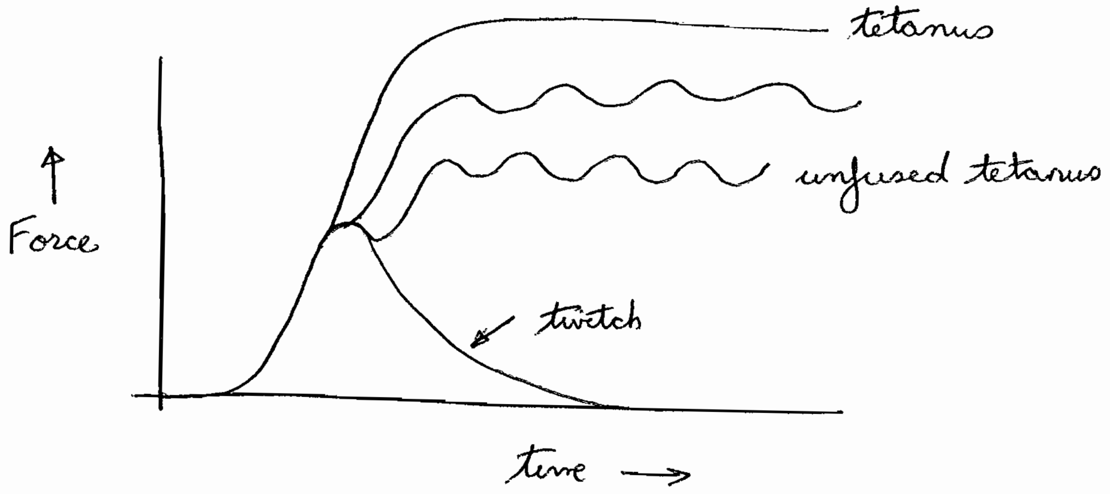
Engineering Tripos IIA & IIB

Year 2005 - 2006

EXAM CRIB for 4C14 :

Mechanics of Biological Systems

1. (a)



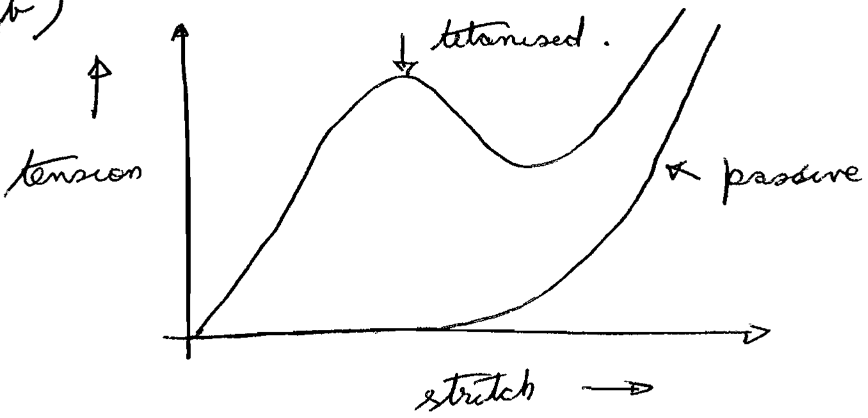
Following ~~on~~ a single electrical pulse a muscle held isometrically responds & produces a transient tensile force known as a twitch. The strength of the stimulating shock must be sufficent to polarise the muscle membran & the peak force developed increases with the strength of the shock as more muscle fibres are recruited into the force generating enterprise.

When a train of stimulations is given, the force has a steady magnitude with a little ripple superimposed at t stimulation frequency & is known as unfused tetanus

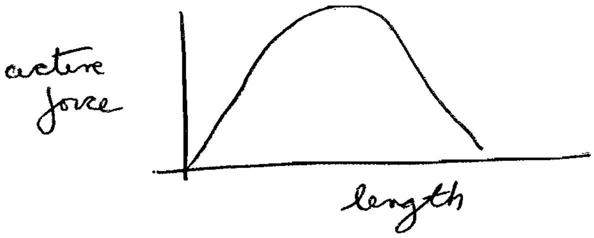
As the frequency is raised the force rises & the ripple finally reaches a low level. Further increases in frequency produce no increases in the mean force & this is called tetanic fusion or tetanus. This occurs at about 30 Hz in a frog's muscle at 0 °C

[25%]

1. (b)



The local maximum occurs in the tetanised force versus length relation as the active component has maximum in its force versus length relation

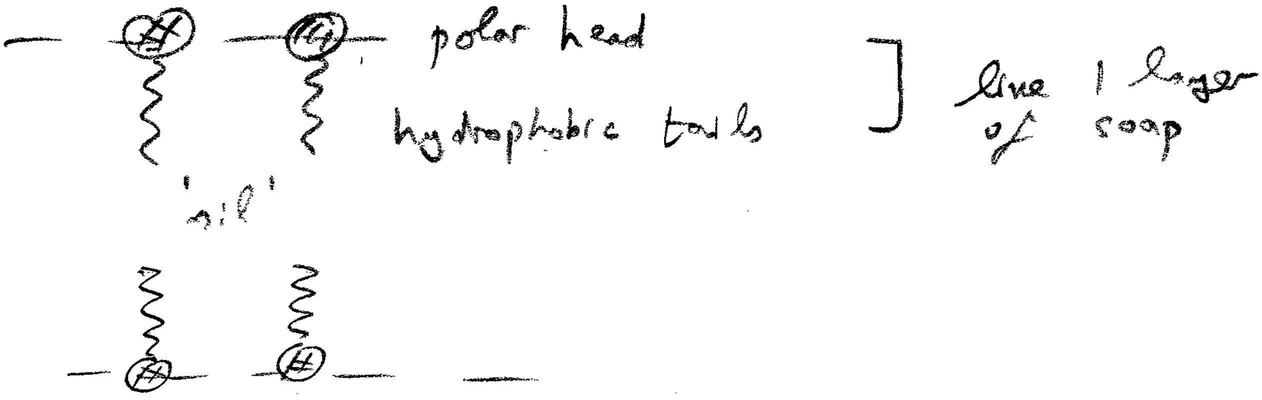


This is due to the structure of the sarcomere. At short lengths, overlap of the thin filaments causes a drop in tension, but as the overlap decreases (as the length increases), the tension rises. However, when the length is large there is less overlap between the thick & thin filaments so fewer crossbridges bind & less tension develops. When there is no overlap between the thick & thin filaments the muscle cannot actively develop any tension.

[25%]

1. (c) Animal cells

The plasma membrane is a lipid bi-layer, under biaxial tension and almost zero in-plane shear stiffness.

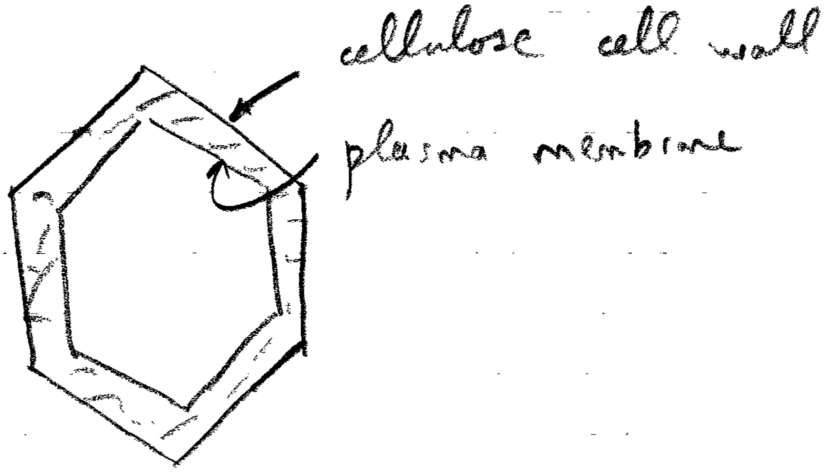


It is a semi-permeable membrane and Na^+ diffuses into the cell. It is pumped back out again by Na^+/K^+ ionic pumps. Otherwise, the high concentration of organic molecules in the cell would lead to water absorption by the cell driven by osmosis, and the cells would explode!

About 25% of the cell's energy drives these pumps.

1(c) contd.

Plant cells



A 0/90° cellulose grid exists. The cellulose is cross-linked, and is glued by pectin. Cellulose molecules are long, unbranched chains of glucose units. The cell wall provides mechanical stiffness and strength – it is a composite laminate.

[25%]

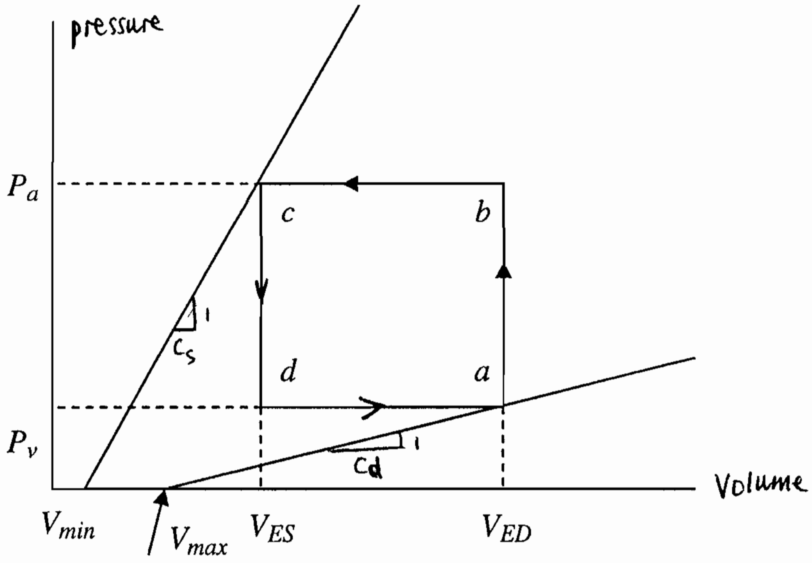
1.(d)

Circulatory System

Cardiac output

Schematic diagram of pressure-volume loop

during heartbeat cycle



a: inflow valve closes	b: outflow valve opens
c: outflow valve closes	d: inflow valve opens

$$V_{ES} = V_{\min} + C_s P_a$$

$$V_{ED} = V_{\max} + C_d P_v$$

where C_s and C_d are the compliance of the heart during systole and diastole, whereas P_a and P_v are the arterial and venous pressures. The *stoke volume* is

$$V_{stroke} = V_{\max} - V_{\min} + C_d P_v - C_s P_a$$

Total cardiac output $Q = FV_{stroke}$

where F is the heart rate in beats per unit time.

[25%]

2. (a) The first order rate equation governing the attachment & detachment is

$$v \frac{dn}{dx} = (1-n)f - ng$$

where n is the fraction of attached cross-bridges & x is the position of an actin binding site from the equilibrium position of a myosin head

$\underline{x \geq h}$

In this region f & $g = 0 \Rightarrow n(x) = n(h) = 0$

$\underline{h - x_0 \leq x \leq h}$

$$-\frac{v\,dn}{dx} = (1-n)k_1$$

$$v \ln(1-n) = k_1 x + C$$

Employing $n(h) = 0$

$$n = 1 - e^{\frac{k_1(x-h)}{v}}$$

$\underline{0 \leq x \leq h - x_0}$

f & $g = 0 \Rightarrow n(x) = n(h-x_0) = n(0) = 1 - e^{-\frac{k_1 x_0}{v}}$

$\underline{x < 0}$

$$-\frac{v\,dn}{dx} = -k_2 n$$

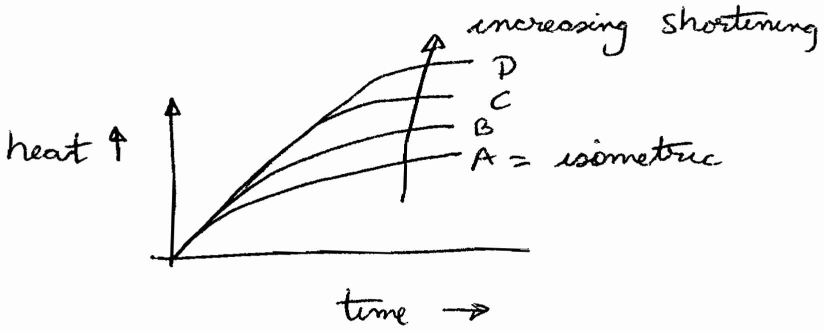
$v \ln n = k_2 x + C$, substitute $n(0) = 1 - e^{-\frac{k_1 x_0}{v}}$

$$v \ln\left[1 - e^{-\frac{k_1 x_0}{v}}\right] = C$$

$$\Rightarrow \quad n = \left(1 - e^{-\frac{k_1 x_0}{v}}\right) e^{\frac{k_2 x}{v}}$$

[60%]

2. (b) A muscle produces more heat while shortening compared to a muscle that is tetanised in its isometric state



In curve A, the muscle remains at constant length while in curves B to D the muscle beginning from a state of isometric contraction, shortens against a fixed load by increasing amounts. The vertical difference betwee curve A & the others is the shortening heat.

The shortening muscle liberates more energy on 2 counts (i) it does external work & (ii) it develops more heat.

[40%]

3. (a)

Waviness $w(x) = a \sin \frac{2\pi x}{\lambda}$

$$M(x) = P.\,w = EI \frac{d^2u}{dx^2}$$

$$\Rightarrow \quad \frac{d^2u}{dx^2} = \frac{Pa}{EI} \sin \frac{2\pi x}{\lambda}$$

$$\Rightarrow \quad u = -\left(\frac{\lambda}{2\pi}\right)^2 \frac{Pa}{EI} \sin \frac{2\pi x}{\lambda}$$

Now assume that $P << \frac{EI\lambda^2}{4\pi^2}$

$$\Rightarrow \quad |u(x)| << |w(x)|$$

Extensional strain is

$$e = -\frac{1}{\lambda} \int_0^\lambda \left(1 + \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial x}\right)^2\right)^{\frac{1}{2}} - \left[1 + \left(\frac{\partial w}{\partial x}\right)^2\right]^{1/2} dx$$

$$\frac{\partial u}{\partial x} = -\frac{\lambda}{2\pi} \frac{Pa}{EI} \cos \frac{2\pi x}{\lambda}$$

$$\frac{\partial w}{\partial x} = \frac{2\pi a}{\lambda} \cos \frac{2\pi x}{\lambda}$$

$$\Rightarrow \left(1 + \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial x}\right)^2\right)^{\frac{1}{2}} \sim 1 + \frac{1}{2}\left(\left(\frac{\partial u}{\partial x}\right)^2 + 2\frac{\partial u}{\partial x}\frac{\partial w}{\partial x} + \left(\frac{\partial w}{\partial x}\right)^2\right)$$

$$\left(1 + \left(\frac{\partial w}{\partial x}\right)^2\right)^{\frac{1}{2}} \sim 1 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$

(a) <u>contd.</u>

3.

$$\Rightarrow \quad e\lambda \approx -\int_0^\lambda dx \left(\frac{\partial u}{\partial x} \frac{\partial w}{\partial x} \right)$$

$$= -\int_0^\lambda dx \quad - \frac{Pa^2}{EI} \cos^2 \frac{2\pi x}{\lambda}$$

$$= \frac{\lambda}{2} \frac{Pa^2}{EI}$$

$$\Rightarrow \quad \underline{e = \frac{Pa^2}{2EI}} \qquad EI = E_s \frac{\pi}{4} \left(\frac{d}{2} \right)^4$$

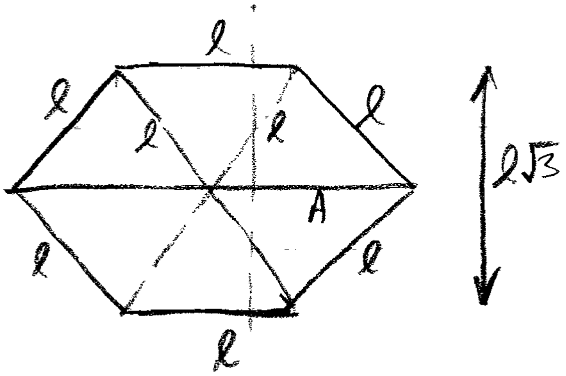
$$\Rightarrow EI = \frac{\pi}{4} \frac{E_s d^4}{16}$$

$$\Rightarrow \quad e = Pa^2 \frac{2}{\pi} \frac{16}{E_s d^4}$$

$$\Rightarrow \quad \underline{e = \frac{32}{\pi} \frac{a^2 P}{E_s d^4}}$$

[30%]

(b) Symmetry $\Rightarrow$ $T_B = T_F = T_E = T_C$.



$T_B = T_C = T_E = T_F = 0$

$T_D = T_A$

$$\bar{\rho} = \frac{\frac{3}{2} \frac{\pi d^2}{4} l}{\frac{1}{2} \cdot l \frac{\sqrt{3}}{2} l \, d}$$

$\Sigma_{11} \cdot d \, l\sqrt{3} = T_A$

$$\Rightarrow \quad \bar{\rho} = \frac{2\sqrt{3}\,\pi d}{l}$$

[30%]

3. (c) $E_1 = \frac{\Sigma_{11}}{e}$ $\Sigma_{11} = \frac{T_A}{\sqrt{3}\, d\ell}$ $T_A = P$

$$\Rightarrow \Sigma_{11} = \frac{P}{\sqrt{3}\, d\ell} = \frac{\pi E_s d^4}{32 a^2} \frac{e}{\sqrt{3}\, d\ell}$$

$$\Rightarrow E_1 = \frac{\pi}{32\sqrt{3}} \frac{E_s d^3}{a^2 \ell}$$

$$= \frac{\cancel{\pi}}{32\sqrt{3}} E_s \left(\frac{d}{a}\right)^2 \frac{\bar{\rho}}{2\sqrt{3}\,\cancel{\pi}}$$

$$\underline{E_1 = \frac{1}{192} \left(\frac{d}{a}\right)^2 E_s \bar{\rho}}$$

[40%]

4C14

4. (a) Blood circulation in the body is composed of two parts. Blood from the right side pump, dark red and low in oxygen, travels along pulmonary arteries to the lungs where it receives fresh oxygen and becomes bright red. It then flows to the left side pump of the heart along pulmonary veins. The blood pressure is the force per unit area that the blood exerts on the walls of blood vessels.

Systolic pressure is the highest surge of pressure in the artery.

Diastolic pressure is the lowest pressure reached during ventricular relaxation and filling. [20%]

(b) This is because the compliance of the veins is about 24 times bigger than that of arteries, due to the fact that the veins are both larger and softer than the arteries.

Thus large amounts of blood can be stored in the veins with only slight changes in venous pressure.

At any one moment, the veins carry about 70% of the blood. [15%]

(c) Respiratory pigments increase the oxygen-carrying capacity of the blood. In human beings, the red-coloured pigment hemoglobin act as the respiratory pigment, which increase the oxygen-carrying capacity of the blood between 65 and 70 times.

[15%]

4.(d) $$U(x) = \sigma P_g + \sigma(P_0 - P_g) e^{-D_m x / v}$$

$\Rightarrow$ Total flux of gas across the wall of a capillary is

$$Q = vA\left[U(L) - U(0)\right]$$
$$= vA\sigma(P_g - P_0)(1 - e^{-D_m L/v})$$

In the limit $D_m L / v \to \infty$, we have

$$Q \to vA\sigma(P_g - P_0) \qquad (*)$$

Test data on diffusion of CO_2 is consistent with the above solution, because the solubility of CO_2 is quite high and the difference between the partial pressure P_{CO_2} for the entering blood and the alveolar air is small, about 5 mm Hg.

In contrast, the solubility of O_2 in blood is about 20 times smaller than CO_2, and although the difference in P_{O_2} is larger, this is not adequate to account for the balance of O_2 inflow and CO_2 outflow. In other words, if eqn. $(*)$ is relevant, then a decrease in σ by a factor of 20 requires a corresponding increase by a factor of 20 for the partial pressure differences to maintain similar transport. That is, the difference in $P_g - P_0$ for O_2 should be about 20 times larger than for CO_2. In practice, this is not the case.

[30%]

4.

(e) Removal of CO_2

In the alveoli capillaries, bicarbonate combines with a hydrogen ion (proten) to form carbonic acid, which breaks down into carbon dioxide and water:

$$CO_2 + H_2O \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} HCO_3^- + H^+$$

CO_2 then diffuses into the alveoli and out of the body with the next exhalation. The conversion of bicarbonate to CO_2 continually replenishes the CO_2 that is lost to alveolar air.

Uptake of O_2

The binding of O_2 with hemoglobin is:

$$Hb + 4O_2 \underset{k_{-2}}{\overset{k_2}{\rightleftharpoons}} Hb(O_2)_4$$

The improvement of O_2 uptake by hemoglobin is substantial, as for normal blood, hemoglobin is 97% saturated, and the hemoglobin of 100 ml of blood carries ~ 20 ml of Oxygen. By contrast, the same 100 ml of blood contains only 0.3 ml of dissolved oxygen.

[20%]