V_O = Mean of Difference Readings + Mean of Value of References — (A)

For 4 references :– As $V_O = [(V_O - V_A) + (V_O - V_B - - -]/4 + (V_A + V_B + V_C + V_D)/4$

4 V_O terms but one V_A term etc.

from last calibration

[20%]

(b) Assuming V_A is unchanging in the test, $(V_O - V_A)$ changes give TC of $+3.2\,mV/(26.1-15.2) = 0.2936\ mV/^\circ C$

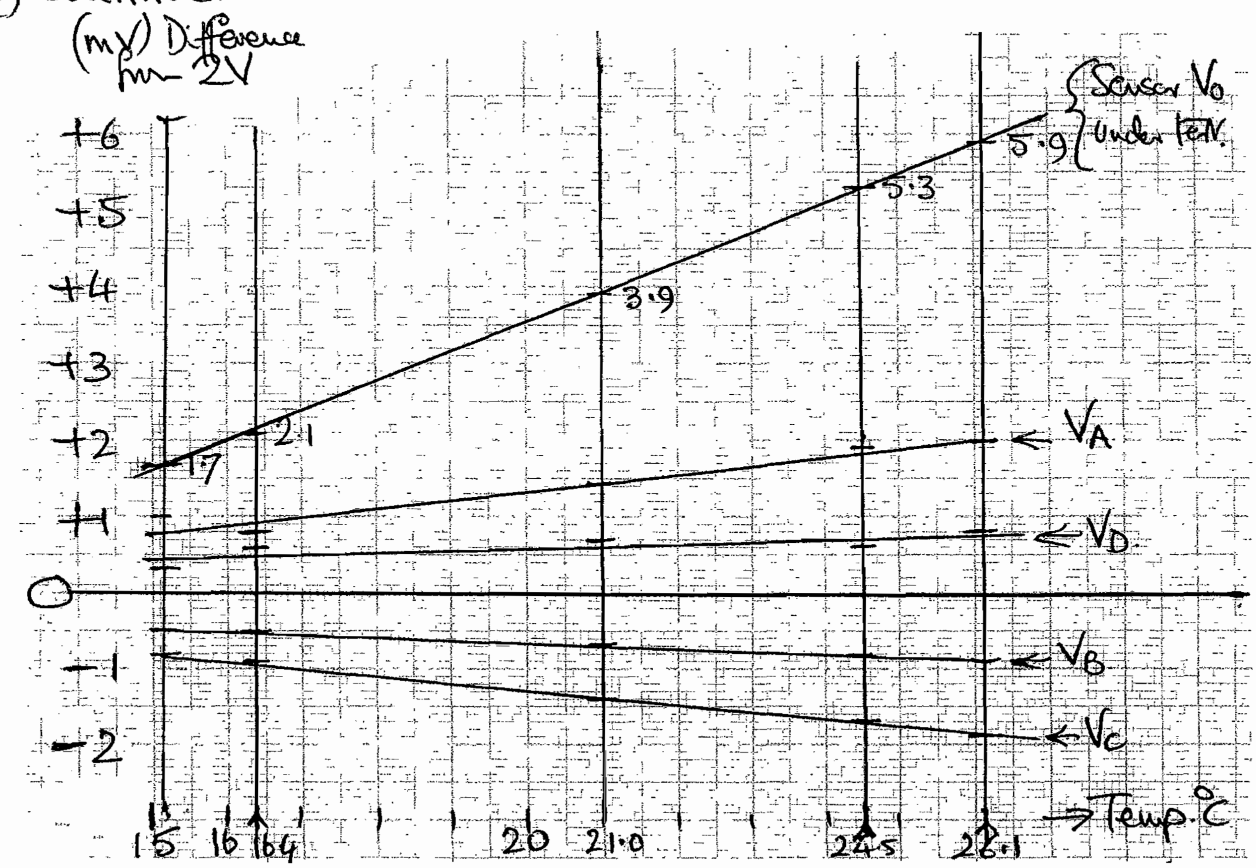
(As $2mV/2V = 0.1\%$) $\rightarrow$ $= 0.015\ \%/^\circ C$ for V_O

Determining the Mean of the Differences for each temperature:–

Temperature °C	15.2	16.4	21.0	24.5	26.1	= Diff of ... from ...
Mean of Diffs (mV)	+1.7	+2.1	+3.9	+5.3	+5.9	
$V_A = V_O$ − Difference measured	(1.7−0.7) = +1.0	(2.1−1.3) +0.8	(3.9−2.5) +1.4	+1.9	2.0	← Diff of ... from 2V
V_B	−0.5	−0.5	−0.7	−0.8	−0.9	Diffs from 2V
V_C	−0.8	−0.9	−1.4	−1.7	−1.9	
V_D	+0.3	+0.6	+0.7	+0.6	+0.8.	

(Continued)

2 (b) continued.



Points for sensor output, V_O, lie exactly on a line and slope is $\frac{5.9-1.7}{26.1-15.2} = +0.385$ mV/°C

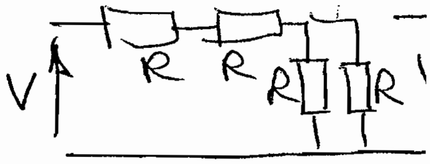
$= +0.019$ %/°C

Points for V_C and V_B lie not closely to a line – both appear to have small negative temperature coefficients.

V_A and V_D have points not so close to a line, both appear to have small positive temperature coefficients.

(60

(c) A Hamon 5:1 divider uses 4 resistors, connected as:–



The output V_O is $\left(\frac{\frac{1}{2}R}{2R+\frac{1}{2}R}\right)V = \frac{V}{5}$. when resistors really are equal.

(Continued)

2 (c) continued

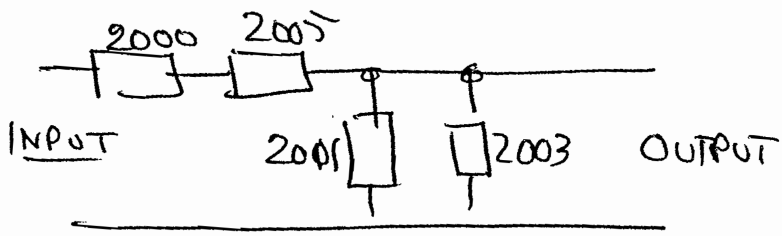
So advantages are :-

(i) Uses only 4 resistors and not 5 for divider

(ii) By interchanging the two series resistors and putting them as the parallel pair (& vice versa), a second V/5 is given. The Mean of the two divider outputs is VERY CLOSE to ideal — ⊛

(iii) Using resistors of the same type & material & batch, their temperature coefficients should be very close so giving negligible error as temp changed.

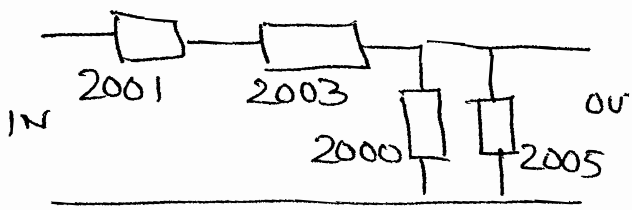
⊛ Illustration



$2001\Omega \,//\, 2003\Omega = 1000.99975$

So Ratio is $\dfrac{1000.99975}{1000.99975 + 4005} = \dfrac{1}{5.001000}$

Interchanging resistors gives



Similar calculation gives ratio $\dfrac{1}{4.99900748}$

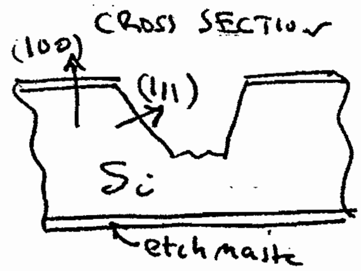
Average of 2 readings removes the 1 part in 5000 error of one reading. Division by $\dfrac{1}{5.00000374}$ (which is an error from $\frac{1}{5}$ of only 7.5 in 10^7)

(~30%)

(Expected only in brief)

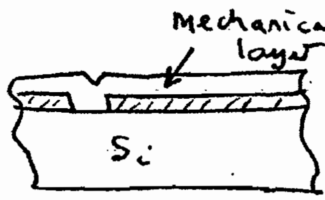
4B13 Q3. (a) The well known and predictable properties of single crystal silicon make it attractive for pressure, acceleration and other mechanical sensors because the flexing structures can be made with great precision and in some cases integrated with electronics.

DJm 1/2006 *



bulk micromachined.

In bulk micromachining the silicon substrate is masked by a resistant layer such as CVD silicon nitride, and anisotropic etchants such as KOH in water are used to etch deep into or through the silicon.



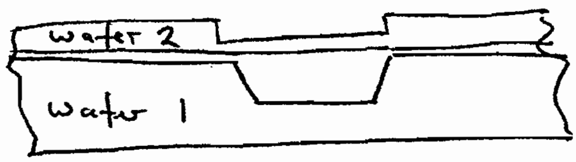
Surface micromachined

In surface micromachining a sacrificial layer such as SiO_2 or polyimide is deposited and patterned (shaded layer in the diagram) on the silicon substrate prior to depositing the layer with the mechanical function.
A selective etchant is then used to remove the sacrificial layer leaving the flexing beams or other freestanding mechanical structures.

Low cost manufacturing is achieved by processing a whole wafer and then dicing individual components – for example hundreds of bulk micromachined pressure sensors are made on one substrate.

Bonding two wafers together can also be cost effective to produce for example sealed cavities in absolute pressure sensors.

cross section of bonded wafer.



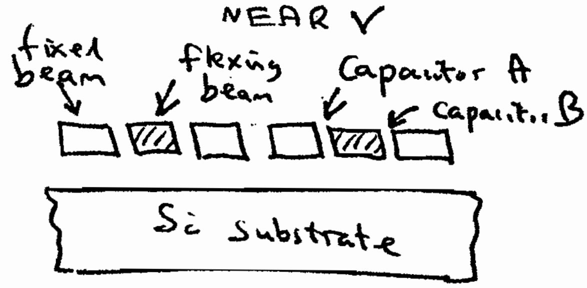
Surface micromachining processes can be made compatible with the presence of conventional CMOS circuitry on the same chip. This is desirable to improve reliability and reduce assembly costs (no wire bonding).

[50%]

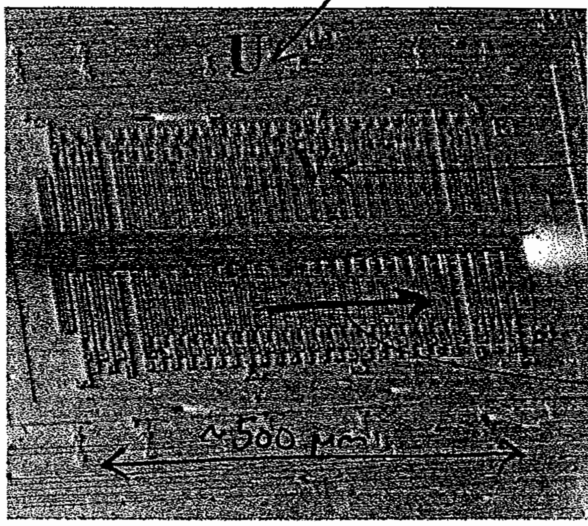
4B13 Q3 (cont.) DPM 1/2006

Surface micromachined polycrystalline silicon capacitance accelerometer

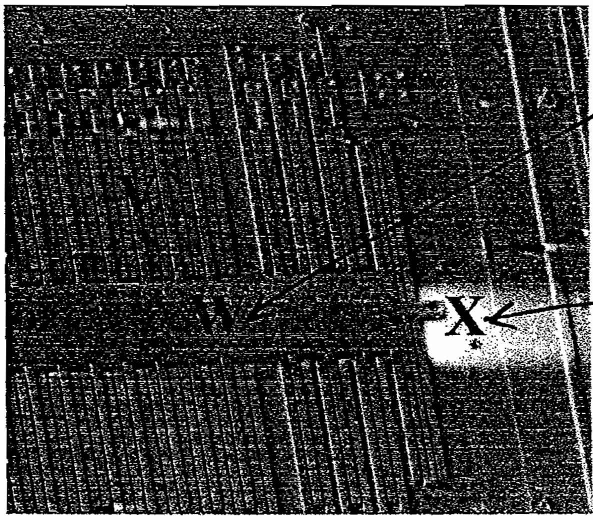
CROSS SECTION NEAR V



(I)



(II)

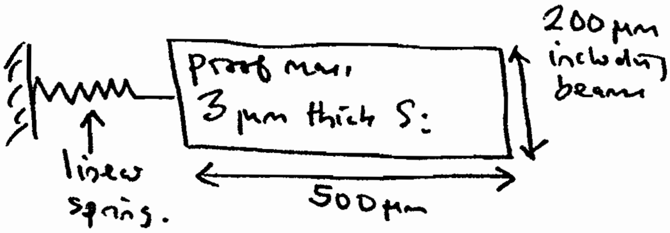


One set of fixed beams forms the capacitor A with the proof mass beams and the other set forms capacitor B.

The difference A−B is monitored and gives the readout in the open loop operation.

For closed loop operation a small fraction of the capacitor beams are used for electrostatic actuation rather than sensing. The proof mass is maintained in one position and the feedback voltage is the output signal.

approximate mechanical equivalent model



Order of magnitude calculation.

Total mass of suspended Si: $m_1 = 3\times10^{-6} \times 500\times10^{-6} \times 200\times10^{-6} \times 2300$ kg (2300 = density)

$$= 6900\times10^{-13} \simeq 7\times10^{-10}\ \text{kg}$$

Estimate of initial gap 10^{-6} metres (from Figure 1)

Given that an acceleration of $5\,\text{ms}^{-2}$ closes this gap on one side by 1%

Force = mass × acceleration = k_1 × distance, where the force constant is k_1,

$$k_1 = \frac{7\times10^{-10}\times5}{0.01\times10^{-6}} = 35\times10^{-2} = \underline{0.35\ \text{newton/metre}}$$

[50%]

4 (a) $v = f\lambda$ $\therefore$ in water $\lambda = \frac{1500}{20\times10^{6}} = 75\times10^{-6}$ m

(75 μm) resolution

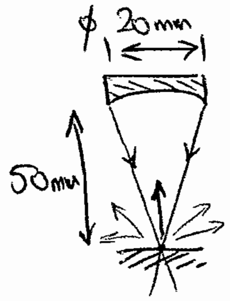
Pressure Reflection Coeff; $p = \frac{Z_e - Z_w}{Z_e + Z_w}$ where $Z_e = 2100\times2400 = 5.04\times$

$Z_w = 1000\times1500 = 1.50\times$

[15%] Z = density $\times$ speed of sound

$\therefore p = 0.541$ $\therefore$ power reflect. $= p^2 = P_r = 0.293$

(b)



If surface smooth, beam is reflected straight back.

If surface rough, beam is scattered into ½sphere

Total path length $\simeq$ 100mm $\therefore$ attenuation = 3.5 dB $\equiv \times 0.447$

Power reflection coeff. (from (a)) $\equiv \times 0.293$

Area of ϕ20mm as fraction of 50mm radius ½ sphere $= \frac{\pi 10^2}{\frac{4}{2}\pi 50^2} \equiv \times 0.02$

$\therefore$ Fraction returned from smooth surface $= 0.447\times0.293 = 0.13$

[45%] —— " —— rough surface $= 0.13\times0.02 = 2.6\times10^{-3}$

(c) Epoxy-water power refl. coeff. = 0.293 from (a)

Epoxy-copper/solder power refl. coeff $= \left(\frac{5.04-32.2}{5.04+32.2}\right)^2 = 0.532$

$Z_c = 9200\times3500 = 32.2\times10^6$

PZT - water power ref. coeff $= \left(\frac{30-1.5}{30+1.5}\right)^2 = 0.82$

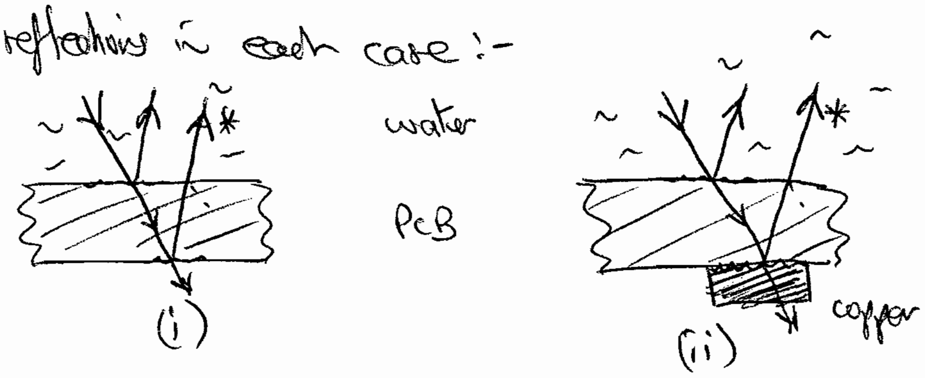
$Z_p = 7500\times4000 = 30\times10^6$

$\therefore$ Initial power launched into water $= \frac{V^2}{R}\cdot 0.05\cdot(1-0.82)$

$= \frac{100}{50}\cdot 0.05\cdot 0.18 = 0.018$W

4(c) contd.

Consider reflections in each case :-



The only change is the power in beam * where case (ii) has increased power by a factor of $\frac{0.532}{0.293} = 1.82$.

So considering case (i),

water → epoxy : × 0.293 reflected ∴ × 0.707 transmitted
epoxy internal reflection : × 0.293
epoxy → water : × 0.707 transmitted (could neglect this)

∴ Power received back from interface * ⇒

$$P_r = \underset{\text{launch power}}{0.018} \times \underset{\text{atten. in water}}{0.447} \times \underset{\text{top interface trans.}}{0.707} \times \underset{\text{bottom interface reflc.}}{0.293} \times \underset{\text{top interface trans.}}{0.707} \times \underset{\text{scattered collection factor}}{0.02}$$

$\therefore P_r = 2.36 \times 10^{-5}$ W this then couples to the PZT transducer with factor $(1-0.82)$ and converted 5% to electrical power

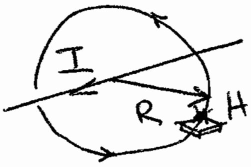
$\therefore P_{elec.} = 2.36 \times 10^{-5} \times 0.18 \times 0.05 = 2.12 \times 10^{-7}$ W $= V_r^2/R$ ← $= 50\Omega$

$\therefore V_r = 3.26 \times 10^{-3}$ V into matched load
or 6.52 mV open-circuit

This is then scaled by a factor of $\sqrt{1.82}$ for copper/epoxy reflection
$= 8.79$ mV ∴ change in signal $= \underline{2.27\text{ mV}}$

[40%]

5 (a)



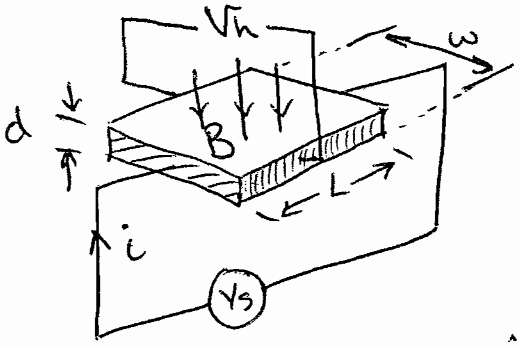
$2\pi R H = I$, $R = 0.05\text{m}$

$\therefore H = 31.8$ A/m

[10%]

$B = \mu_0 H = 0.40$ mT $\qquad \mu_0 = 4\pi \times 10^{-7}$ H/m

(b)



$i = n A v_d q$ (carrier density: n; $w \times d$: A)

$v_d = \mu \frac{V_s}{L}$ (mobility: μ)

$\therefore i = n w d \mu \frac{V_s}{L} q$

$= 10^{15} . 0.2 . 0.001 . 50\times10^{3} . \frac{5}{0.2} . 1.6\times10^{-19}$ (cm units in eqn.)

$i = 0.04$ A

Hence, resistance $R = \frac{V_s}{i} = \underline{125\,\Omega}$

Force on carriers, $F = B q v_d = \frac{V_h}{w} q$

$\therefore V_h = w B v_d = \cancel{w} B \mu \frac{V_s}{\cancel{L}}$ with $w = L$

$= 10^{-3} . 50\times10^{3} \times 10^{-4} . 5$

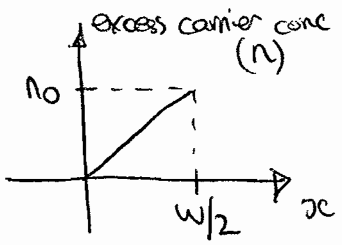
$\underline{V_h = 25\times10^{-3}\text{ V} = 25\text{mV}}$

[40%]

(c)

To calculate response time, estimate diffusion of carriers across width of slice.

Assume linear $\underline{\text{excess}}$ carrier conc. due to magnetic field, symmetric each 1/2 of slice width



$\therefore \frac{dn}{dx} = \frac{2n_0}{w}$ and $n = \frac{2n_0}{w} x$

Then diffusion flux, $f = -D \frac{dn}{dx}$, $D = \frac{\mu k T}{q}$

Fick's Law $\qquad$ Einstein relation

5(c) contd.

If $N =$ total excess carriers one side $= Ld \int_0^{w/2} \frac{2n_0}{w} x \, dx$

$$\therefore \quad N = \frac{n_0 w L d}{4}$$

Considering one side, $\frac{dN}{dt} = f L d$ ← = carriers/sec. crossing centre line

$$\therefore \frac{dN}{dt} = -D \frac{2n_0}{w} Ld = -D \frac{2}{w} \frac{4N}{w\cancel{Ld}} \cancel{Ld} = -\frac{8DN}{w^2}$$

This has a soln. of form $N = N_0 e^{-t/\tau}$

$$\therefore -\frac{N_0}{\tau} e^{-t/\tau} = -\frac{8DN_0 e^{-t/\tau}}{w^2} \qquad \text{with } \tau = \frac{w^2}{8D}$$

$$\therefore \tau = \frac{(2\times10^{-3})^2 \cdot 1.6\times10^{-19}}{8 \cdot 50\times10^{3}\times10^{-4} \cdot 1.38\times10^{-23} \cdot 300} = 3.86\times10^{-6}\text{ s} = 3.86\,\mu\text{s}$$

10%–90% rise-time, $t_r = 2.2\tau = \underline{8.5\,\mu s}$

Bandwidth (−3dB) $\approx \frac{1}{2\pi\tau} = \underline{41.2\text{ kHz}}$ [30%]

(d) $v_{n\,\text{thermal}} = \sqrt{4kTRB}$ where $B = 41.2\times10^{3}$ Hz

$R = 125\,\Omega$

$T = 300$ K

$$\therefore v_n = 2.92\times10^{-7}\text{ V} = 0.29\,\mu\text{V}$$

with a sensitivity from part (b) of 25 mV/mT = 25 V/T and a flux density of 0.4 mT/100 A from part (a), the system responsivity $= \frac{25\times10^{-3}\times0.4}{100}$ V/A $= 10^{-4}$

$\therefore 0.29\,\mu\text{V} \equiv \underline{2.9\text{ mA}_{\text{rms}}\text{ current}}$ I_{noise}

(In practice, it would be much more than this as Hall sensors have significant 1/f noise)

[20%]