ENGINEERING TRIPOS PART IIB

Tuesday 25 April 2006 9 to 10.30

Module 4A8

ENVIRONMENTAL FLUID MECHANICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4A8 Data Card (5 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Explain the meaning of *geostrophic flow*, the assumptions made in deriving the equations governing this flow and where it occurs. A high pressure system is centred above City A that lies in the northern hemisphere. City B is due west from A. In which direction do you expect the geostrophic wind component due to this system to be in City B? [25%]

(b) What are *anabatic* and *katabatic flows*? Toxic gas is released on a slope and is carried towards you by the wind. Would you prefer the wind to be due to an anabatic flow or to a katabatic flow? Explain your reasoning. [25%]

(c) (i) What is the importance of the Monin-Obukhov length? [25%]

(ii) On a particular day after sunset, the Monin-Obukhov length is 10 m. Measurements of the vertical gradient of the streamwise velocity in the atmospheric boundary layer are made at a height of 1 m above the ground. These are used to calculate the wall-shear velocity $u*$ by using the assumption that the measurement point is in the logarithmic layer and that the flow is neutrally stable. What is the error in the calculated value of $u*$ (in percentage points)? [25%]

2 A steady free convection flow is set up near the ground on a hot day due to heat flux Q from the ground to the air above. There is no mean velocity in any direction and the characteristic size of the turbulent eddies is L. The general form of the enthalpy equation for a turbulent flow is given by

$$\rho c_p \frac{DT}{Dt} = \frac{\partial}{\partial x_3}\left(\lambda \frac{\partial T}{\partial x_3} - \rho c_p \overline{u_3 \theta}\right)$$

where T is the mean temperature of the air, c_p is the heat capacity at constant pressure, ρ is the density, λ is the conductivity, x_3 is the vertical direction, u_3 is the vertical velocity fluctuation, θ is the temperature fluctuation, and an overbar denotes a time average.

(a) By considering the boundary condition at the surface and the above equation for the particular problem examined here, show that

$$\lambda \frac{\partial T}{\partial x_3} - \rho c_p \overline{u_3 \theta} = -Q$$

[20%]

(b) Show that the kinetic energy dissipation rate ε is given approximately by

$$\varepsilon = \frac{g}{T}\frac{Q}{\rho c_p}$$

[30%]

(c) What is the characteristic turbulent velocity? [20%]

(d) If the ground were suddenly cooled to ambient so that the heat flux was removed, derive an expression for the time taken for the turbulent velocity to decrease to 10% of its original value, assuming that the lengthscale remains constant. [30%]

3 (a) Briefly discuss the main constituents of photochemical pollution including the key relevant chemical reactions. [50%]

(b) Assume a "Box Model" of length L, width W, height H, that the concentration c of a pollutant is uniform, the wind speed is U, and that b is the incoming pollutant concentration. Derive from first principles a governing equation for the time evolution of c. Include chemical reactions and ground emissions. [25%]

(c) By considering steady-state solutions of the expression you derived in Part (b) and by considering how the various parameters may change during the day, discuss qualitatively the likely diurnal variation of the concentration of an inert pollutant above a city. [25%]

4 (a) A stack of height H emits Q kg s^{-1} of an inert pollutant in a uniform wind of speed U. Assuming that the dispersion coefficient σ is uniform in all directions and is equal to αx, where x is the distance from the stack and α is a constant, find an expression for the distance from the stack where the maximum ground concentration occurs. [50%]

(b) In reality, the horizontal and the vertical dispersion coefficients are not equal. Which one is usually greater and why? For the same wind speed, how do you expect the ratio between the two dispersion coefficients to vary with atmospheric stability? [50%]

END OF PAPER

4A8: Environmental Fluid Mechanics

Part I: Turbulence and Fluid Mechanics

DATA CARD

Rotating Flows

Geostrophic Flow

$$-\frac{1}{\rho}\nabla p = 2\underline{\Omega} \times \underline{u}$$

Ekman Layer Flow

$$-2\Omega_z v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2}$$

$$2\Omega_z u = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu \frac{\partial^2 v}{\partial z^2}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

OR

$$-2\Omega_z v = \nu \frac{\partial^2 u}{\partial z^2}$$

$$-2\Omega_z v(u_0 - u) = \nu \frac{\partial^2 v}{\partial z^2}$$

GEOSTROPHIC VELOCITY

Solution is

$$u = u_0\left[1 - e^{-z/\Delta}\cos\frac{z}{\Delta}\right]$$

$$v = u_0 e^{-z/\Delta}\sin\frac{z}{\Delta}$$

$$\Delta = \left(\frac{\nu}{\Omega_z}\right)^{1/2}$$

Turbulent Flows – Incompressible

Continuity Equation $\nabla \bullet \underline{U} = \frac{\partial U_i}{\partial x_i} = 0$

Momentum Equation $\rho \frac{D\underline{U}}{Dt} = -\nabla P + \mu \nabla^2 \underline{U} + \underline{F}$

$$\rho \frac{DU_i}{Dt} = -\frac{\partial P}{\partial x_i} + \mu \frac{\partial^2 U_i}{\partial x_j^2} + F_i$$

Enthalpy Equation $\rho c_p \frac{DT}{Dt} = -k \frac{\partial^2 T}{\partial x_i^2}$

Reynolds Transformation $U_i = \overline{U_i} + u_i$ etc

Reynolds Stress $= -\rho \overline{u_i u_j}$

Reynolds Heat Flux $= -\rho c_p \overline{u_j \theta}$

Turbulent Kinetic Energy Equation $\frac{D}{Dt} \frac{q^2}{2} = -\overline{u_i u_k} \frac{\partial \overline{U_i}}{\partial x_k} - \varepsilon + \frac{\overline{f_i u_i}}{\rho} +$ transport terms

$$\varepsilon = \nu \overline{\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) \left(\frac{\partial u_i}{\partial x_k} \right)} \quad , \quad \frac{q^2}{2} = \frac{1}{2}\left(u_1^2 + u_2^2 + u_3^2 \right)$$

In flows with thermally driven motion

$$\frac{f_i u_i}{\rho} = \frac{g}{T} \bullet \overline{\theta u_i}$$

i = Vertical direction

Dissipation of turbulent kinetic energy $\varepsilon \approx \frac{u'^3}{\ell}$

Kolmogorov microscale $\eta = \left(\frac{\nu^3}{\varepsilon} \right)^{1/4}$

Taylor microscale (λ) $\varepsilon = 15 \nu \frac{u'^2}{\lambda^2}$

(ν is the kinematic viscocity)

Density Influenced Flows

Atmospheric Boundary Layer

$$\left.\frac{dT}{dz}\right|_{\text{NEUTRAL STABILITY}} = -\frac{g}{C_p} = \left.\frac{dT}{dz}\right|_{\text{DALR}}$$

$$R_i = \frac{g}{T}\frac{\frac{dT}{dz} - \left.\frac{dT}{dz}\right|_{\text{DALR}}}{\left(\frac{dU}{dz}\right)^2} = \text{RICHARDSON NUMBER}$$

Neutral Stability

$$U = \frac{u_*}{\kappa}\ln\frac{z}{z_0}; \qquad \frac{dU}{dz} = \frac{u_*}{\kappa z}$$

$$u_* = \sqrt{\frac{\tau_w}{\rho}} \quad ; \quad \kappa = \text{von Kármán Constant} = 0.40$$

Non-Neutral Stability

$$\text{L} = \text{Monin-Obukhov length} = -\frac{u_*^3}{\kappa \frac{g}{T}\frac{Q}{\rho c_p}}$$

Q = surface heat flux

$$\frac{dU}{dz} = \frac{u_*}{\kappa z}\left(1 - 15\frac{z}{L}\right)^{-1/4} \qquad \text{Unstable}$$

$$= \frac{u_*}{\kappa z}\left(1 + 4.7\frac{z}{L}\right) \qquad \text{Stable}$$

Buoyant plume for a point source

$$\frac{d}{dz}\left(\pi R^2 w\right) = 2\pi R u_e \qquad \text{(i)}$$

$$\frac{d}{dz}\left(\rho \pi R^2 w\right) = \rho_a 2\pi R u_e \qquad \text{(ii)}$$

$$\frac{d}{dz}\left(\rho \pi R^2 w^2\right) = g\left(\rho_a - \rho\right)\pi R^2 \qquad \text{(iii)}$$

(i) and (iii) give

$$\pi R^2 w \left(\frac{\rho_a - \rho}{\rho_a}\right) g = \text{constant} = F_0 \text{ (buoyancy flux)}$$

$$u_e = \alpha w$$

(α = Entrainment coefficient)

Buoyancy Frequency

$$N^2 = -\frac{g}{\rho}\frac{d\rho}{dz} = \frac{g}{T}\frac{dT}{dz}$$

Actually $\frac{g}{T}\left(\frac{dT}{dz} - \left.\frac{dT}{dz}\right|_{\text{DALR}}\right)$

N = Brunt – Väisälä Frequency or Buoyancy Frequency

4A8: Environmental Fluid Mechanics

Part II: Dispersion of Pollution in the Atmospheric Environment

DATA CARD

Transport equation for the mean of the reactive scalar ϕ:

$$\frac{\partial \overline{\phi}}{\partial t} + \overline{u}_j \frac{\partial \overline{\phi}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(K \frac{\partial \overline{\phi}}{\partial x_j} \right) + \overline{\dot{w}}$$

Transport equation for the variance of the reactive scalar ϕ:

$$\frac{\partial g}{\partial t} + \overline{u}_j \frac{\partial g}{\partial x_j} = \frac{\partial}{\partial x_j}\left(K \frac{\partial g}{\partial x_j} \right) + 2K\left(\frac{\partial \overline{\phi}}{\partial x_j} \right)^2 - \frac{2}{T_{turb}} g + 2\overline{\phi' \dot{w}'}$$

Mean concentration of pollutant after instantaneous release of Q kg at t=0:

$$\overline{\phi}(x, y, z, t) = \frac{Q}{8(\pi t)^{3/2}(K_x K_y K_z)^{1/2}} \exp\left[-\frac{1}{4t}\left(\frac{(x - x_0)^2}{K_x} + \frac{(y - y_0)^2}{K_y} + \frac{(z - z_0)^2}{K_z} \right) \right]$$

Gaussian plume spreading in two dimensions from a source at $(0,0,z_0)$ emitting Q kg/s:

$$\overline{\phi}(x, y, z) = \frac{Q}{2\pi} \frac{1}{U \sigma_y \sigma_z} \exp\left[-\left(\frac{y^2}{2\sigma_y^2} + \frac{(z - z_0)^2}{2\sigma_z^2} \right) \right]$$

One-dimensional spreading from line source emitting Q/L kg/s/m :

$$\overline{\phi}(x, y) = \frac{Q}{UL} \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left(-\frac{y^2}{2\sigma_y^2} \right)$$

Relationship between eddy diffusivity and dispersion coefficient:

$$\sigma^2 = 2\frac{x}{U} K$$