4D6

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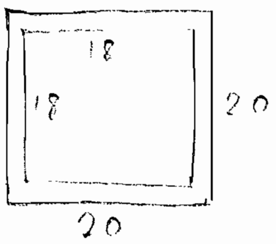
1) (a)



$$I = \frac{20^4 - 18^4}{12} : 1585\tfrac{1}{3}\ m^4$$

Mode shape: $\bar{u} = 1 - \cos\frac{\pi x}{2h}$

$$M_{eq} = \int_0^h m\bar{u}^2\,dx = m\int_0^h \left(1 - \cos\frac{\pi x}{2h}\right)^2 dx$$

$$= m\int_0^h 1 - 2\cos\frac{\pi x}{2h} + \cos^2\frac{\pi x}{2h}\ dx$$

$$\cos^2\frac{\pi x}{2h} = \frac{1}{2}\left(1 + \cos\frac{\pi x}{h}\right)$$

$$= m\left[x - \frac{4h}{\pi}\sin\frac{\pi x}{2h} + \frac{1}{2}\left(x + \frac{h}{\pi}\sin\frac{\pi x}{h}\right)\right]_0^h$$

$$= m\left\{h - \frac{4h}{\pi} + \frac{1}{2}h\right\}$$

$$= \left(\frac{3}{2} - \frac{4}{\pi}\right)mh$$

$$= \left(\frac{3}{2} - \frac{4}{\pi}\right) \times \frac{200\times10^6}{200} \times 200$$

$$= 45.352\times10^5\ kg$$

$$K_{eq} = \int_0^h EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx \qquad \frac{d^2\bar{u}}{dx^2} = \left(\frac{\pi}{2h}\right)^2 \cos\frac{\pi x}{2h}$$

$$= EI\int_0^h \left(\frac{\pi}{2h}\right)^4 \cos^2\frac{\pi x}{2h}\, dx$$

$$= EI\left(\frac{\pi}{2h}\right)^4 \int_0^h \frac{1}{2}\left(1 + \cos\frac{\pi x}{h}\right) dx$$

$$= \frac{EI\pi^4}{32h^4}\left[x + \frac{h}{\pi}\sin\frac{\pi x}{h}\right]_0^h$$

$$= \frac{EI\pi^4}{32h^3}$$

$$= \frac{30\times10^9 \times 4585\tfrac{1}{3} \times \pi^4}{32\times 200^3}$$

$$= 52.342\times10^6 \ \text{N/m}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} = \frac{1}{2\pi}\sqrt{\frac{52.342}{45.352}} = 0.171 \ \text{Hz}$$

$$\therefore T = 1/f = 5.85\,\text{s}$$

(b) $$F_{eq} = \int_0^h w\bar{u}\, dx = w\int_0^h 1 - \cos\frac{\pi x}{2h}\, dx$$

$$= w\left[x - \frac{2h}{\pi}\sin\frac{\pi x}{2h}\right]_0^h$$

$$= w\left\{h - \frac{2h}{\pi}\right\} = 100\times10^3\times200\left(1 - \frac{2}{\pi}\right) = 7.268\times10 \ \text{N}$$

$$u_{STATIC} = F_{eq}/K_{eq} = \frac{7.268}{52.342} = 0.139\,m$$

$$t_d/T = 1.75/5.85 = 0.30 \quad \therefore\ DAF \simeq 1.6$$
(data sheet)

$$u_{DYNAMIC} = DAF \times u_{STATIC} = 1.6 \times 0.139 = 0.22\,m$$

(c) Earthquake ground acceleration pulse $\ddot{v} = 0.1\,m/s^2$
duration 1.75s (hence same DAF as wind load)

Method 1:

Equivalent to a U.D.L. of $m\ddot{v} = 10^6 \times 0.1$
$= 100 \times 10^3$ N/m

i.e. the same U.D.L. as the wind loading

Hence $u_{DYNAMIC\,(EARTHQUAKE)} = 0.22\,m$ as above

Method 2:

Equivalent S.D.O.F. system for ground acceleration:

$$M_{eq}\ddot{u} + K_{eq}u = -\Gamma M_{eq}\ddot{v} = F_{eq\,(EARTHQUAKE)}$$

$$\Gamma = \frac{\int m\bar{u}\,dx}{\int m\bar{u}^2\,dx} = \frac{1 - 2/\pi}{\frac{3}{2} - 4/\pi} = 1.602 \text{ (from integrals above)}$$

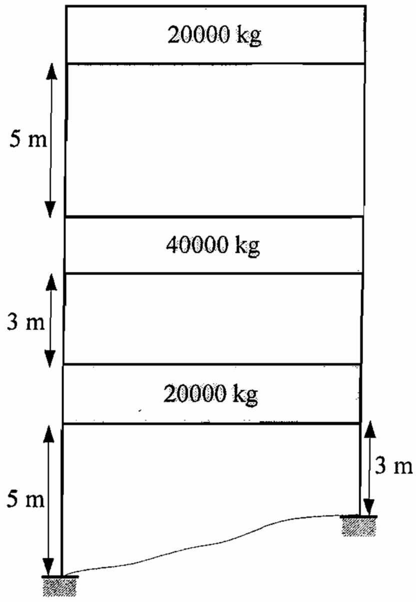
$$\therefore\ \left|F_{eq\,(EARTHQUAKE)}\right| = \frac{(1 - 2/\pi)}{\left(\frac{3}{2} - 4/\pi\right)}\left(\frac{3}{2} - \frac{4}{\pi}\right) \times 200 \times 10^6 \times 0.1$$

$$= \left(1 - 2/\pi\right) \times 20 \times 10^6$$

$$= 7.268 \times 10^6\ N = F_{eq}\ (WIND)$$

Hence $u_{DYNAMIC\,(EARTHQUAKE)} = u_{DYNAMIC\,(WIND)} = 0.22\,m$

2 a) Consider the 3 storey building:



Ground floor stiffness $k_1 = 12 EI \left[\frac{1}{h_1^3} + \frac{1}{h_2^3}\right]$

$k_1 = 13.6732 \times 10^6$ N/m

Mass $M_1 = 20000$ kg

I floor stiffness $k_2 = 12EI \times \frac{2}{3^3}$

$= 22.49 \times 10^6$ N/m.

Mass $M_2 = 40000$ kg.

II floor stiffness $= k_3 = 12EI \times \frac{2}{5^3}$

$= 12 \times 25.3 \times 10^6 \times \frac{2}{5^3}$.

$k_3 = 4.8576 \times 10^6$ N/m

Mass $M_3 = 20000$ kg.

Consider the suggested mode shape $\bar{U} = \begin{bmatrix} 1.00 \\ 0.57 \\ 0.38 \end{bmatrix}$

Consider the KE $= \frac{1}{2} \sum_{i=1}^{3} M_i \dot{U}_i^2 = \frac{1}{2} \sum M_i \, U_i^2 \, \dot{U}^2(t)$

$\therefore \; M_{eq} = \sum M_i \bar{U}_i^2$

$= 20000 \times 0.38^2 + 40000 \times 0.57^2 + 20000 \times 1^2$

$M_{eq} = 35884$ kg.

Consider the strain energy $= \frac{1}{2} \sum_{i=1}^{3} K_i U_i^2$

$= \frac{1}{2} \left[K_1 \bar{U}_1^2 + K_2 (\bar{U}_2 - \bar{U}_1)^2 + K_3 (\bar{U}_3 - \bar{U}_2)^2 \right] U(t)$

$\therefore \; K_{eq} = k_1 \bar{U}_1^2 + k_2 (\bar{U}_2 - \bar{U}_1)^2 + k_3 (U_3 - U_2)^2$

$= 10^6 \times \left[13.6732 \times 0.38^2 + 22.49 \times (0.57 - 0.38)^2 + 4.8576 \times (1 - 0.57)^2 \right]$

$= 3.6845 \times 10^6$ N/m.

$\therefore$ The natural frequency of the equivalent SDOF is

$$\omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}} = \sqrt{\frac{3.6845 \times 10^6}{35884}}$$

$$= 10.133 \text{ rad/s}$$

or $f_n = \frac{\omega_n}{2\pi} = \underline{\underline{1.612 \text{ Hz}}}$

Natural Period $= \frac{1}{f_n} = \underline{\underline{0.62 \text{ s}}}$. [40%]

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2 b) Using the natural period from part a) $T = 0.62$ s.

From Fig. 2.. $\frac{S_{da}}{a_g} = 2.5$ for $T = 0.62$ s.

The peak grund acceleration at this site $a_g = 0.65g$.

$$\therefore \quad S_{da} = 2.5 \times 0.65 \times 0 \text{ m/s}^2$$
$$= 16.25 \text{ m/s}^2$$

As the shaking is due to grund motion. we need to find modal participation factor Γ

$$\Gamma = \frac{\Sigma m\bar{U}}{\Sigma m\bar{U}^2} = \frac{20000 \times 0.38 + 40000 \times 0.57 + 20000 \times 1}{20000 \times 0.38^2 + 40000 \times 0.57^2 + 20000 \times 1^2}$$

$$= 1.4045$$

$$\therefore \quad S_{da} = \Gamma \times 2.5 \times a_g = 1.4045 \times 16.25$$
$$= 22.823 \text{ m/s}^2$$

$\therefore$ Floor accelerations

I floor $\ddot{y}_{1\,max} = 0.38 \times 22.823 = \underline{8.67 \text{ m/s}^2}$ $(0.87g)$

2b) Floor accelerations (contd)

II floor $= \ddot{y}_{2\,max} = 0.57 \times 22.823$

$= \underline{\underline{13}}\ m/s^2 \quad (1.3g)$

III floor $= \ddot{y}_{3\,max} = 1 \times 22.823$

$= 22.823\ m/s^2 \quad (2.28g)$

(These are quite large floor accelerations) [30%]

2c) Shear force at the bases of the ground floor columns.

Use $F = ma$.

$\therefore$ Total $F_H = \Sigma ma$

$= 20000 \times 22.823 + 40000 \times 13 + 20000 \times 8.67$

$= 1149860\ N$

$= 1149.86\ kN$

$\therefore$ Each column base will carry a shear force of $\underline{\underline{574.93}}$ kN

Again these are quite large shear forces. [30%]

Q3) a) During earthquake loading, horizontally polarised shear waves (S_h) are propagated from bedrock towards the soil surface. Any structure that is founded near the surface can be excited and undergo vibrations due to the S_h waves. The structure can undergo horizontal, vertical or rocking vibrations. As the structure begins to vibrate say horizontally it will induce additional stress waves in the surrounding soil which will superpose onto incoming S_h waves from the base. Thus the input motion is modified due to the presence of the structure and its vibrational mode. This is termed as dynamic soil-structure interaction.

Soil liquefaction occurs due to the cyclic shear strains induced in saturated, loose soil deposit by earthquake loading. During this the stiffness of the soil can be degraded quite severely. Thus the vibrational characteristics of a soil structure system can change significantly with the onset of liquefaction. For example a stiff soil-structure system with high natural frequency can be degraded to a system of lower natural frequency due to the softening of the soil. This may lead the natural frequency of the soil-structure system to move towards the earthquake 'driving' frequencies causing resonant vibrations. This can lead to severe structural damage. [20%]

b) The stiffness of the soil is a function of the shear modulus. It is known that shear modulus of soil decreases with the increase in amplitude of cyclic shear strain. Stronger the earthquake, larger will be the amplitude of cyclic shear strain and consequently larger will be the decrease of shear modulus.

Secondly, the soil stiffness can be reduced with build up of excess pore water pressure. Loose saturated soils have a tendency to densify during earthquake loading. Pore water in such soils is unable to drain quickly as earthquake loading is quite quick. As a result the excess pore pressure builds up, reducing the effective stress. This leads a drop in the soil stiffness. [10%]

3c) Unit weight of Saturated soil $\gamma_{sat} = 20\ kN/m^3$.

Poisson's Ratio $\nu = 0.3$, $G_s = 2.65$

$\therefore$ Co-efficient of earth pressure at rest $K_o = \frac{\nu}{1-\nu} = 0.4287$.

Void Ratio of the soil e :- From Data sheets

$$\gamma = \frac{(G_s + e S_r)\gamma_w}{1+e}$$

For $\gamma = \gamma_{sat}$ $S_r = 1$ (Degree of Saturation)

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e}$$

$$20 = \frac{(2.65+e)10}{1+e} \Rightarrow 2+2e = 2.65+e$$

Void ratio $e = 0.65$

Vertical stress on Reference plane $= \sigma_v = \gamma_{sat} \times z = 20 \times 6 = 120$ kpa

Hydrostatic water pressure here $= \gamma_w z = 10 \times 6 = 60$ kpa.

$\therefore$ Effective vertical stress $(\sigma_v') = 120 - 60 = 60$ kpa

From Data sheets: $p' = \sigma_v' \left(\frac{1+2K_o}{3}\right) = 60\left(\frac{1+2\times0.4287}{3}\right)$

Effective mean confining stress $= 37.148$ kpa.

$$G_{max} = 100 \frac{[3-e]^2}{1+e} (p')^{0.5}$$

$$= 100 \frac{[3-0.65]^2}{1.65} \left(\frac{37.148}{1000}\right)^{0.5}$$

$$= 64.51\ MPa$$

Shear wave velocity $= v_s = \sqrt{\frac{G_{max}}{\rho_{sat}}}$.

$$\rho_{sat} = \frac{20 \times 10^3}{10} = 2000\ kg/m^3$$

$$\therefore v_s = \sqrt{\frac{64.51 \times 10^6}{2000}} = 179.596\ m/s$$

$$\simeq 180\ m/s.$$

[30%]

3d) Shear wave velocity $v_s = 180$ m/s

Natural frequency of a horizontal soil layer $f_n = \frac{v_s}{4H}$.

$$\therefore \; f_n = \frac{180}{4 \times 12} = \underline{3.75 \text{ Hz}}.$$ [10%]

3e) During the Study Kobe earthquake $u_{excess} = 35$ kpa on the reference plane.

$$\therefore \; p' = p'_{old} - u_{excess}$$
$$= 37.148 - 35$$
$$= 2.148 \text{ kpa}$$

This is quite a small mean effective confining stress.

$$G = 100 \frac{[3-e]^2}{1+e} (p')^{0.5}$$

$$= 100 \frac{[3-0.65]^2}{1.65} \sqrt{\frac{2.148}{1000}}$$

$$G = \underline{15.51} \text{ MPa}$$

Using This, the shear wave velocity

$$v_s = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{15.51 \times 10^6}{2000}} = \underline{88.06 \text{ m/s}}$$

Comment :- Although the excess pore water pressure nearly caused 'full liquefaction' the Hardin-Drnevich equation for Shear Modulus gives fairly large shear modulus & hence high shear wave velocity. Therefore these values are unreliable at low p' values. Shear wave velocity of about 20 m/s were measured in centrifuge experiments that caused near liquefaction of sandy deposits. [30%]

4. SOLUTION
and approximate marking scheme

4 (a)i Vortex shedding behind a bluff body [2.5%]. Caused by the interaction of two approximately-parallel shear layers [2.5%]. The frequency is a function of the diameter of the body (or, more exactly, the distance between the shear layers) and the flow velocity [2.5%]. It beats at its own frequency and is independent of external disturbances, unless that disturbance is very near its natural frequency [2.5%].

4 (a)ii

•Vortex shedding and lock-in, caused by vortex shedding near the natural frequency of the structure [2.5%]. The transient response of the structure is at its own resonant frequency [2.5%]. If this is near the vortex shedding frequency, the v-s frequency locks in [2.5%].

•Galloping of a non-circular cross section [2.5%]. When the structure moves in the cross-stream direction, there is a force in the direction of motion for some cross-section shapes [2.5%]. For small oscillations this force is proportional to the structure's velocity and can be thought of as negative damping [2.5%]. If the net damping (the negative damping subtracted from the actual damping) is negative then the structure will be unstable to soft excitation [2.5%]. Bonus for discussion of hard excitation [2.5%].

•Flutter [2.5%]. When the structure has a torsional and a translational mode [2.5%] it will flutter if the net damping of one of the modes is negative [2.5%]. Flutter depends on the relative positions of the centre of mass, the aerodynamic centre and the elastic axis [2.5%].

4 (b) The cross-section is circular so there will be no gallop or flutter [5%]. However, it will potentially suffer from vortex shedding when the water moves across the walkway [5%]. This may be dangerous in itself but will be particularly dangerous if this locks into its resonant frequency [5%]. To calculate its resonant frequency in water, f_w, we can use the resonant frequency in air, f_a, but must consider the added mass of the water around

(TURN OVER for continuation of Question 2

the structure.

$$f_w = f_a\sqrt{\frac{m+m_a}{m+m_w}}$$

where the mass per unit length of structure is m, the added mass of the air is m_a and the added mass of the water is m_w. The added mass is equal to $1 \times \rho\pi D^2/4$, since the added mass coefficient for a cylinder cross-section is 1. Therefore [20%]:

$$f_w = 0.5\sqrt{\frac{885+1.2\pi 3^2/4}{885+1027\pi 3^2/4}} = 0.166\text{Hz}$$

The pipe's diameter is much smaller than the water depth and it is neither near the surface nor the sea floor. Therefore we do not need to consider the effects of the floor or the surface on the added mass coefficient [5%]. If we did, the floor would increase the added mass coefficient and the surface would decrease it.

The Reynolds number is much greater than 1000 [5%] so the Strouhal number of vortex shedding is 0.2. The water velocity at which this will be the same as the resonant frequency of the structure is 2.49ms^{-1} [10%]. The maximum cross-stream velocity is 3ms^{-1} so lock in can potentially occur [5%]. This would be dangerous.

END OF PAPER