ENGINEERING TRIPOS PART IIB

Tuesday 9 May 2.30 to 4

Module 4A14

SILENT AIRCRAFT INITIATIVE

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment
Data sheet for 4A14 (2 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 The wheels of an undercarriage of an aircraft flying with speed $V = 60\text{ms}^{-1}$ have diameter $D = 0.8\text{m}$ and exert an unsteady force f on the air of density ρ_0, where $f_{rms} = k\frac{1}{2}\rho_0 V^2 D^2$, $k = 0.1$ and the characteristic frequency is 45Hz.

(a) Estimate the radiated sound power, stating clearly the assumptions made. [60%]

(b) What additional contributions would there be to the noise of an undercarriage? [20%]

(c) What changes can be made to reduce undercarriage noise? [20%]

[Note that $\int_0^{\pi} \sin^3\theta\, d\theta = 4/3$.]

2 (a) Derive Lighthill's eighth-power law for the sound intensity of a low-speed jet. (Any formulae from the data sheet may be quoted without proof.) [45%]

(b) Explain this result can be modified to include the Doppler effect so as to treat the case of a high-speed jet. [35%]

(c) Show that the sound power produced by a low-speed jet of a given thrust is proportional to A^{-3}, where A is the jet pipe area. [20%]

3 An acoustic liner is made of a honeycomb structure with a perforated front plate and a rigid backing plate. A single cell of the honeycomb is illustrate in Fig. 1 and has an opening of cross-sectional area A_1, widening into a duct of length d and cross-sectional area A_2. The opening has a resistance described by $\alpha = \rho_0 c_0 k$, i.e. the pressure drop across the opening $p_2' - p_1'$ is related to $u_1'(t)$, the flow velocity through the opening, by

$$p_2' - p_1' = \alpha u_1'.$$

(a) By considering plane waves within the duct show that the pressure perturbation at the neck is related to the outward flow velocity by

$$p_1'(t) = \rho_0 c_0 \frac{u_1'(t)}{A_2}\left(\mathrm{i}A_1 \cot\left(\frac{\omega d}{c_0}\right) - A_2 k \right)$$

where ρ_0 and c_0 are the mean density and speed of sound. [50%]

(b) An acoustic liner is formed of closely packed identical cells. State what honeycomb depth d you would choose to optimally absorb sound of 1kHz at an ambient temperature of 600K. [25%]

(c) For normally incident sound, $k = 0.1$ and porosity 0.05 (i.e. $A_1 / A_2 = 0.05$) determine the proportion of incident sound of 1k Hz that is absorbed by this optimal liner. [25%]

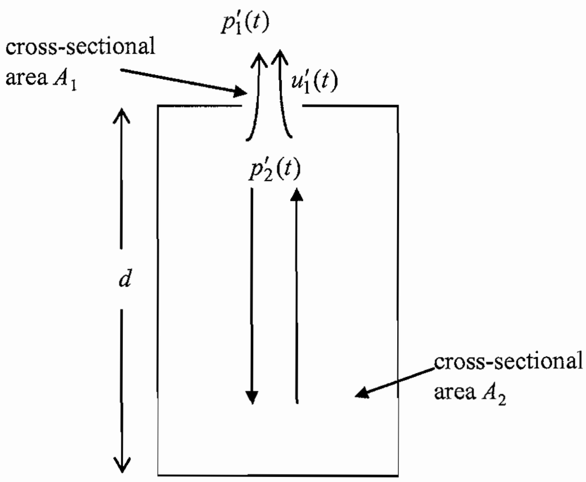

Fig. 1

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4 In two dimensions, a high-frequency point source is located in a uniform medium at the point *(0, 0),* and a thin rigid plate of length $2L$ lies parallel to the x axis, with its ends at $(\pm L, h)$.

(i) Using simple geometrical acoustics, estimate what fraction of the acoustic energy emitted by the source crosses the plane $y = 2h$. [10%]

(ii) By now including the diffracted field for sound of radian frequency ω, estimate the amplitude of the acoustic pressure measured at *(0, 2h)* as a fraction of the acoustic pressure which would have been measured there if the intervening plate had not been present. Express your answer in terms of the wavenumber k_0, where k_0 is ω divided by the speed of sound.
[Recall that for an incident plane wave of pressure amplitude p_i propagating at an angle θ_0, the diffracted pressure from a sharp edge is

$$p_i\left(\frac{2}{\pi k_0 r}\right)^{1/2}\frac{\sin(\theta_0/2)\sin(\theta/2)}{\cos\theta_0+\cos\theta}\quad\exp\left(-ik_0 r - i\pi/4\right),$$ [60%]

see Fig. 2]

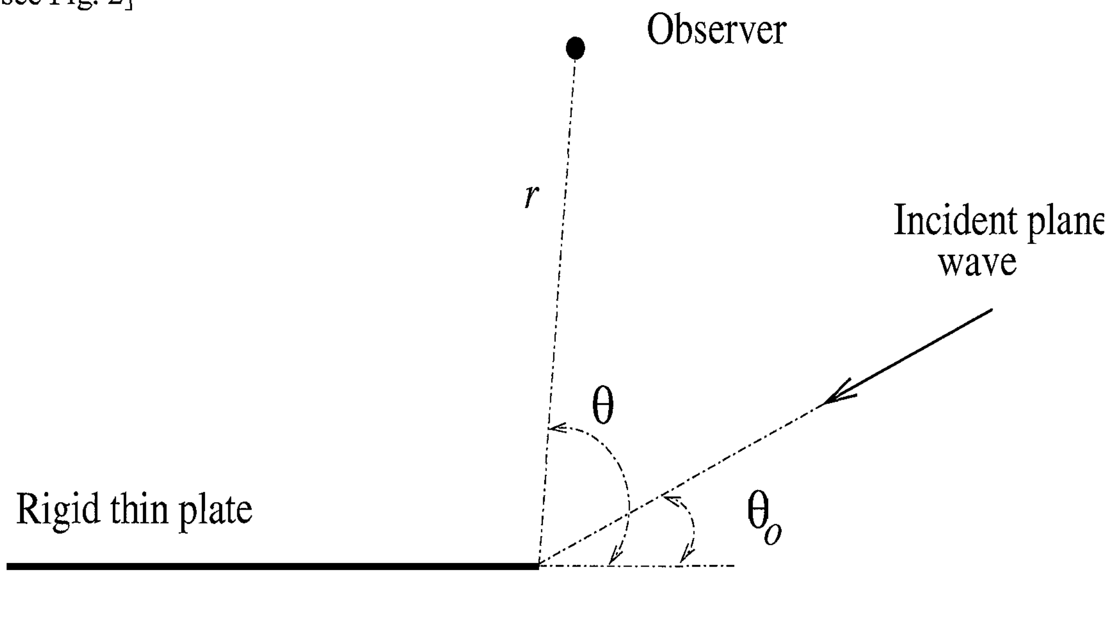


Fig. 2

(CONT.

(iii) Without detailed calculation, describe how your answers to (i) and (ii) may change if:

(a) the sound speed increases linearly in the positive y direction

(b) the plate has rounded edges whose radii of curvature are much larger than the wavelength. [30%]

END OF PAPER

USEFUL MATHEMATICAL FORMULAE

In spherical polar coordinates (r,θ,ϕ)

$$\nabla p' = \left(\frac{\partial p'}{\partial r}, \frac{1}{r}\frac{\partial p'}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial p'}{\partial \phi}\right)$$

For $\mathbf{v}' = (v'_r, v'_\theta, v'_\phi)$, $\nabla\cdot\mathbf{v}' = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v'_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v'_\theta) + \frac{1}{r\sin\theta}\frac{\partial v'_\phi}{\partial\phi}$.

$$\nabla^2 p' = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p'}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial p'}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 p'}{\partial\phi^2}.$$

In cylindrical polar coordinates (r,θ,x_3)

$$\nabla p' = \left(\frac{\partial p'}{\partial r}, \frac{1}{r}\frac{\partial p'}{\partial \theta}, \frac{\partial p'}{\partial x_3}\right)$$

For $\mathbf{v}' = (v'_r, v'_\theta, v'_3)$, $\nabla\cdot\mathbf{v}' = \frac{1}{r}\frac{\partial}{\partial r}(r v'_r) + \frac{1}{r}\frac{\partial v'_\theta}{\partial\theta} + \frac{\partial v'_3}{\partial x_3}$, $\nabla^2 p' = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p'}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p'}{\partial\theta^2} + \frac{\partial^2 p'}{\partial {x_3}^2}$.

Heaviside function $H(t-\tau) = 1 \quad t>\tau \;;\; = 0 \quad t<\tau$

δ-functions

Kronecker delta $\delta_{ij} = 1$ if $i=j$; 0 if $i \neq j$

1D δ-function: $\delta(t) = 0$ for $t \neq 0$; $f(t)\delta(t-\tau) = f(\tau)\delta(t-\tau)$

$$\int_{-\infty}^{\infty}\delta(t)\,dt = 1 \text{ and } \int_{-\infty}^{\infty}\delta(t-\tau)f(t)\,dt = f(\tau) \text{ for any function } f(t).$$

3D δ-function: $\delta(\mathbf{x}) = \delta(x_1)\,\delta(x_2)\,\delta(x_3)$; $\delta(\mathbf{x}) = 0$ for $|\mathbf{x}| \neq 0$; $f(\mathbf{x})\delta(\mathbf{x}-\mathbf{y}) = f(\mathbf{y})\delta(\mathbf{x}-\mathbf{y})$

$\int_V \delta(\mathbf{x})\,dV = 1$ for any volume V that includes the origin

and

$\int_V \delta(\mathbf{x}-\mathbf{y})f(\mathbf{x})\,dV = f(\mathbf{x})$ for any function $f(\mathbf{x})$ and volume V that includes $\mathbf{x}$.

$$\nabla^2\left(\frac{1}{|\mathbf{x}|}\right) = -4\pi\delta(\mathbf{x}).$$

Autocorrelation

$F(\xi)$, the autocorrelation of $f(y) = \overline{f(y)f(y+\xi)}$

$$F(0) = \overline{f^2}$$

Integral lengthscale ℓ $\quad \ell\overline{f^2} = \int_{-\infty}^{\infty}F(\xi)\,d\xi.$

SOURCES

Point sources

monopole of strength $Q(t)$ at the origin generates a pressure field

$$p'(\mathbf{x},t) = \frac{Q(t-|\mathbf{x}|/c_0)}{4\pi|\mathbf{x}|}.$$

dipole of strength $\mathbf{F}(t)$ at the origin generates a pressure field

$$p'(\mathbf{x},t) = -\frac{\partial}{\partial x_i}\left[\frac{F_i(t-|\mathbf{x}|/c_0)}{4\pi|\mathbf{x}|}\right] = \frac{x_i}{4\pi}\left[\frac{1}{|\mathbf{x}|^3}F_i(t-|\mathbf{x}|/c_0) + \frac{1}{|\mathbf{x}|^2 c_0}\frac{\partial F_i}{\partial t}(t-|\mathbf{x}|/c_0)\right]$$

Distributed sources

Monopole, strength $q(\mathbf{x},t)$, wave equation $\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p' = q$, pressure field $p'(\mathbf{x},t) = \int\frac{q(\mathbf{y},t-|\mathbf{x}-\mathbf{y}|/c_0)}{4\pi|\mathbf{x}-\mathbf{y}|}dV$

Dipole, strength $\mathbf{f}(\mathbf{x},t)$, wave equation $\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p' = -\nabla\cdot\mathbf{f}$, $\quad p'(\mathbf{x},t) = -\frac{\partial}{\partial x_i}\int\frac{f_i(\mathbf{y},t-|\mathbf{x}-\mathbf{y}|/c_0)}{4\pi|\mathbf{x}-\mathbf{y}|}dV$

Quadrupole, strength $T_{ij}(\mathbf{x},t)$, equation $\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p' = \frac{\partial T_{ij}}{\partial x_i \partial x_j}$, $\quad p'(\mathbf{x},t) = \frac{\partial^2}{\partial x_i\partial x_j}\int\frac{T_{ij}(\mathbf{y},t-|\mathbf{x}-\mathbf{y}|/c_0)}{4\pi|\mathbf{x}-\mathbf{y}|}dV$

Far-field form $|\mathbf{x}| >> |\mathbf{y}|$, $\mathbf{y}$ near origin

$$|\mathbf{x}-\mathbf{y}| \sim |\mathbf{x}| - \frac{\mathbf{x}\cdot\mathbf{y}}{|\mathbf{x}|} + 0(|\mathbf{x}|^{-2}),\quad \frac{1}{|\mathbf{x}-\mathbf{y}|} \sim \frac{1}{|\mathbf{x}|} + 0(|\mathbf{x}|^{-2}),\quad \frac{\partial}{\partial x_i} \sim -\frac{x_i}{|\mathbf{x}|c_0}\frac{\partial}{\partial t} + 0(|\mathbf{x}|^{-1}).$$

Physical sources

Lighthill's theory shows that jet noise is generated by quadrupoles of strength $T_{ij} = \rho v_i v_j + (p' - c_0^2\rho')\delta_{ij} - \tau_{ij}$.

The Ffowcs-Williams Hawkings equation shows that foreign bodies in linear motion generate far-field sound

$$p'(\mathbf{x},t) = \frac{1}{4\pi|\mathbf{x}|}\frac{\partial}{\partial t}\int_S \rho_0 d\mathbf{S}\bullet\mathbf{v}\left(\mathbf{y}, t-\frac{|\mathbf{x}|}{c_0}+\frac{\mathbf{x}\cdot\mathbf{y}}{|\mathbf{x}|c_0}\right) + \frac{x_i}{4\pi|\mathbf{x}|^2 c_0}\frac{\partial}{\partial t}\int_S dS_i p\left(\mathbf{y}, t-\frac{|\mathbf{x}|}{c_0}+\frac{\mathbf{x}\cdot\mathbf{y}}{|\mathbf{x}|c_0}\right)$$

SOUND POWER

Sound power from a source, $P = \int_S \overline{\mathbf{I}}\bullet d\mathbf{S} = \int_{S_\infty}\frac{\overline{p'^2}}{\rho_0 c_0}dS$ for a statistically stationary source.

For a spherically symmetrical sound field $P = \frac{\overline{p'^2}}{\rho_0 c_0}4\pi r^2$ where p' is the pressure at radius r.

For a sound field, which is a function of spherical polar coordinates r, θ only, and independent of ϕ,

$$P = 2\pi r^2\int_0^\pi \frac{\overline{p'^2}}{\rho_0 c_0}\sin\theta\,d\theta$$

USEFUL DATA AND DEFINITIONS

Physical Properties

Speed of sound in an ideal gas $\sqrt{\gamma RT}$, where T is temperature in Kelvins

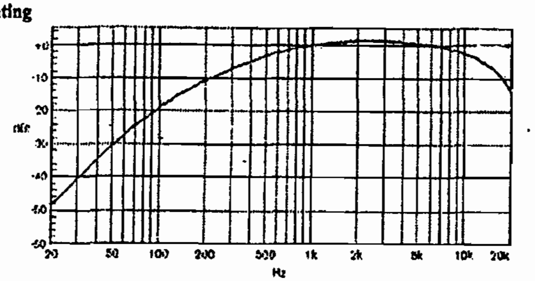
Units of sound measurement

SPL (sound pressure level) $= 20\log_{10}\left(\dfrac{p'_{rms}}{2.10^{-5}\,\text{Nm}^{-2}}\right)$ dB

IL (intensity level) $= 10\log_{10}\left(\dfrac{\text{intensity}}{10^{-12}\,\text{watts m}^{-2}}\right)$ dB

PWL (power level) $= 10\log_{10}\left(\dfrac{\text{sound power output}}{10^{-12}\,\text{watts}}\right)$ dB

A-weighting



SPL mode A-Weighting Curve

Definitions

Surface impedance, Z_n, relates the pressure perturbation applied to a surface, p', to its normal velocity v_n; $p' = Z_n v_n$.

Characteristic impedance of a fluid $\rho_0 c_0$

Nondimensional surface impedance of a surface $Z/\rho_0 c_0$

Transmission loss $= 10\log_{10}\left(\dfrac{\text{incident sound power}}{\text{transmitted sound power}}\right)$

Absorption coefficient of a sound absorber $= \dfrac{\text{sound power absorbed}}{\text{incident sound power}}$

Wavelength, λ, for sound waves with angular frequency ω, $\lambda = 2\pi c_0/\omega$

Wave-number $k_0 = \omega/c_0 = 2\pi/\lambda$

Phase speed $= \omega/k$

Group velocity $= \dfrac{\partial\omega}{\partial k}$

Helmholtz number (or compactness ratio) $= k_0 D$, where D is a typical dimension of the source.

Strouhal number $= \omega D/(2\pi U)$ for sound of frequency ω, produced in a flow with speed U, length scale D.

BASIC EQUATIONS FOR LINEAR ACOUSTICS

Conservation of mass $\dfrac{\partial\rho'}{\partial t} + \rho_0\,\nabla\cdot\mathbf{v}' = 0$

Conservation of momentum $\rho_0\dfrac{\partial\mathbf{v}'}{\partial t} + \nabla p' = 0$

Isentropic $c_0^2 = \left.\dfrac{dp}{d\rho}\right|_s$

These equations combine to give the wave equation $\dfrac{1}{c_0^2}\dfrac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = 0$

Energy density $e = \frac{1}{2}\rho_0 v'^2 + \frac{1}{2}p'^2/\rho_0 c_0^2$

Intensity $\mathbf{I} = p'\mathbf{v}'$

$\text{div}\,\bar{\mathbf{I}} = 0$ for statistically stationary (in time) sound fields.

Velocity potential $\phi'(\mathbf{x},t)$ satisfies the wave equation and $p' = -\rho_0\dfrac{\partial\phi'}{\partial t}$, $\mathbf{v} = \nabla\phi'$.

SIMPLE WAVE FIELDS

1D or plane wave

The general solution of the 1D wave equation is $p'(x,t) = f(t - x/c_0) + g(t + x/c_0)$, where f and g are arbitrary functions. In a plane wave propagating to the right $p' = \rho_0 c_0 u'$; in a plane wave propagating to the left $p' = -\rho_0 c_0 u'$, u' being the particle velocity.

Spherically symmetric sound fields

The general spherically symmetric solution of the 3D wave equation is $\phi'(r,t) = \dfrac{f(t - r/c_0)}{r} + \dfrac{g(t + r/c_0)}{r}$

where r is the distance from the source; f and g are arbitrary functions.

cos θ dependence

The general solution of the 3D wave equation with $\cos\theta$ dependence is

$$p'(\mathbf{x},t) = \frac{\partial}{\partial x}\left[\frac{f(t - r/c_0)}{r} + \frac{g(t + r/c_0)}{r}\right] = \cos\theta\frac{\partial}{\partial r}\left[\frac{f(t - r/c_0)}{r} + \frac{g(t + r/c_0)}{r}\right]$$

In a cylindrical duct, radius a_0

$p'(\mathbf{x},t) = e^{i(\omega t + n\theta)} J_n(z_{mn} r/a_0)(Ae^{-ikx_3} + Be^{ikx_3})$, where z_{mn} is the mth zero of $\dot{J}_n(z)$ and $k = (k_0^2 - z_{mn}^2)^{1/2}$.

SCATTERING

For an incident plane wave of amplitude p_I, propagating at an angle θ_0, the diffracted pressure a distance r from a sharp edge is

$$p_I\left(\frac{2}{\pi k_0 r}\right)^{1/2}\frac{\sin(\theta_0/2)\sin(\theta/2)}{\cos\theta_0 + \cos\theta}\exp(-ik_0 r - i\pi/4)$$