ENGINEERING TRIPOS PART IIB

Mon 24th April 2006 9 to 10.30

Module 4B5

NANOTECHNOLOGY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering data book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the invigilator

1 (a) A premise of quantum mechanics is the wave-particle duality. Briefly discuss this concept with reference to experimental evidence which supports it. Under which circumstances should one represent electrons using (i) plane waves and (ii) wave-packets? [30%]

(b) In what way are the waves which are used to represent matter particles different to those which describe electromagnetic waves? [15%]

(c) Consider the 1-dimensional case where we have a stream of particles moving in the x-direction, described by the wave-function $\psi(x, t) = Ae^{i(kx-\omega t)}$. What exactly does $\psi(x, t)$ represent, and what properties must it have? How can we find the momentum and energy of the particles? By introducing the energy and momentum operators, deduce Schrodinger's equation. [40%]

(d) There are many situations in which quantum and classical mechanics disagree and predict different behaviour. How can we reconcile these two ways of looking at the world? [15%]

2 Electrons at an energy E are incident from the left on the one-dimensional potential step of height V (where $E>V$), as shown below in Fig. 1.

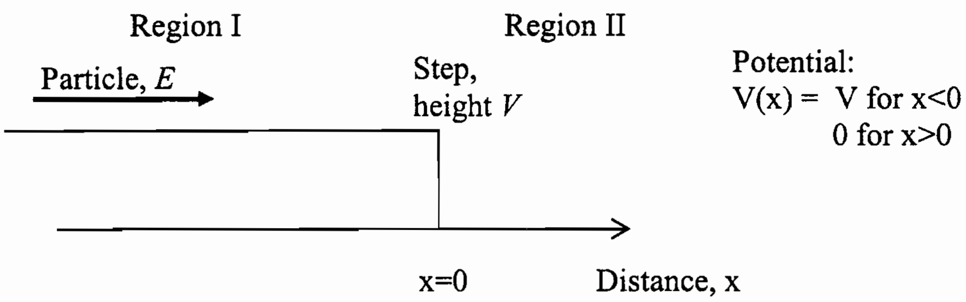

Fig. 1

(a) Solve Schrödinger's equation for regions I and II and find the corresponding wave-functions. Hence derive an expression for the reflection probability, R for this potential step. Explain what the term "reflection coefficient" means. [40%]

(b) For the case where E = 5.5 eV and V = 5 eV, calculate R. Is there any value of E where R will equal exactly 1 or 0? [10%]

(c) Now consider the case where the potential profile is as shown in Fig. 2. Derive an expression for the reflection probability for electrons encountering this potential. Calculate R for the same conditions as in part (b), where L = 2 nm. [50%]

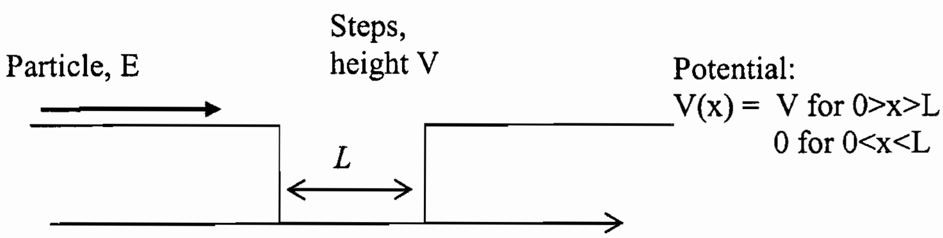

Fig. 2

(TURN OVER

3 (a) If we bring a number of quantum wells into close proximity with each other, then the individual energy levels within those wells start to become modified. How and why does this happen? [20%]

(b) State the main assumptions used in the nearly-free electron model. For an atomic potential of the form $V(x) = V_0 + V_1 e^{iG_1 x} + V_{-1} e^{iG_{-1} x}$ where $G_n = 2\pi n/a$ are the reciprocal lattice vectors, show that

$$\left(\frac{\hbar^2 k^2}{2m} + E - V_0\right)\left(\frac{\hbar^2 (k + G_{-1})^2}{2m} + E - V_0\right) = V_1 V_{-1} = |V_1|^2$$

where E is the electron energy. [50%]

(c) Sketch the dispersion relationship (E *vs* k) for a nearly-free electron, in the reduced-zone scheme, clearly labelling the salient features. Write down the wave-functions for electrons at the valence- and conduction-band edges. Do these electrons contribute to conduction? [30%]

4 (a) Sketch the configuration of an atomic force microscope (AFM), and describe the three most commonly used modes in which it can operate. [30%]

(b) In atomic force microscopy, different properties of samples can be imaged and measured through the use of different types of tip. Discuss this statement using two examples to illustrate your answer. [25%]

(c) Discuss the limitations of atomic force microscopy, and the efforts that are being made to overcome some of them, with particular reference to improvements in scan rate and scan area. [30%]

(d) If the amount of thermal energy in an AFM cantilever is $\frac{1}{2}k_B T$, where k_B is Boltzmann's constant and T is temperature, estimate the amount by which the cantilever will be vibrating at room temperature, for a cantilever spring constant of 1 Nm^{-1}.

[15%]

(TURN OVER

5 (a) What is a quantum well? Sketch the potential profile associated with a typical finite quantum well, and describe how you would fabricate such a structure. [20%]

(b) Derive expressions for the wave-functions and the allowed energy levels in an infinitely-deep quantum well of width, L. Why are the wave-functions symmetric or antisymmetric? [30%]

(c) Consider an infinitely-deep quantum well with the following properties: well width, L = 1 nm, electron mass = 9.1×10^{-31} kg. Calculate the first three energy levels, and sketch their corresponding wave-functions. [20%]

(d) What will happen to the wave-functions and energy levels if the quantum well becomes finite in depth? When is it reasonable to approximate a finite well to an infinite one? [30%]

END OF PAPER

4B5 2006 short/numerical Answers

1. (a) Matter waves: $E = \hbar^2k^2/2m$, dispersion; electromagnetic waves, $E = \hbar\omega$, no dispersion unless in a dispersive medium.

   (b) Momentum operator = $-i\hbar d/dx$

   Energy operator = $i\hbar d/dt$

2. (a) $R = ((k_1-k_2)/(k_1+k_2))^2$.

   (b) R = 0.29

   (c) R = 0.627

3.

4. (d) noise = 64 pm

5. E_1 = 373 meV, E_2 = 1.492 eV, E_3 = 3.357 eV