ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Friday 5 May 2006 2.30 to 4

Module 4A1

NUCLEAR POWER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4A1 datasheet (8 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) (i) Americium-241 decays by alpha emission into neptunium-237 with a half-life of 432 years. Given that the atomic masses of $^{241}\mathrm{Am}$ and $^{237}\mathrm{Np}$ are 241.05682 u and 237.04817 u respectively, calculate the energy of the alpha particle (in MeV) assuming it takes up all the energy released in the decay. [10%]

(ii) In an Am-Be neutron source, alpha particles from the decay of $^{241}\mathrm{Am}$ interact with beryllium-9 nuclei to release neutrons through the following reaction

$$^{9}\mathrm{Be} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \mathrm{n}\,.$$

Estimate the maximum energy (in MeV) of the neutron released in this reaction.

Relevant atomic masses can be found in the 4A1 datasheet. [15%]

(b) An Am-Be neutron source contains 10 g of $^{241}\mathrm{Am}$. Assuming that 50% of the alpha particles emitted give rise to a neutron, that all neutrons released escape the source and that they are emitted equally in all directions, calculate the neutron flux at a distance of 1 m from the source. [30%]

(c) The one-group, steady-state, source-free neutron diffusion equation for a non-multiplying medium can be written as

$$\nabla^2\phi - \frac{\phi}{L^2} = 0\,,$$

where L is the diffusion length.

Treating the diffusion of neutrons as one-dimensional, find the thickness of a lead shield that will stop 99% of an incoming flux of neutrons. Take the neutron diffusion length in lead to be 0.12 m.

Comment on the practical implications of this result. [25%]

(d) Discuss the components needed for a more effective neutron shield than the lead one analysed in part (c). [20%]

2 The equations governing the behaviour of xenon-135 in a 'lumped' reactor model can be written as

$$\frac{dI}{dt} = \gamma_i \Sigma_f \phi - \lambda_i I$$

$$\frac{dX}{dt} = \gamma_x \Sigma_f \phi + \lambda_i I - \lambda_x X - \phi \sigma X \, .$$

(a) Explain the meaning of each symbol in these equations. [15%]

(b) A thermal reactor operates in steady state with a neutron flux of 15×10^{16} $\mathrm{n\,m^{-2}s^{-1}}$. Find the steady-state poisoning effect due to xenon-135. Take the average number of neutrons released in fission (ν) to be 2.4, $\gamma_i = 0.061$, $\gamma_x = 0.003$, $\lambda_i = 2.874 \times 10^{-5}\ \mathrm{s^{-1}}$, $\lambda_x = 2.027 \times 10^{-5}\ \mathrm{s^{-1}}$ and $\sigma = 2.75$ Mbarns. [20%]

(c) The reactor is shut down after a prolonged period of operation at this flux level. Additional reactivity is available equal to 25% of the loss of reactivity to xenon-135 during the previous steady-state operation. Show that it would be possible to restart the reactor two hours after shutdown, but not three hours after shutdown. [65%]

3 (a) Which types of nuclear reactor currently in service worldwide are conventionally refuelled on-line and which off-line? Describe the advantages and disadvantages of on-line refuelling. [20%]

(b) The fuel of a reactor is subdivided into M batches of standard fuel elements which lose reactivity linearly with burn-up. The reactor is operated at constant power until the reactivity of the core as a whole reaches zero, whereupon one batch of fuel elements is replaced by a fresh batch, and so on, in cycles. Show that the length of the second such cycle will be $1/M$ of the first cycle length. Assume that all fuel elements operate at the same power. [20%]

(c) A 3-batch reloading scheme is chosen for such a reactor. The first cycle lasts 18 months. Derive a recurrence relationship governing the lengths of successive cycles, and hence find the lengths of the third, fourth and fifth cycles. What is the eventual steady-state cycle length? [30%]

(d) Show that the fuel utilisation in this steady-state cycle is only 75% of the utilisation available with continuous on-line refuelling. [15%]

(e) Explain how equilibrium operation (in terms of cycle length) can be established immediately from start-up for M-batch refuelling. [15%]

4 (a) Describe the basic steps in the reprocessing of spent nuclear fuel, and discuss the main waste streams arising and how they are handled. [30%]

(b) What were the reasons for the UK and France establishing nuclear fuel reprocessing programmes? What are the disadvantages of reprocessing? [20%]

(c) A large utility operating a number of pressurised water reactors requires 100 tonnes (as U metal) of fuel per year at an enrichment of 3.5% U-235. Taking the U-235 content of natural uranium to be 0.7% and assuming an enrichment plant tails of 0.3% U-235, calculate the amount of natural uranium and the number of separation work units required per year. Losses in processing can be neglected. [20%]

(d) Estimate the savings in fresh natural uranium and separation work units if the fuel was reprocessed and the uranium recycled. Assume that the fuel leaves the reactor containing 96% uranium at an enrichment of 0.8% U-235. Take the total losses in the reprocessing plant to be 1%. [20%]

(e) Suggest ways of using the plutonium recovered in recycling the fuel. [10%]

END OF PAPER

MODULE 4A1

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	299.792458×10^{6} m s^{-1}
Magnetic permeability in vacuum	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Planck constant	h	6.626176×10^{-32} J s
Boltzmann constant	k	1.380662×10^{-23} J K^{-1}
Elementary charge	e	$1.6021892 \times 10^{-19}$ C

Definitions

Unified atomic mass constant	u	$1.6605655 \times 10^{-27}$ kg (931.5016 MeV)
Electron volt	eV	$1.6021892 \times 10^{-19}$ J
Curie	Ci	3.7×10^{10} Bq
Barn	barn	10^{-28} m^2

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}Ne$	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}Ne$	20.99385 u
99.985%	$^{1}_{1}H$	1.00783 u	8.94%	$^{22}_{10}Ne$	21.99138 u
0.015%	$^{2}_{1}H$	2.01410 u	10.1%	$^{25}_{12}Mg$	24.98584 u
0%	$^{3}_{1}H$	3.01605 u	11.1%	$^{26}_{12}Mg$	25.98259 u
0.0001%	$^{3}_{2}He$	3.01603 u	0%	$^{32}_{15}P$	31.97391 u
99.9999%	$^{4}_{2}He$	4.00260 u	96.0%	$^{32}_{16}S$	31.97207 u
7.5%	$^{6}_{3}Li$	6.01513 u	0%	$^{60}_{27}Co$	59.93381 u
92.5%	$^{7}_{3}Li$	7.01601 u	26.2%	$^{60}_{28}Ni$	59.93078 u
0%	$^{8}_{4}Be$	8.00531 u	0%	$^{87}_{35}Br$	86.92196 u
100%	$^{9}_{4}Be$	9.01219 u	0%	$^{86}_{36}Kr$	85.91062 u
18.7%	$^{10}_{5}B$	10.01294 u	17.5%	$^{87}_{36}Kr$	86.91337 u
0%	$^{11}_{6}C$	11.01143 u	12.3%	$^{113}_{48}Cd$	112.90461 u
98.89%	$^{12}_{6}C$	12.00000 u		$^{226}_{88}Ra$	226.02536 u
1.11%	$^{13}_{6}C$	13.00335 u		$^{230}_{90}Th$	230.03308 u
0%	$^{13}_{7}N$	13.00574 u	0.72%	$^{235}_{92}U$	235.04393 u
99.63%	$^{14}_{7}N$	14.00307 u	0%	$^{236}_{92}U$	236.04573 u
0%	$^{14}_{8}O$	14.00860 u	99.28%	$^{238}_{92}U$	238.05076 u
99.76%	$^{16}_{8}O$	15.99491 u	0%	$^{239}_{92}U$	239.05432 u
0.04%	$^{17}_{8}O$	16.99913 u		$^{239}_{93}Np$	239.05294 u
0.20%	$^{18}_{8}O$	17.99916 u		$^{239}_{94}Pu$	239.05216 u
				$^{240}_{94}Pu$	240.05397 u

Simplified Disintegration Patterns

Isotope	$^{60}_{27}Co$	$^{90}_{38}Sr$	$^{90}_{39}Yt$	$^{137}_{55}Cs$	$^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

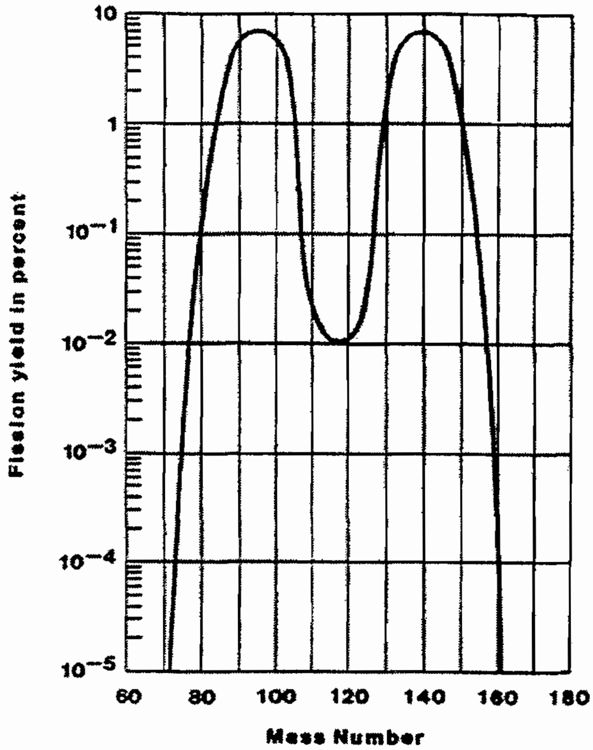
	"Nuclear" graphite	$^{16}_{8}O$	$^{113}_{48}Cd$	$^{235}_{92}U$	$^{238}_{92}U$	$^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half lives) from thermal-neutron fission of U-235 are:

$$\text{Kr-90} \xrightarrow{33\text{ s}} \text{Rb-90} \xrightarrow{2.7\text{ min}} \text{Sr-90} \xrightarrow{28\text{ yr}} \text{Y-90} \xrightarrow{64\text{ hr}} \text{Zr-90}$$

Sr-90 is a serious health hazard, because it is bone-seeking.

$$\text{Sb-131} \xrightarrow{23\text{ min}} \text{Te-131} \xrightarrow{24\text{ min}} \text{I-131} \xrightarrow{8\text{ days}} \text{Xe-131}$$

I-131 is a short-lived health hazard. It is thyroid-seeking.

$$\text{I-135} \xrightarrow{6.7\text{ hr}} \text{Xe-135} \xrightarrow{9.2\text{ hr}} \text{Cs-135 (nearly stable)} \qquad \text{Fission yield } \gamma = 0.064$$

Xe-135 is a strong absorber of thermal neutrons, with $\sigma_a = 3.5$ Mbarn .

$$\text{Pm-149} \xrightarrow{54\text{ hr}} \text{Sm-149} \qquad \text{Fission yield } \gamma = 0.014$$

Sm-149 is a strong absorber of thermal neutrons, with $\sigma_a = 53$ kbarn .

$$\text{Se-87} \xrightarrow{17\text{ s}} \text{Br-87} \xrightarrow{55\text{ s}} \text{either } \{\text{Kr-86} + \text{n}\} \text{ or } \{\text{Kr-87} \xrightarrow{78\text{ min}} \text{Rb-87}\}$$

This chain leads to a "delayed neutron".

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	
Mean life time of precursor ($1/\lambda_i$) / s	80	32	8.0	3.1	0.65	0.22	Total
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy ⇒ ~200 MeV / fission

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\dfrac{\text{production}}{\text{removal}} = \dfrac{P}{R}$
Excess multiplication factor	k_{ex}	$\dfrac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\dfrac{P-R}{P} = \dfrac{k_{ex}}{k_{eff}}$
Lifetime	l	$\dfrac{1}{R}$
Reproduction time	Λ	$\dfrac{1}{P}$

Reactor Kinetics Equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where n = neutron concentration

c = precursor concentration

β = delayed neutron precursor fraction $= \sum \beta_i$

λ = average precursor decay constant

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla . \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$$D = \frac{1}{3\Sigma_s (1 - \overline{\mu})}$$

with $\overline{\mu}$ = the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$

Cylindrical geometry: $\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r \dfrac{\partial}{\partial r} \right) + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial \theta^2} + \dfrac{\partial^2}{\partial z^2}$

Spherical geometry: $\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial}{\partial r} \right) + \dfrac{1}{r^2 \sin\theta}\dfrac{\partial}{\partial \theta}\left(\sin\theta \dfrac{\partial}{\partial \theta} \right) + \dfrac{1}{r^2 \sin^2\theta}\dfrac{\partial^2}{\partial \psi^2}$

Bessel's Equation of 0th Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+R=0$$

Solution is: $$R(r)=A_1J_0(r)+A_2Y_0(r)$$

$J_0(0)=1;\ Y_0(0)=-\infty;$

The first zero of $J_0(r)$ is at $r=2.405$.

$J_1(2.405)=0.5183$, where $J_1(r)=\frac{1}{r}\int_0^r xJ_0(x)dx$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density g cm^{-3}	Σ_a cm^{-1}	D cm	$L^2=D/\Sigma_a$ cm^2
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $$\theta=\frac{T_M}{T_1}=\frac{2}{M+1}$$

Enrichment of Isotopes

Value function: $$v(x)=(2x-1)\ln\left(\frac{x}{1-x}\right)\approx-\ln(x)\text{ for small }x$$

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $$R_n\equiv\frac{L_n''}{P}=\frac{x_p-x_{n+1}'}{x_{n+1}'-x_n''}$$

Stripping section reflux ratio: $$R_n=\left[\frac{x_p-x_f}{x_f-x_w}\right]\left[\frac{x_{n+1}'-x_w}{x_{n+1}'-x_n''}\right]$$

Bateman's Equation

$$N_i = \lambda_1 \lambda_2 \ldots \lambda_{i-1} P \sum_{j=1}^{i} \frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j \tau)}{\lambda_j \prod_{\substack{k=1 \\ k \neq j}}^{i} (\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide i

λ_j = decay constant of nuclide j

P = parent nuclide production rate

T = filling time

τ = decay hold-up time after filling

Temperature Distribution

For axial coolant flow in a reactor with a chopped cosine power distribution, Ginn's equation for the non-dimensional temperature is:

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L = fuel half-length

L' = flux half-length

$T_{c1/2}$ = coolant temperature at mid-channel

T_{co} = coolant temperature at channel exit

$Q = \frac{\pi \dot{m} c_p}{UA} \frac{L}{L'}$

with $\dot{m}$ = coolant mass flow rate

c_p = coolant specific heat capacity (assumed constant)

$A = 4\pi r_o L =$ surface area of fuel element

and for radial fuel geometry:

$$\frac{1}{U} = \underbrace{\frac{1}{h}}_{\text{bulk coolant}} + \underbrace{\frac{1}{h_s}}_{\text{scale}} + \underbrace{\frac{t_c}{\lambda_c}}_{\text{thin clad}} + \underbrace{\frac{r_o}{h_b r_i}}_{\text{bond}} + \underbrace{\frac{r_o}{2\lambda_f}\left(1 - \frac{r^2}{r_i^2}\right)}_{\text{fuel pellet}}$$

with h = heat transfer coefficient to bulk coolant

h_s = heat transfer coefficient of any scale on fuel cladding

t_c = fuel cladding thickness (assumed thin)

λ_c = fuel cladding thermal conductivity

r_o = fuel cladding outer radius

r_i = fuel cladding inner radius = fuel pellet radius

h_b = heat transfer coefficient of bond between fuel pellet and cladding

λ_f = fuel pellet thermal conductivity

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Numerical Results

1 (a) (i) 5.636 MeV
(ii) 11.337 MeV
(b) $5.06 \times 10^{10}\ \mathrm{n\,m^{-2}s^{-1}}$
(c) 0.55 m
(d) -

2 (a) -
(b) –0.01788
(c) After two hours poisoning is –0.02178
After three hours poising is –0.02306
Maximum reactivity available is 0.02235

3 (a) -
(b) -
(c) 8 months, $10\frac{2}{3}$ months, $8\frac{2}{9}$ months, 9 months
(d) -
(e) -

4 (a) -
(b) -
(c) 800 tonnes, 432.1 tonnes SWU
(d) 118.8 tonnes, 7.5 tonnes SWU
(e) -