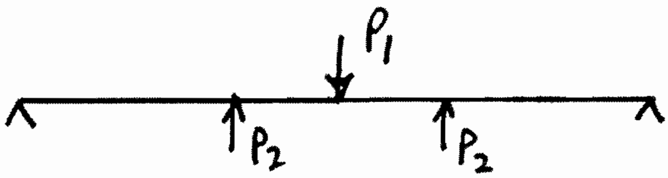
4D8 Prestressed Concrete – Examination 2006

Solutions

Examiner's comment in italics.

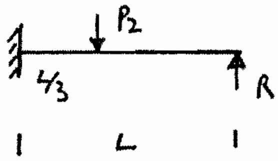
1. (a)

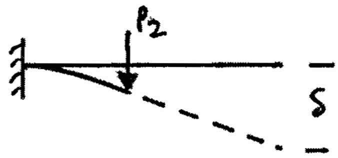


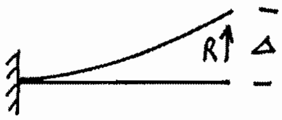
Need to find relationship between P_1 and P_2 to give zero displacement at centre.

Can treat the centre as a support and analyse under these loads. *Completely correct answers were given by candidates using Macaulay (as taught in 3D4), Virtual Work (as taught in IB) or as here by Data Book coefficients.*

Symmetrical, so consider as a pair of cantilevers and analyse separately

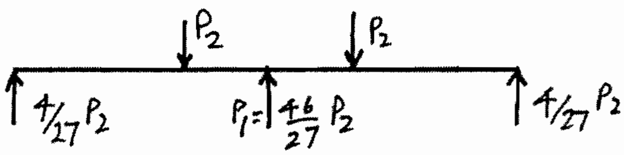






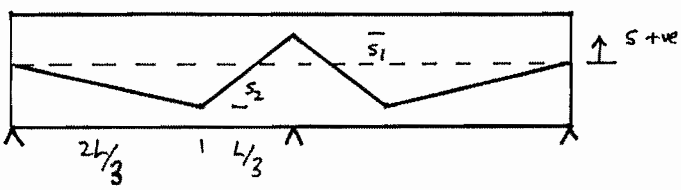
$$\delta = \frac{P_2}{3EI}\cdot\left(\frac{L}{3}\right)^3 + \frac{P_2}{2EI}\cdot\left(\frac{L}{3}\right)^2\cdot\frac{2L}{3} = \frac{4}{81}\cdot\frac{P_2L^3}{EI}$$

equate to Δ $\qquad \Delta = \frac{R}{3EI}.L^3 = \delta = \frac{4}{81}.\frac{P_2L^3}{EI} \qquad$ so $\qquad R = \frac{4}{27}P_2$

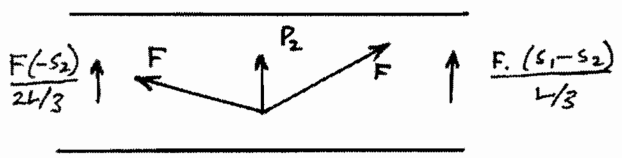


So by overall equilibrium, $P_1 = \frac{46}{27}P_2$

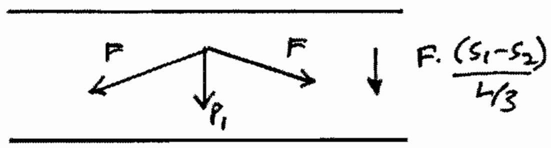
(b)



Cable will be concordant if the forces exerted by cable cause no deflection (or reaction at centre). $\therefore$ They must be in the same proportion as P_1 and P_2 in (a).



$$P_2 = \frac{F}{L}\left(-\frac{3s_2}{2} + 3s_1 - 3s_2\right) = \frac{F}{L}(3s_1 - 4.5s_2)$$



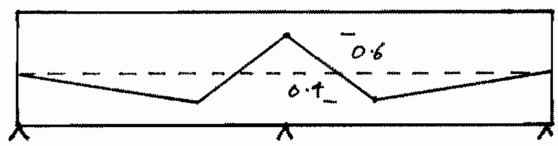
$$\therefore P_1 = \frac{2F}{L}(3s_1 - 3s_2) = \frac{46}{27}P_2 \qquad \text{(from (a))}$$

Solve for s_1 and s_2 in terms of P_2. What is important is the ratio s_1/s_2.

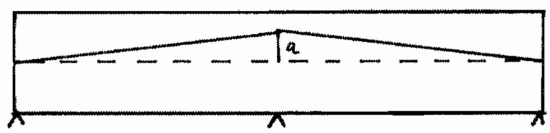
$$\begin{pmatrix} 3 & -4.5 \\ 6 & -6 \end{pmatrix}\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \frac{P_2 L}{F}\begin{pmatrix} 1 \\ 46/27 \end{pmatrix}$$

so $\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \frac{1}{9}\begin{pmatrix} -6 & 4.5 \\ -6 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 46/27 \end{pmatrix}\frac{P_2 L}{F} = \begin{pmatrix} 5/3 \\ -8/9 \end{pmatrix}\frac{P_2 L}{9F}$ and $\frac{s_1}{s_2} = \frac{5/3}{-8/9} = -\frac{15}{8}$

(c) Actual profile



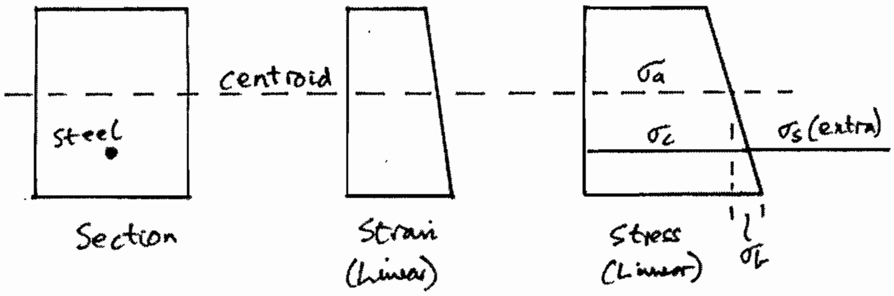
Add a linear transformation to get s_1/s_2 from (b), which would then be concordant line of thrust.



$$\frac{0.6 + a}{-0.4 + 2a/3} = -\frac{15}{8} \qquad \text{so } a = 1/15 = 0.0667.$$

Actual cable lower than line of thrust so beam would try to deflect upwards causing sagging M_2 of 1000/15 = 66.7 kNm at centre support.

Probably the easier of the long questions but it required clarity of thought. The first part required analysis of a once-indeterminate symmetrical structure, which could be (and was) done successfully by various methods. Several candidates got full marks and some missed out only because of minor numerical errors. Others confused the shape of the bending moment diagram and the shape of the cable profile when trying to determine concordancy. The question was either done well or badly, with few middling attempts, 2 (a) Bookwork.



Change in strain in concrete next to tendon = change in strain in steel

$$\left(\frac{\sigma}{E_{ce}}+\varepsilon_{cs}\right)=\frac{(\sigma_s+\sigma_c)}{E_s}-\frac{f}{E_s} \quad \text{where } E_{ce}=\frac{E_c}{(1+\phi)} \text{ (the age adjusted effective modulus)}$$

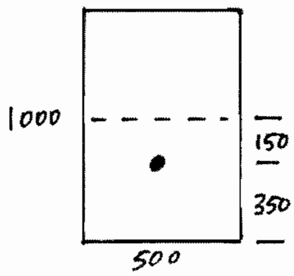
Rearrange $$\sigma_s=\left(\frac{E_s}{E_{ce}}-1\right)\sigma_c+E_s\varepsilon_{cs}+f \qquad (1)$$

Linear variation of stress and strain $$\sigma_c=\sigma_a+\frac{e}{y_2}\sigma_b \qquad (2)$$

Axial Equilibrium $$N=\sigma_a A_c+\sum\sigma_s A_s \qquad (3)$$

Moment Equilibrium
(summed over all steel layers) $$M=\sigma_b Z_2+\sum\sigma_s A_s e \qquad (4)$$

(b) (i) In this example



$Z_2=\dfrac{bd^2}{6}$ = 0.0833 m^3 $\qquad A_c$ = 0.5 m^2 $\qquad e$ = 0.150 m $\qquad A_s$ = 2500 mm^2

Initially E_{ce} = 25 kN/mm^2

$$\therefore \sigma_s=\left(\frac{200}{25}-1\right)\sigma_c+(-800) \quad \text{(From (1))} \qquad \textit{(N.B. –800 because compression +ve)}$$

$$\sigma_c=\sigma_a+\frac{150}{500}.\sigma_b \qquad \text{(From (2))}$$

From (3) (units N, mm)

$$0=0.5.10^6.\sigma_a+2500(7\sigma_c-800)=517500.\sigma_a+5250.\sigma_b-2.10^6$$

From (4)

$$0=.0833.10^9.\sigma_b+2500(7\sigma_c-800).150=2.625.10^6\sigma_a+84.12.10^6.\sigma_b-300.10^6$$

$$\begin{pmatrix} 517500 & 5250 \\ 2.625.10^6 & 84.12.10^6 \end{pmatrix}\begin{pmatrix} \sigma_a \\ \sigma_b \end{pmatrix} = \begin{pmatrix} 2.10^6 \\ 3.10^8 \end{pmatrix}$$

solve gives σ_a=3.83 N/mm^2 and σ_b = 3.45 N/mm^2, so σ_c = 4.86 N/mm^2 and σ_s = -765.9 N/mm^2.
Bottom fibre concrete stress = 7.28 N/mm^2; total steel stress = 761 N/mm^2 (tension)

(b) (ii) Now repeat with E_{ce} = 10 kN/mm^2 and shrinkage = 0.0003 (N.B. +ve)

$$\therefore \sigma_s = \left(\frac{200}{10} - 1\right)\sigma_c + (-800) + 200.10^9.300.10^{-6}$$

Substitute revised values into (3) and (4) gives

$$\begin{pmatrix} 547500 & 14250 \\ 7125000 & 85.47.10^6 \end{pmatrix}\begin{pmatrix} \sigma_a \\ \sigma_b \end{pmatrix} = \begin{pmatrix} 185000 \\ 278.10^6 \end{pmatrix}$$

solve gives σ_a=3.30 N/mm^2 and σ_b = 2.97 N/mm^2, so σ_c = 4.19 N/mm^2 and σ_s = -660.3 N/mm^2.
Bottom fibre concrete stress = 6.27 N/mm^2; total steel stress = 656 N/mm^2 (tension)

Bottom fibre stress has changed from 7.28 to 6.27 N/mm^2, so loss of 14%

(c) If there is a thermal gradient the effective modulus will change with depth, so several changes will be needed.

It would no longer be true that linear variation of strain (which must still be true) would mean linear variation of stress, which is the underlying assumption for Equation (2). This would thus need to be rewritten so that the equations were written in terms of strains, with appropriate values of Young's modulus used at different depths.

It would also be necessary to use a transformed section for A_c and Z_2 in equations (3) and (4) (or to go back and work completely from first principles.

Parts (a) and (b) were fairly standard and should have been familiar, while section (c) required some thought about thermal effects that was far from routine. There were a number of numerical errors which were not heavily penalised, and there were a number of completely correct answers, but a significant number fell down on the earlier parts. A small minority appeared not to have attended the lectures on creep nor to have attempted any of the past questions on the subject, and yet inexplicably still chose to do this question. They must have been desperate!

3. (a) Precast beam area = 0.75 m^2; Eccentricity e = 0.72-0.37 = 0.35 m:

Z_2 = 0.203/0.72 = 0.282 m^3

Insitu concrete area = 2. 0.2 = 0.4 m^2. Weight of concrete = 1.15. 24 = 27.6 kN/m

Span = 30 m, so moment at mid-span = $\dfrac{27.6 \cdot 30^2}{8}$ = 3105 kNm

Stress in bottom fibre $= \frac{P}{A} + \frac{Pe}{Z_2} - \frac{M}{Z_2} = \frac{7500.10^{-3}}{0.75} + \frac{7500.10^3.350}{0.282.10^9} - \frac{3105.10^6}{.282.10^9}$

$= 10.0 + 9.3 - 11.0 = 8.3$ N/mm^2

Similarly, stress in top fibre = 11.8 N/mm^2

(b) Need section properties of composite section

Modular ratio of insitu concrete = 0.8, so effective breadth of top flange for calculating section properties = 1.6 m (*but not for calculating weight as some candidates did*!)

Height of composite centroid $\bar{y}' = \frac{0.75 \cdot 0.72 + 1.6 \cdot 0.2 \cdot 1.6}{0.75 + 0.32} = 0.983\,\text{m}$

Composite 2nd moment of area

$$I' = 0.203 + \frac{1.6 \cdot 0.2^3}{12} + 0.75 \cdot (0.983 - 0.72)^2 + 0.32 \cdot (1.6 - 0.983)^2 = 0.378\,\text{m}^2$$

New $Z_2' = \frac{0.378}{0.983} = 0.384\,\text{m}^3$

Stress needed to cause tension = 8.3 N/mm^2 (from (a))

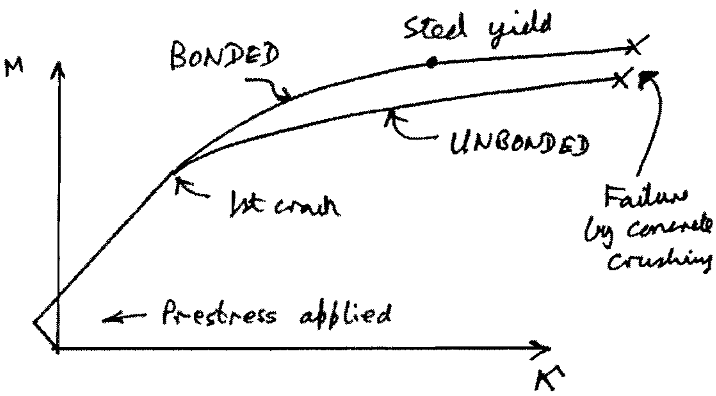
Moment needed to cause this = $8.3 \cdot 0.384.10^9$ Nmm = 3187 kNm

Load needed to cause this moment = $\frac{3187 \cdot 8}{30^2} = 28.3\,\text{kN/m}$ = 14.2 kN/m^2

(c) At mid-span there is almost always a sagging bending moment which allows the tendon to be placed below the lower Kern point, but at the support the beam has no bending moment. The tendon therefore has to lie within the Kern points and this can often be critical. The designer is either forced to use a larger section than would otherwise be required, of has to arrange the prestressing tendons so that some of the lower ones (but not all) can be curtailed near the support, thus raising the centroid of the prestress.

Most popular question and on the whole done well. Most of the errors were relatively trivial but there were a significant number who recalculated the existing stresses when the in-situ concrete acted compositely with the precast. This is the classic error and one that was emphasised several times in lectures. A few assumed that the weight of the in-situ concrete altered in proportion to the modular ratio!

4. (a)



For both beams the behaviour up to first cracking will be very similar. There will normally be some negative curvature when prestressed due to the eccentricity of the tendons. The beams will behave linearly elasticly until the tensile stress capacity is first exceeded.

After cracking, the unbonded beam loses stiffnesses more quickly than the bonded beam, because the tendon does not pick up stress so quickly at crack locations. Unbonded beams only have to satisfy compatibility in global terms, not locally.

The bonded beam, if properly designed, will probably reach a point where the tendon yields, although the moment capacity may continue to increase due to changes in lever arm between the tension and compression forces.

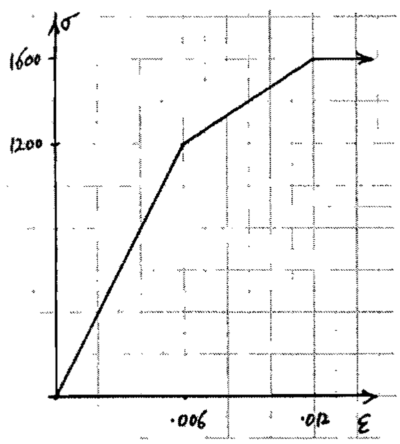
The unbonded beam will continue to increase in moment capacity, but usually without yielding of the steel.

Final failure in both beams will occur when concrete crushes, at similar strains.

(b) A complete analysis if the bonded beam can be carried out by satisfying compatibility locally, but for an unbonded beam this is not possible. The designer must calculate the strains in the tendon and in the concrete adjacent to the tendon all along the beam, and then integrate both separately. The two should be equal, satisfying the global compatibility condition.

What is normally of interest is to determine the peak moment capacity in the beam. This can be achieved by assuming that the change in strain in the tendon at the peak moment position is a certain fraction of the change in strain in the adjacent concrete. A factor of 0.25 is normally used for this but the exact value depends on the shape of the bending moment diagram and is higher for constant moments but lower for peaked moment diagrams.

(c) First plot steel stress-strain curve to scale



Need to make an initial guess for position of neutral axis. This would normally be between 0.25 and 0.5 of the effective depth, which in this case is at 600 mm, so a value of 200 mm seems reasonable. (*Full credit was given for any realistic value.*) Then iterate.

The strain factor β is here defined as 0.25.

	1st Guess	2nd Guess	3rd Guess
Neutral axis x (mm)	200	180	182
Additional steel strain $\beta\varepsilon_s$	$0.25 \cdot 0.0035 \cdot \frac{400}{200} = .00175$	.0024	.00201
Prestrain	0.004	0.004	0.004
Total Steel strain	0.00575	0.00604	0.00601
Steel stress (N/mm^2) from stress-strain curve	1150	1206	1200
Tension (kN) T	2070	2171	2160
Compression (kN) C	$30 \cdot 200 \cdot 400 = 2400$	2160	2184
	$C > T$	$C < T$	$C > T$
	increase x	decrease x	

Close enough. x must be between 180 and 182, say 181 mm; Lever arm = 600 – 181/2 = 509 mm

Take T = C = 2165 kN, so ultimate moment = 1102 kNm

Attempted by many candidates but done very well only by a few. There was a tendency to reproduce random thoughts from their notes for part (a) with little thought about producing a coherent answer. The numerical part caused few problems.

5. (a) Prestressed concrete developed largely in WWII due to the need to reduce amount of steel. Examples should include the mine supports at Monkton Farleigh Mine and the development of Emergency Bridge Beams. The use of concrete railway sleepers reduced need for timber but reinforced concrete could not take fatigue loads so prestressed concrete developed. Post war there were advantages because of lack of a maintenance requirement for prestressed concrete, but the difficulties of protecting tendon should be acknowledged.

(b) Winterton House is unusual in that the outer brick walls were added and placed in compression to allow a tensile prestress to be applied to the internal lightweight steel columns when heavier floors and internal walls added to refurbish the building.

(c) The Athenian Trireme is the earliest known example of prestressing. The vessel was prestressed by means of a rope twisted together, thus applying a compressive prestress from stem to stern. The rope was placed at a carefully chosen position inside the vessel (*not wrapped around the outside as some answers suggested*). This allowed the vessel to resist hogging and sagging moments induced by wave action without opening the transverse joints between the longitudinal members.

(d) Steel structures are not usually prestressed because they are strong in tension but are deficient in compression (because they are susceptible to buckling). Thus, any prestress would usually need to be a tensile prestress, which would mean that the reaction force would be in compression. It is difficult to place a tendon in compression. They should have noted that this is the exact opposite of prestressing in concrete, where the reacting tendon is in tension.

Foyle Bridge is one of the few examples of prestressed steel. The prestress is applied in the form of an external set of reactions which alters the bending moment diagram. The outer supports are pushed down while the inner ones are pushed up. This causes hogging moments in the central span, reducing the sagging bending moments there, which allows a smaller and lighter section to be used. The penalty is larger moments at the internal supports, but added weight near the supports is less of a problem than added weight at mid-span, so the overall result is a saving.

(e) Bicycle wheels rely on tension forces in spokes. When the rider's weight is transmitted to the ground it does so by reducing this tensile prestress, but not eliminating it, so the spokes remain in tension and do not buckle. The tension in the spokes is reacted by compression in the rims. When loaded, only the lower spokes experience a change in force. They could also point out that a pneumatic tyre is also a prestressed structure, putting the side-walls of the tyre into tension so that compressive loads are carried by relieving this tension. The air pressure does not change significantly when loaded, and the inner tube is only needed to keep the tyre airtight – it plays no part in the load-carrying. The tyres are held in place by the tyre bead which is also prestressed to the rim by being made smaller than the rim diameter.

A cartwheel works in the opposite way. A compressive prestress is applied to the wheel by shrinking a metal tyre onto the rim. The effect is to place the spokes and the components of the rim into compression. This keeps the joints secure and stops the wheel shaking itself to pieces.

Candidates were offered the opportunity to write about any one of five different topics relating to prestressing as a structural principle, which had been mentioned fairly briefly in the course. There were two different types of answers – clearly-weak candidates scrabbling around for odd marks in desperation, which had been expected, but also good candidates who had obviously been reading-around the topic and wrote very lucidly on the subject.

C J Burgoyne
1st June 2006