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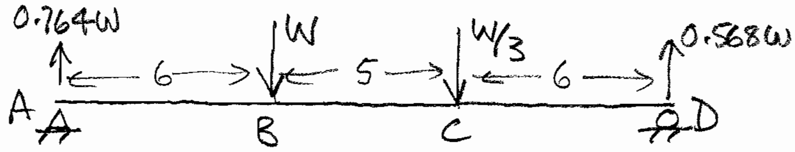
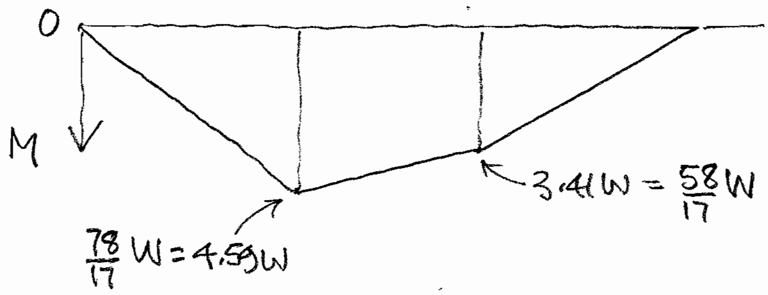
Engineering Tripos Part IIB, 2006
Module 4D10. SOLUTIONS

Q1(a)





Reactions, bending moments by statics. [20%]

(b) Structures Data Book:

$610 \times 229 \times 140$ UB has $Z_p = 4142\,cm^3$

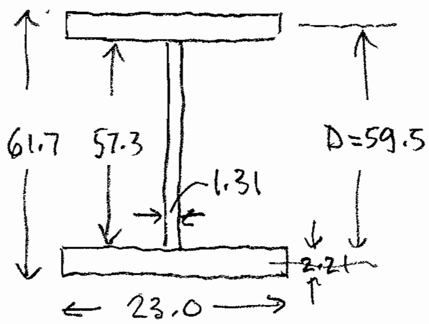
Given $\sigma_y = 355$ MPa,

$M_p = 1470$ kNm.

For plastic collapse, $4.59W = M_p$; $W = 320$ kN

Need to consider Lateral-torsional buckling in spans AB and BC (CD not critical).

For UB : $I_{yy} = 4505\,cm^4$, $J = 216\,cm^4$.



$G = 81$ GPa, $E = 205$ GPa.

Check compactness.

Flange : $\lambda = 5 < 8$

Web : $\lambda = 44 < 56$ } ok.

	Span AB	Span BC	
L	6	5	(m)
$M_1 = \frac{\pi}{L}\sqrt{EI_{yy}GJ}$	666	799	(kNm)
$M_2 = \frac{\pi^2}{L^2}E.D.\frac{I_{yy}}{2}$	753	1085	(kNm)
$M_E = \sqrt{M_1^2 + M_2^2}$	1005	1347	(kNm)

Q1 (cont)

	Span AB	Span BC	
M_p	1470	1470	(kNm)
$\lambda_{LT} = 75\sqrt{\dfrac{M_p}{M_E}}$	91	78	
$\overline{M}_c$	0.51	0.63	
$M_c = \overline{M}_c M_p$	757	926	
β	0	0.75	
M_u/m	0.6	0.9	
m	4.59 W	4.59 W	
M_u	2.76 W	4.13 W	
For stability $M_u < M_c$	$W \leq 271$	$\underline{\underline{W \leq 224}}$	(kNm)
For plastic collapse	$W \leq 320$		(kNm)

Span BC is critical for Lateral-torsional buckling.

Check Shear Limit also:

$$V_{crit} = A_{web} \times \tau_y = 1.31 \times 57.3\ \text{cm}^2 \times \frac{355}{\sqrt{3}}\ \text{MPa}$$
$$= 1540\ \text{kN}$$

$V_{max} = 0.764\,W$ in span AB $= 170$ kN $\ll 1540$
So OK. [70%]

(c) Self-weight of beam $= 140$ kg/m $\times 9.81$ N/kg

$\therefore$ Total self-wt $= 17 \times 140 \times 9.81 = 23$ kN

Total imposed load $= \frac{4}{3} \times 224 = 299$ kN

So self-weight $\simeq 8\%$ of critical imposed load: need to make some allowance for it. [10%]

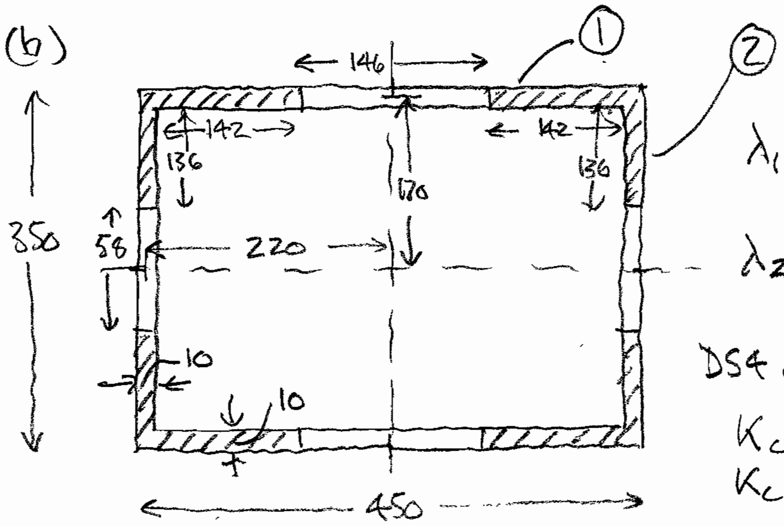
Q2.

(a) Factors are : initial crookedness, residual stresses, boundary conditions, yield stress....

(Euler analysed a pin-ended; elastic column)

[20%]

(b)



$\boxed{\sigma_y = 355}$

$\lambda_1 = \frac{b}{t} = \frac{450-20}{10} = 43$

$\lambda_2 = \frac{350-20}{10} = 33$

DS4 gives

$K_c = 0.66$ for ①
$K_c = 0.82$ for ② **

for ① $b_e = 0.66 \times (450-20) = 284 = 2 \times 142$
for ② $b_e = 0.82 \times (350-20) = 271 = 2 \times 136$ [30%]

$$I_{major} = \frac{1}{12}(350 \times 450^3 - 330 \times 430^3) - 2 \times 580 \times 220^2 - \frac{2 \times 10 \times 146^3}{12}$$
$$= 10^8 [4.71 - 0.56 - 0.05] = 4.1 \times 10^8 \text{ mm}^4$$

$$I_{minor} = \frac{1}{12}(450 \times 350^3 - 430 \times 330^3) - 2 \times 1460 \times 170^2 - \frac{2 \times 10 \times 58^3}{12}$$
$$= 10^8 [3.20 - 0.84 - 0.003] = 2.4 \times 10^8 \text{ mm}^4$$

$A_{eff} = 4 \times (142 + 136 + 10) \times 10 = \underline{11{,}500 \text{ mm}^2}$

(Major) $r = \sqrt{\frac{I_{maj}}{A}} = 190$ mm

(Minor) $r = \sqrt{\frac{I_{minor}}{A}} = 145$ mm

** Note that such values are hard to read from the charts; small differences acceptable

Major $\frac{r}{y} = \frac{190}{225} = 0.84 > 0.7$; Minor $\frac{r}{y} = \frac{145}{175} = 0.83 > 0.7$

So use <u>curve B on DS1</u>

Major: Clamped, pinned; $k = 0.85$, $\lambda = \frac{0.85 \times 15}{0.19} = 67$

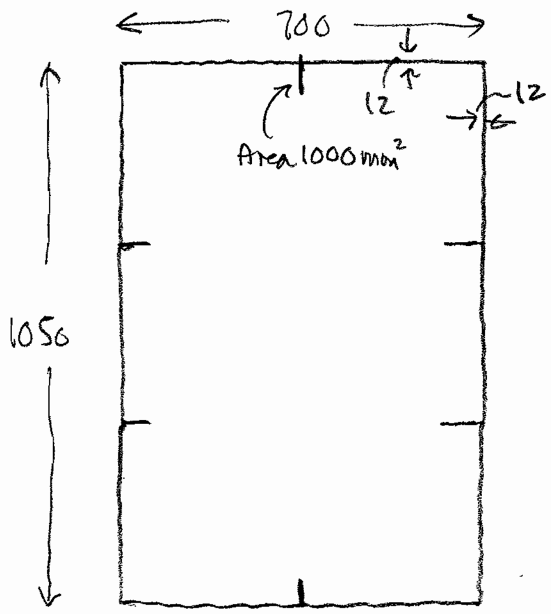
Minor: Clamped, clamped; $k = 0.7$, $\lambda = \frac{0.7 \times 15}{0.145} = \underline{72}$

Minor-axis bending critical.

[50%]

$P = A_{eff} \times \underset{**}{0.62} \times \sigma_y = 11500 \text{ mm}^2 \times 0.62 \times 355 \text{ N/mm}^2$
$= 2500$ kN

Q3



(a) Effective thickness of top plate

$$= 12 + \frac{1000}{700} = 13.4\text{ mm}$$

Eff. thickness of side plates

$$= 12 + \frac{2000}{1050} = 13.9\text{ mm}$$

OD of box $= 1050 \times 700$ mm

ID of box $= (1050 - 2\times 13.4) \times (700 - 2\times 13.9)$

$= 1023.2 \times 672.2$ mm

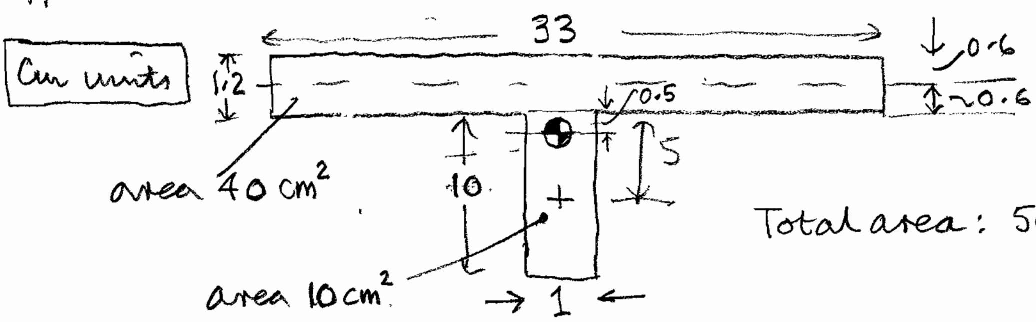
$$I_{major} = \frac{1}{12}\left(700 \times 1050^3 - 672.2 \times 1023.2^3\right) = 7520 \times 10^6\text{ mm}^4$$
$$= 75.2 \times 10^{-4}\text{ m}^4$$

$$A = 700 \times 1050 - 672.2 \times 1023.2 = 47.2 \times 10^3\text{ mm}^2 \quad [40\%]$$

(b) Top flange : $\lambda = \frac{350}{12}\sqrt{\frac{275}{355}} = 26$ (for each half-width)

From DS4 $K_c = 0.95$ (just not compact)

Effective T-section width $= 0.95 \times 350 = 330$ mm $\doteq 33$ cm



[N.B. If λ for top flange is worked out insted for full width, the T-section is 39 cm wide --- but resulting value of λ (see over) is 53 -- hardly different.]

Q3 (cont.)

Find centroid of T : Moments of area about ℄ of flange.

$$10 \times 5.6 = \bar{y}\,(10 + 40) \quad \therefore \; \bar{y} = 1.1$$

Overall CG = 0.5 cm below lower surface of flange.

$$
\begin{aligned}
I &= \tfrac{1}{12} \times 33 \times 1.2^3 &&= 4.75 \\
&+ 33 \times 1.2 \times 1.1^2 &&+41.53 \\
&+ \tfrac{1}{12} \times 1 \times 10^3 &&+83.33 \\
&+ 1 \times 10 \times 4.5^2 &&+202.50 \\
& && \overline{332.0} \text{ cm}^4
\end{aligned}
$$

I about own centroidal axis

Apply ‖-axis theorem

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{332}{50}} = 2.6 \text{ cm}$$

y to extreme fibre = 9.5 cm

$\left.\right\} \dfrac{r}{y} = 0.27$ -- so use Curve C of DS1.

Diaphragms at spacing 150 cm = L for buckling

$$\lambda = \frac{150}{2.6}\sqrt{\frac{275}{355}} = 51 \;\rightarrow\; \bar{\sigma}_c = 0.75$$

$$\therefore \sigma_c = 0.75 \times 275 = 206 \text{ MPa}$$

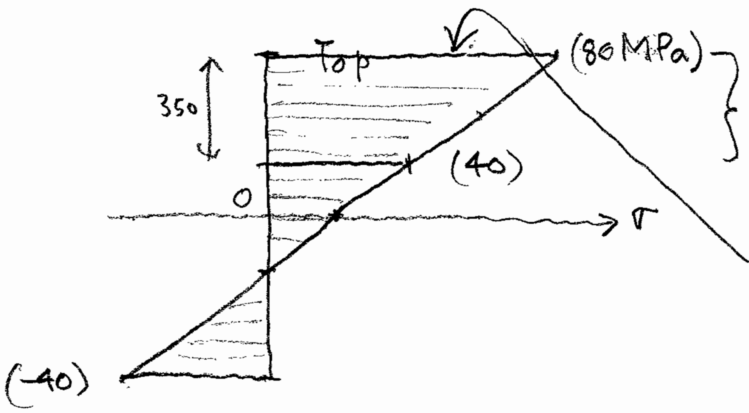
Major axis BM = 860 kNm
Axial compression = 950 kN } given

$$\text{Bending stress} = \frac{860 \times 10^3 \times 0.525}{75.2 \times 10^{-4}} \quad \frac{\text{N m}^2}{\text{m}^4}$$

$$= 60 \times 10^6 \text{ N/m}^2 = 60 \text{ MPa}$$

$$\text{Overall Compressive stress} = \frac{950 \text{ kN}}{47.2 \times 10^3 \text{ mm}^2} = 20 \text{ MPa}$$

Distribution of stress in cross-section:



top panel
Mean stress = 60
Bending stress = 20

Stress in top plate < 206 MPa, ∴ OK. [40%]

Q3 (cont.)

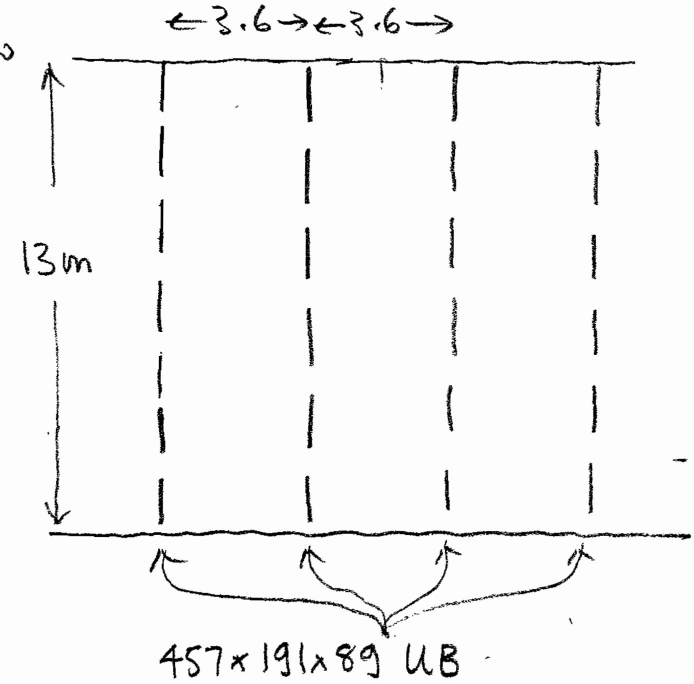
(c) For top web panel: $\lambda = \frac{350}{12}\sqrt{\frac{275}{355}} = 26$

From DS4, $K_c = 1$, $K_b = 1.23$

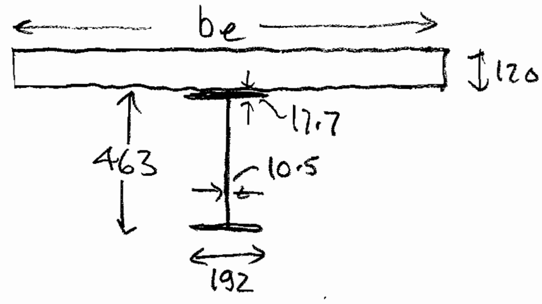
So $\frac{60}{275} + \left(\frac{20}{1.23 \times 275}\right)^2 \simeq 0.2 < 1$ ✓ easily.

Strength check: $80 < 275$ ✓ easily. [20%]

Q4. Plan view



Cross-section



From Structures Data Book: $A_s = 114\,cm^2$
Mass/length = 89.3 kg/m
$I_{xx} = 41020\,cm^4$

Check for compactness:

$\lambda_{flange} = \frac{192 - 10.5}{2 \times 17.7} = 5.1 < 8$ ✓ OK

$\lambda_{web} = \frac{463 - 2 \times 17.7}{10.5} = 41 < 56$ ✓ OK

Q4 (cont.)

b_e = effective width of slab
= smaller of $b = 3.6$ and $\frac{\text{span}}{4} = \underline{3.25}$
↑ b_e.

Permanent loads per unit length of beam:

Concrete: $2400 \times 9.81 \times 0.12 \times 3.6$	=	10.2 kN/m
UB Self weight: 89.3×9.81	=	0.88
Services 0.7×3.6	=	2.52
		13.6 kN/m

Multiply by partial factor 1.4 → $\underline{19.0}$

Imposed load $= 7 \times 3.6 \times 1.6 = 40.3$

Total design load $= 59.3$ kN/m $= w$

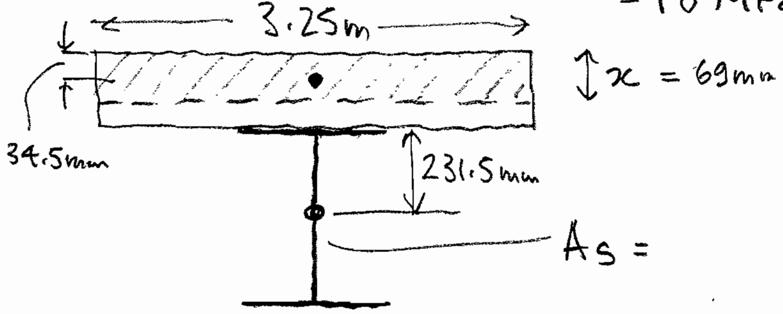
Over span 13 m, Max. BM $= \frac{wL^2}{8} = \frac{59.3 \times 13^2}{8}$

$= \underline{1250 \text{ kNm}}$

(a) Plastic calculation.

Assume NA is in concrete

Eff. yield stress of concrete $= 0.6 f_{cd} = 0.6 \times 30$
$= 18$ MPa.



Full plastic tension in steel = full plastic compression in depth x of concrete.

$114 \times 10^{-4} \times 355 \times \cancel{10^6} = 18 \times \cancel{10^6} \times 3.25\, x$

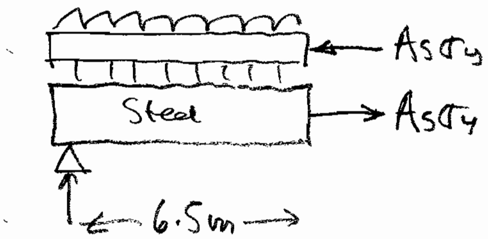
$\therefore x = 0.069$ m < 0.12 m ✓ assumption ok

Lever arm $= 231.5 + 120 - 34.5 = 317$ mm $= 0.317$ m

$M_p = \underset{m^2}{114 \times 10^{-4}} \times \underset{N/m^2}{355 \times 10^6} \times \underset{m}{0.317} = 1.28 \times 10^6$ Nm
$= \underline{1280 \text{ kNm}}$

Q4 (cont) This is just OK – 2% margin. [40%]

(b) Shear studs.



Shear studs together have to carry total force $A_s\sigma$.

If N studs over whole span:

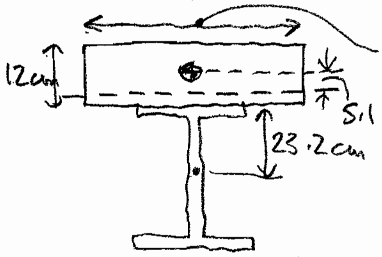
$$\frac{N}{2} \times \underbrace{47{,}000}_{\text{for } 65\times13\text{mm stud (DS6)}} = 114\times10^{-4} \times 355\times10^{6}$$

$\rightarrow$ N = 172 studs

Spacing $= \frac{13000}{172} = 75$ mm ✓ OK [25%]

(C) Elastic deflection. From DS6 Short-term E_{concr} = 28 GPa

$\frac{E_s}{E_c} = \frac{205}{28} = 7.3$. "Transform" concrete to equivalent steel



$\frac{325}{7.3} = 44.5$ cm

Use "see-saw" rule to find centroid. Moments of area about top edge:

$$12\times44.5\times6 + 114(12+23.2) = \bar{y}(12\times44.5+114)$$

$\rightarrow$ $\bar{y} = 11.1$ cm, ie just above the interface.

However, this implies that the bottom 0.9 cm of concrete is cracked/ineffective elastically. Strictly we should do the calculation again, obtaining a quadratic equation for y. But the answer is almost exactly the same, because I of this "lost" concrete is so small. Thus, ignoring the "lost" concrete: $I = \frac{1}{12}\times44.5\times12^3 + 44.5\times12\times5.1^2 + 41020 + 114\times24.1^2$

$= 128{,}000$ cm^4 $= 1280\times10^{-6}$ m^4

Central defl. $= \frac{5}{384}\frac{wL^4}{E_sI_{xx}} = \frac{5\times\overbrace{25.2}^{7\times3.6}\times10^3\times13^4}{384\times205\times10^9\times1280\times10^{-6}}$ ← Unfactored live load

$= 0.036$ m $= 36$ mm.

Check: $\frac{\text{Span}}{250} = 52$ mm : 36 mm is OK. [35%]

CRC
07/05/06