Engineering Tripos Part IIA: Module 4C2
Designing with Composites
CRIB - 2005/6

Examiner / Assessor
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1 (a) $E_1 = 39\,GPa$, $E_2 = 8.3\,GPa$, $G_{12} = 6.1\,GPa$, $\nu_{12} = 0.26$, $\nu_{21} = \nu_{12}\frac{E_2}{E_1} = 0.055$

$Q_{11} = E_1/(1-\nu_{12}\nu_{21}) = 39.6\,GPa$, $Q_{22} = 8.4\,GPa$, $Q_{12} = 2.2\,GPa$, $Q_{66} = 8.4\,GPa$.

Need $\bar{Q}$. As balanced symmetric $A_{16} = A_{26} = 0$, so no need for Q_{16}, Q_{26}

+45° $\bar{Q}_{11} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2$ $\quad (s^2 = c^2 = \frac{1}{2},\ s^4 = c^4 = \frac{1}{4})$

$= 17.2\,GPa$

$\bar{Q}_{22} = \bar{Q}_{11}$ by symmetry.

$\bar{Q}_{12} = 9.0\,GPa$, $Q_{66} = 10.9\,GPa$.

$$\bar{Q}_{45} = \begin{bmatrix} 17.2 & 9.0 & - \\ 9.0 & 17.2 & - \\ - & - & 10.9 \end{bmatrix} GPa$$

$[A] = 4t\bar{Q}_{45}$ [Q_{16}, Q_{26} terms cancel with −45° ply. $Q_{11}, Q_{12}, Q_{22}, Q_{66}$ add]

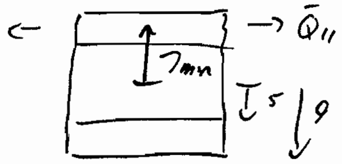
$$= \begin{bmatrix} 68.8 & 36 & 0 \\ 36 & 68.8 & 0 \\ 0 & 0 & 43.6 \end{bmatrix} GPa\,mm.$$

This is the stiffness matrix for in-plane loads/displacements. $B = 0$ is for bending, as is D. [45%]

(b) Axial stiffness $= 2 \times A_{11} = 137.6\,MPa\,m$.

Bending stiffness D, for $M_x = D_{11}K_x$. We assume we can ignore coupling with other bending moments, eg D_{12}.

For this geometry, it may be good enough to ignore bending stiffness of face sheets, ie



$D = 2 \cdot A_{11} \times (7mm)^2 = 6.7 \times 10^3\,Nm$

To include bending, could use $D = \frac{1}{3}\sum \bar{Q}(z_k^3 - z_{k-1}^3)$

$= \frac{1}{3} \cdot \bar{Q}_{11}(9^3 - 5^3) \times 2$

$= 6.9 \times 10^3\,Nm$ [30%]

(c) Glass ⇒ not high cost.

⇒ probably not using autoclave, instead a resin transfer moulding (RTM) process might be more economic, using vacuum bagging for consolidation and oven curing. If the foam is strong enough and OK at the cure temperature it may be possible to cure and bond in one shot. Otherwise bond after making the face sheets. [25%]

[Comments on Q1: Well answered. Discussion in part (c), eg between RTM and autoclave processes required.]

2. (a) We assume that the laminate contains 0°, 90° and 45° plies. The ε is the same in all the plies, so that the ~~ft~~ ply with the smallest ε to failure in a given direction is the one that fails first. The advantage is that only laminate stiffness needs to be calculated, not σ's in individual plies.

	CFRP	GFRP	Kevlar
e^+	0.4	0.3	0.5
e^-	0.5	0.7	0.1
e_{LT}	0.5	0.5	0.3

e^+ smaller due to weak transverse tension failure mode

Low due to poor compression of Kevlar fibres

Shear failure maps to transverse tension of a 45° ply

[25%]

(b) The basic assumption, that we are loading all fibre directions 0, 45 and 90 is now no ~~m~~ longer valid. Assuming the laminate ε is still calculated correctly, then it is possible that the ε allowable estimate is too conservative, as the weakest mode will not be present. For example a $\pm 10°$ laminate won't suffer so badly from transverse tensile failure of a (non-existent) 0° ply. But the details will depend on the loading and lay up, and we can expect serious inaccuracies.

[Comment: I was looking for sensible comments relating the failure to failure mechanisms in different plies.]

[25%]

2 (c) Assume all N_x load carried by 0° plies

$\Rightarrow t_0 E_1 = E_x \;,\; \varepsilon_x = N_x / E_x = e^+$

$\Rightarrow t_0 = \dfrac{N_x}{\varepsilon^+ E_1} = 3.6\,mm \quad \left(\dfrac{2 \times 10^6}{0.004 \cdot 140 \times 10^9} \right)$

N_y loading: $t_{90} = \dfrac{N_y}{e^- E_1} = \dfrac{10^6}{0.005 \cdot 140 \times 10^9} = 1.4\,mm$

Include 10% 45° plies to be on the safe side

(may not be necessary given the actual mix)

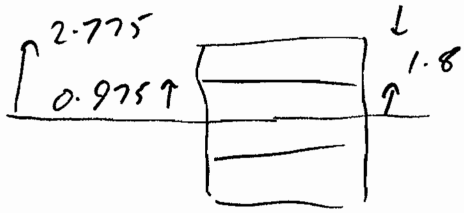
$\Rightarrow t_{total} = \dfrac{5.0}{0.9} = 5.55\,m$ with $0:90:45 = 65:25:10\%$

[25%]

(d) Axial stiffness

~~Axial~~ $\approx E_1 t_0 = 140 \times 3.6 = 500 \times$ MNm^{-1}

Bending stiffness: maximise by putting 0°'s on outside.



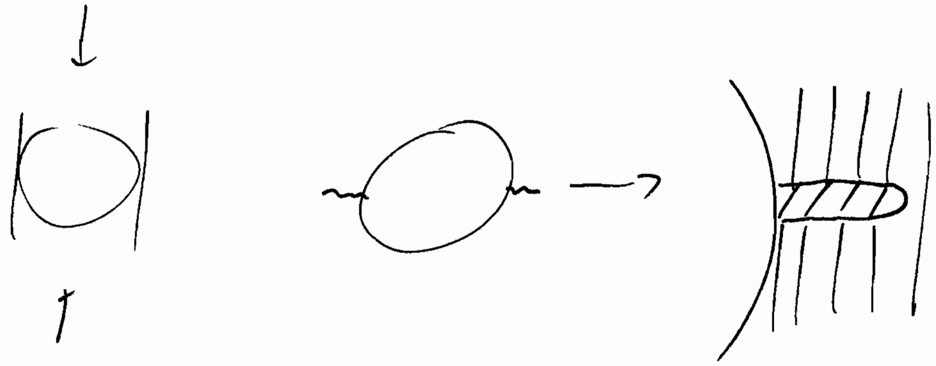
ignore 90° contribution in both cases

$D \approx \frac{1}{3} E_1 \left(2.775^3 - 0.975^3\right) \times 2 = 1.9 \times 10^3$ Nm

[25%]

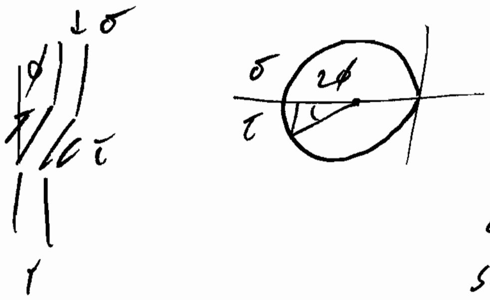
[For stiffness I was looking for a structural property, A or D, rather than a modulus, eg. $\frac{A}{t} = E$. Overall a well answered question, though (b) had some wooly answers and some people made a meal of (d)]

3(a) Compressive loading ⇒ splitting or microbuckling.



Because a multidirectional laminate is considered, we expect splitting to be inhibited by ±45 and 90 plies and microbuckling to dominate.

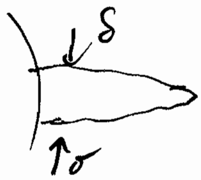
Microbuckling is a plastic instability, governed by shear of the matrix. As the fibres rotate (increasing ϕ)



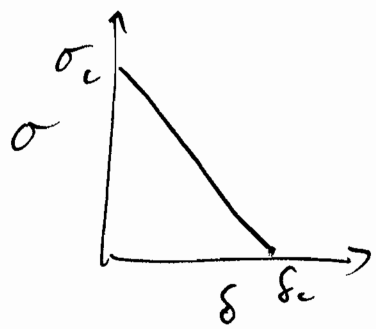
the shear τ increases. In this case the hole represents a stress concentration ($\times \sim 3$) so this is the initiation site.

Delamination and shear of the off axis plies accompanies the microbuckling in the $0°$ plies. [35%]

(b) As the microbuckle grows, the apparent crack overlap



δ increases. Because of the softening response with increasing ϕ we expect the local bridging σ to fall. σ_c is the unnotched strength at the tip, δ_c is some critical displacement where the microbuckle supports no load. The area $\sigma_c \delta_c / 2$ gives the apparent toughness. [20%]



3(c) $$v_c = \frac{\pi d_f}{8}\left(\frac{V_f E_f}{2\tau_y}\right)^{\frac{1}{3}}$$

Put $d_f = 6\times10^{-6}$ m - typical of carbon

$V_f = 0.65$ " CFRP, high performance

$\tau_y = 80$ MPa (say - of the order of the shear yield strength)

$E_f = 200$ GPa (typical of high modulus fibre)

$\Rightarrow v_c = 22\,\mu m$

Now put $\delta_c = v_c \Rightarrow G_c = \frac{1}{2}\sigma_u v_c = \frac{1}{2}\cdot 740\times10^6 \cdot 22\times10^{-6}$

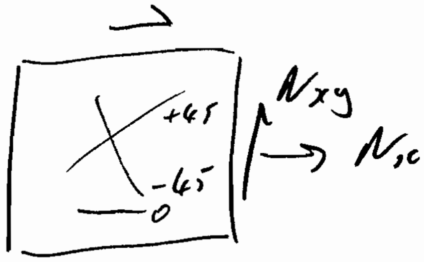
$= 8.1\times10^3$ J/m

$\simeq 8$ kJ/m [20%]

(d) For unnotched strength, compressive failure depends on initiation sites. Hence details of the distribution of fibre waviness, defects or voids will be important. In addition it can be difficult to realise the full strength in test specimens due to stress concentrations eg. associated with load transfer. ~~Finnac~~ Finally stress raisers associated with free edges may be important. [20%]

[Not a popular question. Marks were given in (a) for sensible comments about compressive failure in laminates.]

4. (a)



$e_L^+ = S_L^+/E_1 = 0.0105$

$e_{LT} = S_{LT}/G_{12} = 0.009$

$e_T^+ = S_T^+/E_2 = 0.0056$

$e_T^- = S_T^-/E_2 = 0.0275$

- Find laminate ε's
- Equate these with lamina strains
- rotate into lamina frame of reference
- compare with failure criteria

Told to assume failure is due to transverse tension in one of the 45° plies, using max ε criterion.

$$\begin{pmatrix} N_x \\ 0 \\ N_{xy} \end{pmatrix} = A \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} \Rightarrow \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} 109 & -68 \\ -68 & 368 \end{pmatrix} \begin{pmatrix} N_x \\ 0 \end{pmatrix} \quad \text{where } \Delta = 35488\,(\text{GPa mm})^2$$

$$\gamma_{xy} = N_{xy}/85 \text{ GPa mm}$$

Here ~~[illegible]~~ $\Rightarrow$ $N_x = 1.6\ \text{MN m}^{-1}$

$$\varepsilon_x = 4.91 \times 10^{-3}$$
$$\varepsilon_y = -3.07 \times 10^{-3}$$
$$\gamma_{xy} = N_{xy}/85 \text{ GPa mm}$$

For -45

Transverse failure

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = T_1^{-T} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} - & - & - \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ - & - & - \end{pmatrix} \begin{pmatrix} 4.91 \times 10^{-3} \\ -3.07 \times 10^{-3} \\ N_{xy}/85 \end{pmatrix}$$

$$\Rightarrow \varepsilon_2 = \frac{1}{2}\left(4.91 \times 10^{-3} - 3.07 \times 10^{-3} + \frac{N_{xy}}{85}\right)$$

Put $\varepsilon_2 = \varepsilon_T^+ = 0.0056 \Rightarrow \underline{N_{xy} = 0.76\ \text{MN/m}}$ [55%]

[Comment: much less work if only the critical failure mode is considered]

4 (b) Need to explore different failure modes as a function of applied load. With the different permutations it will be necessary to make a judgement as which ones to consider.

For mixed N_x and N_{xy} loading:

Transverse tension in 45: $\varepsilon_T^+ = 0.0056 = \frac{1}{2}\left(\frac{N_x}{1.6\,MNm^{-1}}(4.91-3.07)\times10^{-3} + \frac{N_{xy}}{85\,MNm^{-1}}\right)$

$\Rightarrow \quad N_{xy} = 0.92 - 0.098\,N_x$

As N_{xy} increases we expect this mode still to be relevant, but also check shear of 0°:

shear in 0°: $\sigma_{12} = \sigma_{xy} = \frac{N_{xy}}{85} = e_{LT} = 0.009 \Rightarrow N_{xy} = 0.765$ MN/m

independent of N_x

As N_x becomes dominant check axial tension of 0's and shear of ±45°

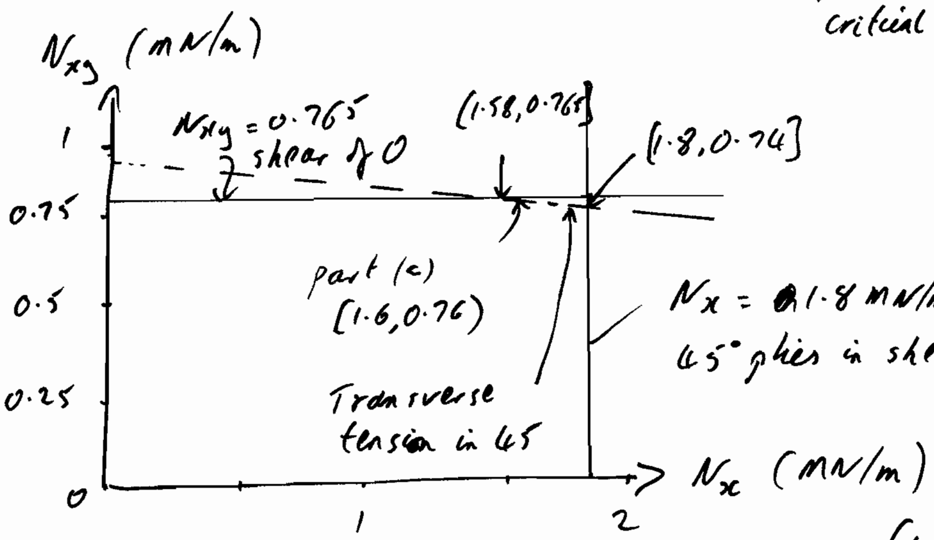
Axial tension of 0's: $\varepsilon_1 = \varepsilon_x = 4.91\times10^{-3}\cdot\frac{N_x}{1.6\,MNm^{-1}} = e_L^+ = 0.0105$

$\Rightarrow N_x = 3.4\,MNm^{-1}$

shear of −45's: $\sigma_{12} = (\varepsilon_x - \varepsilon_y) = \frac{N_x}{1.6\,MNm^{-1}}(4.91-3.07)\times10^{-3} = e_{LT} = 0.009$

$\Rightarrow N_x = 1.8$ MN/m ($\Rightarrow$ Axial tension of 0's not critical)

Combining:



[45%]

[Comment: On reflection too hard (though one student got this correct!). Marks awarded for some sensible analysis and comments.]