ENGINEERING TRIPOS PART IIB

Wednesday 3 May 2006 2.30pm to 4pm

Module 4A10

FLOW INSTABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Data Card for 4A10 (two sides)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Liquid of density ρ is retained between two vertical plates, distance d apart, over air of negligible density. The undisturbed liquid/air interface is at $z = 0$ and has surface tension σ, as shown in Fig. 1.

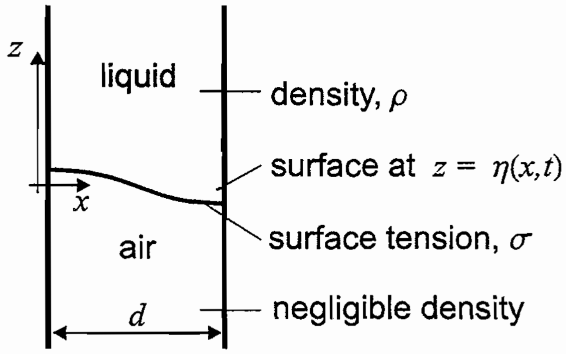

Fig. 1

(a) A velocity potential, ϕ, is given by:

$$\phi(x,z,t) = Ae^{st}\cos\left(\frac{n\pi x}{d}\right)e^{-n\pi z/d}$$

for constants A and s. Show that this velocity potential can describe linear disturbances in the liquid, provided that n is an integer. [25%]

(b) The corresponding displacement of the liquid/air interface is:

$$\eta(x,t) = \eta_0 e^{st}\cos\left(\frac{n\pi x}{d}\right).$$

Determine the relationship between A and η_0 and find s in terms of σ, d and the acceleration due to gravity, g. [40%]

(c) What is the minimum value of surface tension required to ensure that the liquid remains in place despite the influence of gravity? [20%]

(d) What form of surface displacement occurs if the surface tension is suddenly decreased (e.g. by the introduction of contaminants) to just below this critical value? [15%]

2 Consider:

(I) A boundary layer flow over a curved wall

(II) A layer of liquid with a free upper surface and heated from below.

For each flow:

(a) State two different physical mechanisms that can lead to instability. [10%]

(b) For each mechanism:

(i) explain the physical mechanism that drives towards instability; [20%]

(ii) state the stabilizing influences; [20%]

(iii) derive an appropriate non-dimensional number to describe the susceptibility to instability; [20%]

(iv) describe the flow pattern *just* after the onset of instability. [20%]

(c) State when you would expect each mechanism to dominate. [10%]

(TURN OVER

3 A lamp hangs above a radiator as shown in Fig. 2a. It has a D-shaped cross-section and solid boundaries on all sides. Its mass per unit length, m, is 10 kgm^{-1}. The width, W, is much greater than the diameter, D, which is 0.3 m. It is held to the ceiling by rigid struts that can pivot about the point where they meet the ceiling. The angular displacement from the equilibrium position is θ. The distance from the pivots to the centre of mass, L, is 2.45 m. When it is switched on, the radiator causes the air to move vertically upwards at a velocity, U, equal to 1 ms^{-1}. The gravitational acceleration, g, is 9.8 ms^{-2}. The density of air, ρ, is 1.2 kgm^{-3} and its viscosity, μ, is 1.8×10^{-5} kgm^{-1}s^{-1}. The flow-induced cross-stream force per unit length on the D-shaped cross-section is F and the corresponding force coefficient, C_F, is shown in Fig. 2b as a function of the angle of attack, α, in a steady flow.

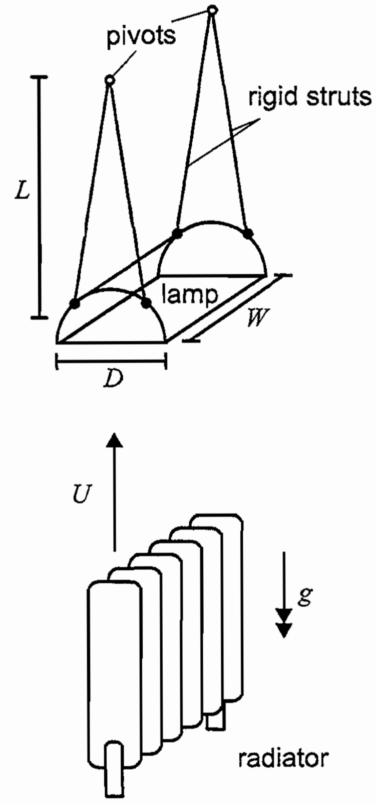

Fig. 2a

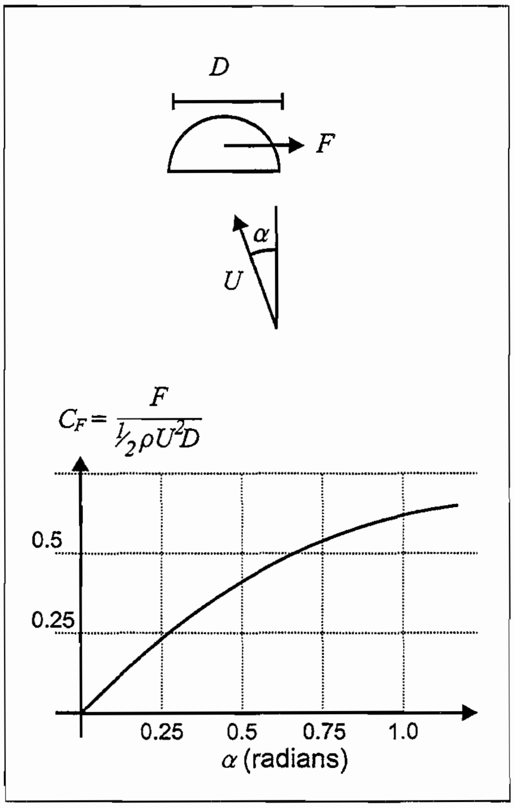

Fig. 2b

(cont.

(a) Assuming that the lamp starts from rest, describe its motion when the radiator is turned on. Explain why this motion occurs. [20%]

(b) When the radiator is off and when friction is ignored, show that the moment about the ceiling pivot is given by $-mgL\sin\theta$. Hence show that the equation of motion for small oscillations about the ceiling pivot is: [10%]

$$L\ddot{\theta} + g\theta = 0.$$

(c) When friction is included and the radiator is off, the equation of motion can be modelled by

$$L\ddot{\theta} + \frac{\gamma}{m}\dot{\theta} + g\theta = 0,$$

where γ is an unknown constant. Determine the forcing term to be added to this equation when the radiator is on, making the following assumptions: any oscillations are small; the quasi-steady assumption is valid; U is uniform across the area swept out by the moving lamp; γ does not change; all transverse forces due to the air are modelled by C_F. Hence find the range of γ at which oscillations of the lamp will grow, given a very small initial push. What is the frequency of these oscillations? [60%]

(d) How could the shape of the lamp's cross-section be altered to prevent this motion. [10%]

(TURN OVER

4 A large television mast is to be built near Cambridge. The main body will be a lattice structure like an electricity pylon. The aerial itself will be a long cylinder of diameter D. The constructors propose to place it on large rubber feet in order to damp vibrations that might cause noise.

(a) Before the mast is built, describe how you would estimate the response of the structure to unsteady forces arising from an unsteady turbulent wind. Include an explanation of the following expression: [20%]

$$S_{FF} = |H_{UF}|^2 S_{UU}$$

(b) The time-varying horizontal velocity, U, has spectral density function:

$$S_{UU} = \left(\frac{U_0^2/\omega_0}{1+\omega^2/\omega_0^2} \right) \qquad \text{for} \quad \omega > 0$$

where U_0 and ω_0 are constants. The aerodynamic admittance takes the form:

$$|H_{UF}|^2 = \left\{ \begin{array}{ll} \dfrac{F_0^2}{U_0^2} & \text{for } 0 \le \omega \le \omega_0 \\ \dfrac{F_0^2}{U_0^2}\left(2 - \dfrac{\omega}{\omega_0}\right) & \text{for } \omega_0 < \omega \le 2\omega_0 \\ 0 & \text{for } 2\omega_0 < \omega \end{array} \right\}$$

Find an expression for the mean square force on the structure in terms of F_0. [60%]

(c) Discuss the generation of unsteady forces on this structure arising from a steady wind of velocity U. [20%]

You may find the following derivative useful:

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

END OF PAPER

Module 4A10 Data Card

UNIVERSITY OF CAMBRIDGE

EQUATIONS OF MOTION

For an incompressible isothermal viscous fluid:

Continuity $\nabla \cdot \boldsymbol{u} = 0$

Navier Stokes $\rho \frac{D\boldsymbol{u}}{Dt} = -\nabla p + \mu \nabla^2 \boldsymbol{u}$

D/Dt denotes the material derivative, $\partial/\partial t + \boldsymbol{u} \cdot \nabla$

IRROTATIONAL FLOW $\nabla \times \boldsymbol{u} = 0$

velocity potential ϕ,

$$\boldsymbol{u} = \nabla \phi \quad \text{and} \quad \nabla^2 \phi = 0$$

Bernoulli's equation

for inviscid flow $\frac{p}{\rho} + \frac{1}{2}|\boldsymbol{u}|^2 + gz + \frac{\partial \phi}{\partial t} =$ constant throughout flow field.

KINEMATIC CONDITION AT A MATERIAL INTERFACE

A surface $z = \eta(x,y,t)$ moves with fluid of velocity $\boldsymbol{u} = (u, v, w)$ if

$$w = \frac{D\eta}{Dt} = \frac{\partial \eta}{\partial t} + \boldsymbol{u} \cdot \nabla \eta \quad \text{on } z = \eta(x,t).$$

For η small and $\boldsymbol{u}$ linearly disturbed from $(U,0,0)$

$$w = \frac{\partial \eta}{\partial t} + U \frac{\partial \eta}{\partial x} \quad \text{on } z = 0.$$

SURFACE TENSION σ AT A LIQUID-AIR INTERFACE

Potential energy

The potential energy of a surface of area A is σA.

Pressure difference

The difference in pressure Δp across a liquid-air surface with principal radii of curvature R_1 and R_2 is

$$\Delta p = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right).$$

For a surface which is almost a circular cylinder with axis in the x-direction, $r = a + \eta(x, \theta, t)$ (η is very small so that η^2 is negligible)

$$\Delta p = \frac{\sigma}{a} + \sigma \left(-\frac{\eta}{a^2} - \frac{\partial^2 \eta}{\partial x^2} - \frac{1}{a^2} \frac{\partial^2 \eta}{\partial \theta^2} \right),$$

where Δp is the difference between the internal and the external surface pressure.

For a surface which is almost plane with $z = \eta(x,t)$ (η is very small so that η^2 is negligible)

$$\Delta p = -\sigma \frac{\partial^2 \eta}{\partial x^2}$$

where Δp is the difference between pressure at $z = \eta^+$ and $z = \eta^-$.

ROTATING FLOW

In steady flows with circular streamlines in which the fluid velocity and pressure are functions of radius r only:

Rayleigh's criterion

The flow is unstable/stable to inviscid axisymmetric disturbances if Γ^2 decreases/increases with r.

$\Gamma = 2\pi r V(r)$ is the circulation around a circle of radius r.

Navier Stokes equation simplifies to

$$0 = \mu \left(\frac{d^2 V}{dr^2} + \frac{1}{r}\frac{dV}{dr} - \frac{V}{r^2} \right)$$

$$-\rho \frac{V^2}{r} = -\frac{dp}{dr}.$$