4C1 2006

1. (a)

Line tension T is the force acting along the dislocation line.

$T \equiv$ change in energy when a dislocation is extended or shortened by unit length

This is the same as the energy / unit length of dislocations, i.e.,

$$T = Gb^2/2$$ (15%)

dislocation lines tend to be straight in the absence of internal stresses to minimise dislocation line energy.

(b) Dislocation energy / unit length $= Gb^2/2$ (10%)

Thus use smallest lattice displacement vector for b to minimise energy. Typically, b = shortest atomic spacing ≈ 0.3 nm

(c) The resistance force acting on a dislocation is (10%)

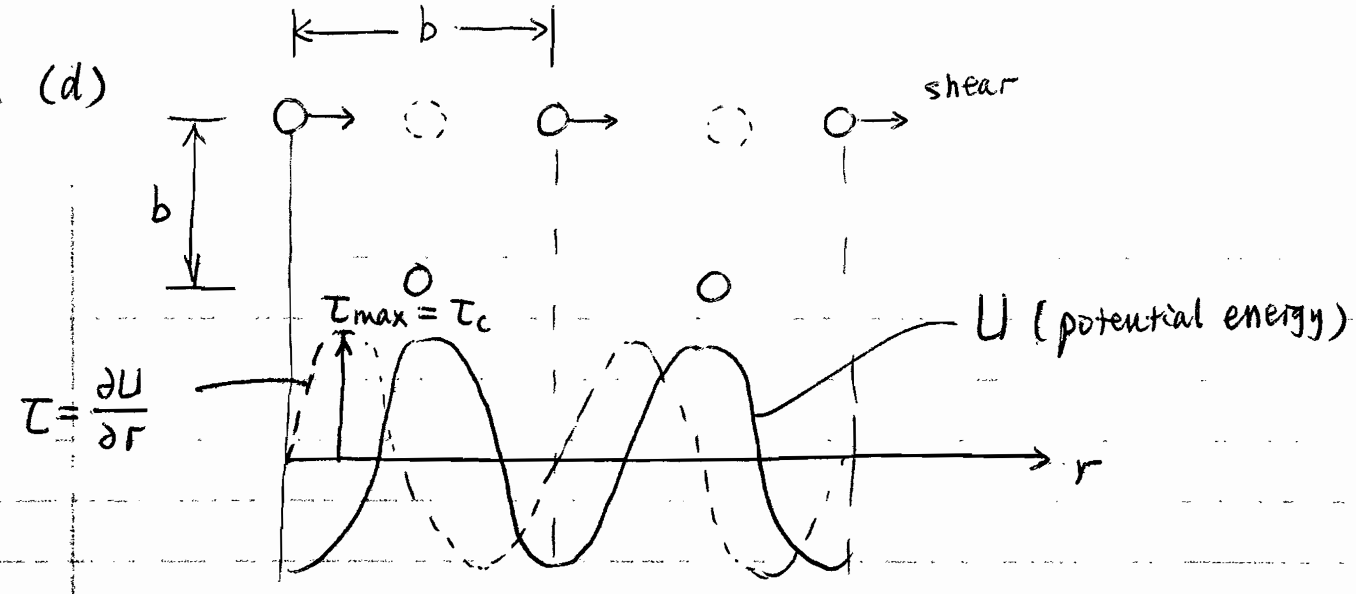
$$f = \tau b \qquad (\text{N/m})$$

Where τ is the shear stress around the dislocation.

This expression is true for edge & screw dislocations or a mixture of both.

The force is always normal to the dislocation line.

1. (d)



Consider the above two neighbouring planes of atoms in a FCC crystal subjected to a shear stress τ:

$$\tau = \tau_c \sin \frac{2\pi r}{b}$$

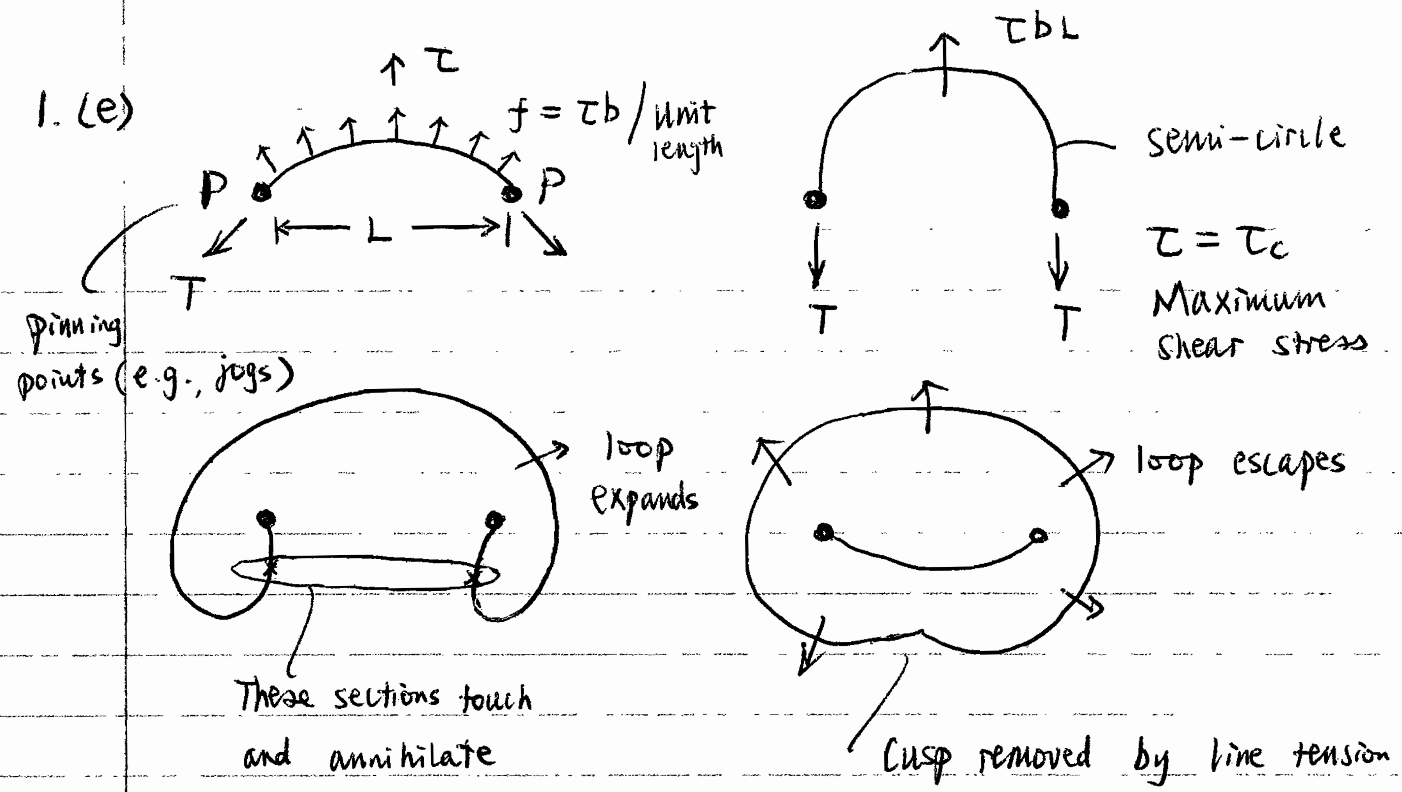
$$\Rightarrow \quad G = \frac{\Delta \tau}{\Delta \gamma}\bigg|_{r=0} = \frac{\partial \tau}{\partial r / b}\bigg|_{r=0} = 2\pi \tau_c$$

$$\Rightarrow \quad \tau_c = G/2\pi$$

With G typically 100 GPa, we get $\tau_c = 16$ GPa, which is orders of magnitude larger than typical measured values of shear strength of metals $\tau_c \approx 100$ MPa.

(25%)

1. (e)



Frank-Read source

Line tension $T = Gb^2/2$ (25%)

At maximum shear, balancing of force : $\tau b L = 2T$

$$\Rightarrow \quad \underline{\tau_c = Gb/L} \qquad \text{Frank-Read eqn.}$$

(f) $G = 100\ GPa = 10^{11}\ N/m^2$

$b = 0.3\ nm = 3 \times 10^{-10}\ m$ (15%)

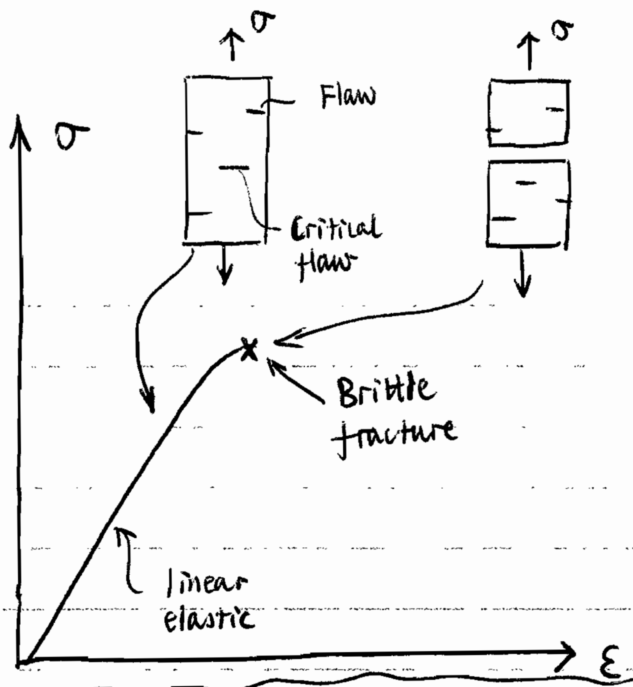
$L \approx 10^{-6} \sim 10^{-7} m$ (jog spacing)

$\Rightarrow \quad \tau_c = 30 \sim 300\ MPa$

This prediction is more reasonable than the ideal shear strength model ($\tau_c \approx 10\ GPa$)

2.

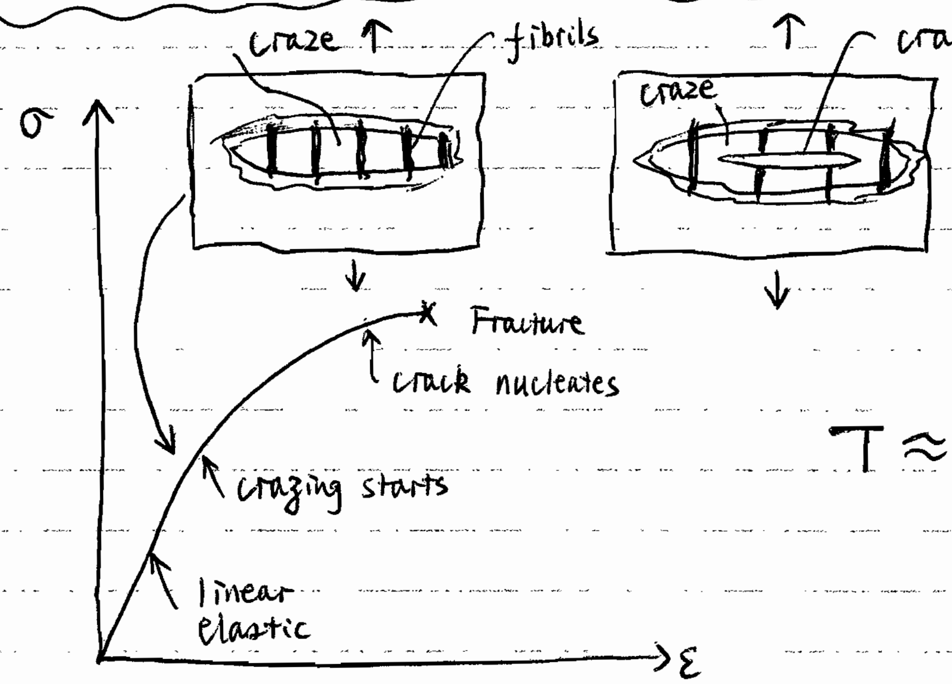
(a)



$T < 0.8\,T_g$

Fast propagation of intrinsic flaws (mainly on surface, similar to cleavage 1) or at localised craze (similar to cleavage 2) (15%)

(b)



(15%)

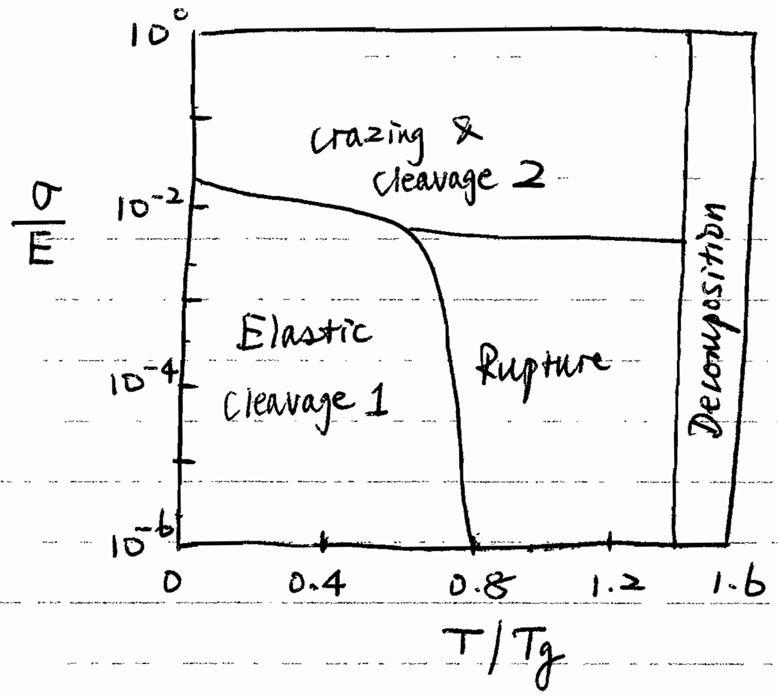
$T \approx 0.8\,T_g$

Localised plastic flow occurs ahead of flaws → leading to stretching out of microfibrils separated by voids (craze).

Crack grows unstably through craze (similar to cleavage 2).

2.

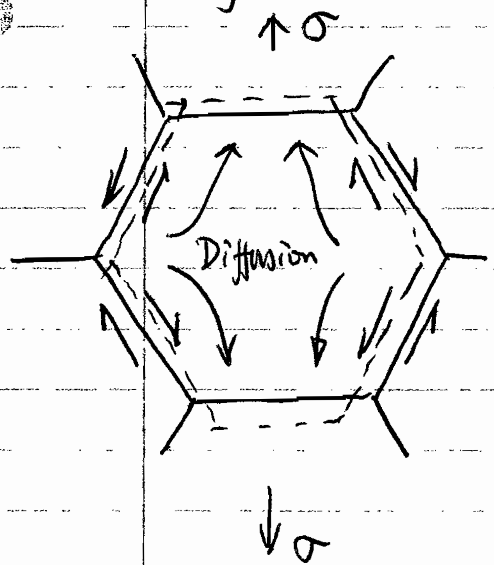
(c)



Amorphous polymers

(20%)

(d) Coble creep and Nabarro–Herring creep are forms of creep by diffusional flow, which occur at low stresses and high tempera ($T > 0.5T_m$). Atoms diffuse from grain boundaries which are at an angle to the tensile stress, and move onto boundaries which are approximately normal to the stress. The grains therefore elongate in the direction of applied stress, whilst grain boundary sliding occurs to accommodate the shape change in grains.



In Coble creep the dominant diffusion path is along the grain boundaries.
In Nabarro–Herring creep it is through the bulk of grains.

(20%)

2.

(e) (i) Plastic yielding

Plastic flow is a shear deformation process, with yielding taking place when the Mises effective stress reaches a critical value, i.e. when

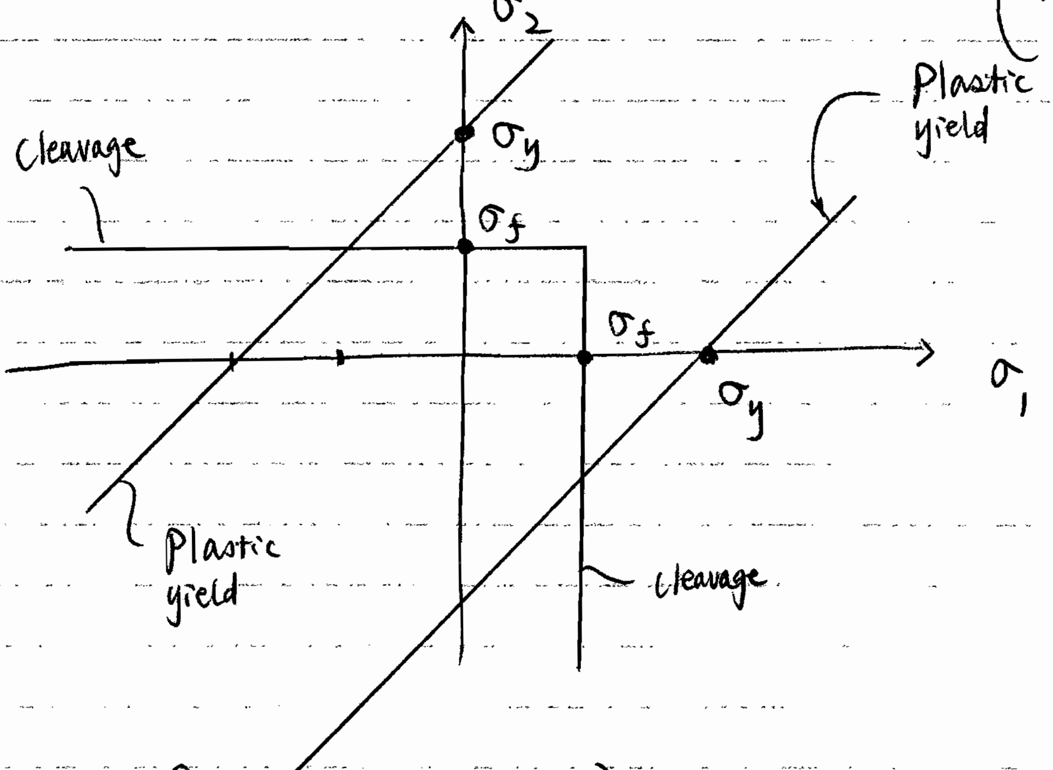
$$\sigma_e = \sqrt{\frac{1}{2}\left\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right\}} = \sigma_y$$ (15%)

$$\Rightarrow \quad \sigma_e = |\sigma_1-\sigma_2| = \sigma_y \quad \text{since} \quad \sigma_2=\sigma_3$$

(ii) Cleavage failure (propagation controlled)

Failure occurs when the stress intensity factor for cracks oriented normal to the direction of maximum principal stress reaches a critical value – fracture toughness. Consequently, the material fails when the maximum principal stress reaches a critical (tensile) value σ_f.

(15%)



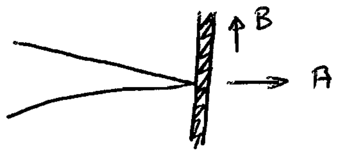
(Assume $\sigma_y > \sigma_f$)

6

Q3

(a) Pure mode I ($K_{II} = 0$)

$\theta = 0°$: $\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}$, $\theta = 90°$: $\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} \cos^3\frac{\pi}{4} = \frac{1}{4}\frac{K_I}{\sqrt{\pi r}}$



Path A $\theta = 0°$: $\rho_A = \frac{K_{IA}}{\sqrt{2\pi(0.01a)}} \Rightarrow K_{IA} = \rho_A\sqrt{0.02\pi a}$

Path B $\theta = 90°$ $\rho_A = \frac{1}{4}\frac{K_{IB}}{\sqrt{\pi(0.01)a}} \Rightarrow K_{IB} = \rho_B\sqrt{0.16\pi a}$

(40%)

Path A followed if $K_{IA} < K_{IB}$

$\Rightarrow \rho_A\sqrt{0.02\pi a} < \rho_B\sqrt{0.16\pi a} \Rightarrow \frac{\rho_A}{\rho_B} < 2\sqrt{2}$

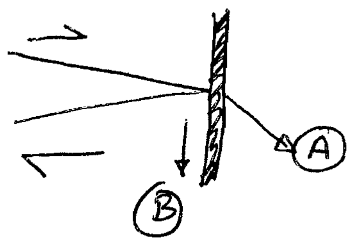
(b)
(i) The crack will branch along the direction for which $\sigma_{r\theta} = 0$ & $\sigma_{\theta\theta} > 0$. In mode II we have

$\sigma_{r\theta} = 0 \Rightarrow (3\cos\theta - 1) = 0$
$\Rightarrow \theta = \pm 70.5°$

$\sigma_{\theta\theta} > 0$ for $\theta = -70.5°$ & hence the crack will propagate into the bulk at $\theta = -70.5°$

(20%)

(b) (ii)



The normal stress is $\sigma_{\theta\theta} = -\frac{3}{2} \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \sin\theta$

<u>Path A:</u> $\theta = -70.5^\circ$, $\sigma_{\theta\theta} = -\frac{3}{2} \frac{K_{II}}{\sqrt{2\pi r}} \cos\left(-\frac{70.5^\circ}{2}\right) \sin(-70.5^\circ)$

$$= 1.16 \frac{K_{II}}{\sqrt{2\pi r}}$$

$$\Rightarrow \quad \rho_A = 1.16 \frac{K_{IIA}}{\sqrt{2\pi(0.01a)}} \quad \Rightarrow \quad K_{IIA} = \frac{\rho_A \sqrt{0.02\pi a}}{1.16}$$

<u>Path B</u> $\theta = -90^\circ$: $\sigma_{\theta\theta} = -\frac{3}{2} \frac{K_{II}}{\sqrt{2\pi r}} \cos 45^\circ \sin(-90^\circ) = \frac{3}{4} \frac{K_{II}}{\sqrt{\pi r}}$

$$\Rightarrow \quad \rho_B = \frac{3}{4} \frac{K_{IIB}}{\sqrt{\pi(0.01)a}} \quad \Rightarrow \quad K_{IIB} = \frac{4}{3} \rho_B \sqrt{0.01\pi a}$$

Path A followed if $K_{IIA} < K_{IIB}$

$$\Rightarrow \quad \frac{\rho_A \sqrt{0.02\pi a}}{1.16} < \frac{4}{3} \rho_B \sqrt{0.01\pi a} \quad \Rightarrow \quad \underline{\frac{\rho_A}{\rho_B} < 1.09}$$

(40%)

Q4

(a) The critical energy release rate ~~for~~ fracture G_{IC} is much greater than twice the surface energy for engineering alloy typically fail by ductile fracture. The energy dissipated in crack tip plasticity, per unit area of fracture surface is very much greater than the surface energy. (15%)

(b)

(i) deflection of indenter $v = \varepsilon l$

where $\varepsilon = \dfrac{P}{\pi a^2 E}$

hence $C = \dfrac{v}{P} = \dfrac{l}{E\pi a^2}$ (15%)

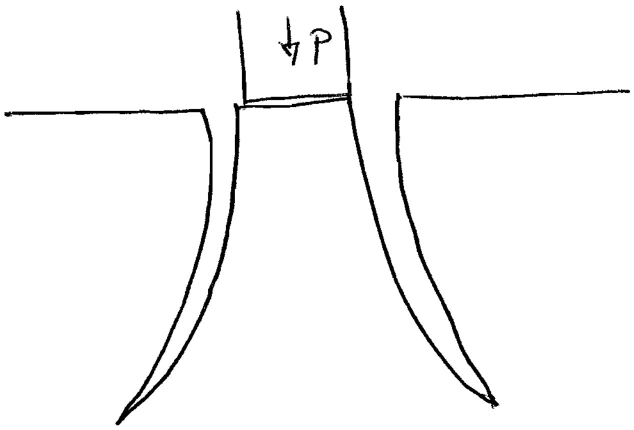
(ii) $G = \dfrac{1}{2} \dfrac{P^2}{2\pi a} \dfrac{dc}{dl}$

[Note that the crack length is l, & the thickness B has been replaced with $2\pi a$, since a crack advance δl, produces a new crack area $2\pi a \, \delta l$].

$\dfrac{dc}{dl} = \dfrac{1}{E\pi a^2} \Rightarrow G = \dfrac{P}{4\pi a} \dfrac{1}{E\pi a^2} = \dfrac{P^2}{4\pi^2 E a^3}$ (50%)

Under fixed P, G is independent of crack length l, ie "neutral" stability - it is not necessary to increase the load to cause further crack propagation, neither does the crack run away since the energy release rate is always just sufficient for crack propagation.

(iii) There is a mode II component of loading at the crack tip. Hence a will increase with l



(20%)