ENGINEERING TRIPOS PART IIB

Friday 28 April 2006 9.00 to 10.30

Module 4C1

DESIGN AGAINST FAILURE

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Elasticity and Plasticity formulae (2 pages)
Fracture Mechanics Datasheet (8 pages)

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Define line tension T along a dislocation line and relate it to the dislocation line energy. Comment on why a dislocation line tends to be straight in the absence of stresses. [15%]

(b) Explain how to determine the magnitude of a typical Burger's vector. [10%]

(c) Define the force acting on a dislocation. [10%]

(d) Derive an expression for the ideal shear strength, and hence explain why metals do not typically achieve this ideal strength. [25%]

(e) Explain the operation of a Frank-Read dislocation source, and hence derive an expression for the yield stress in shear. Using estimates for the relevant quantities, calculate the value of the shear yield strength of a typical metal. Comment on how this compares with the ideal shear strength. [25%]

(f) Assuming that the pinning points in the Frank-Read model are dislocation jogs, comment on how the shear yield strength derived with this model compares with the ideal shear strength. [15%]

2 (a) Sketch the uniaxial tensile stress versus strain curve of an amorphous polymer tested *well below* $0.8T_g$ where T_g is the glass transition temperature, and describe the underlying failure mechanisms. [15%]

(b) Sketch the uniaxial tensile stress versus strain curve of an amorphous polymer tested *at* $0.8T_g$, and describe the underlying failure mechanisms. [15%]

(c) Construct a failure mechanism map for an amorphous polymer. [20%]

(d) Describe briefly what is meant by Coble creep and Nabarro-Herring creep. [20%]

(e) Figure 1 shows a cylindrical volume element subjected to axisymmetric loading, with axial stress σ_1 and radial stress σ_2. For each of the following room temperature deformation and/or failure processes, sketch the failure surfaces in axisymmetric stress space.

(i) Plastic yielding; [15%]

(ii) Cleavage failure (propagation controlled). [15%]

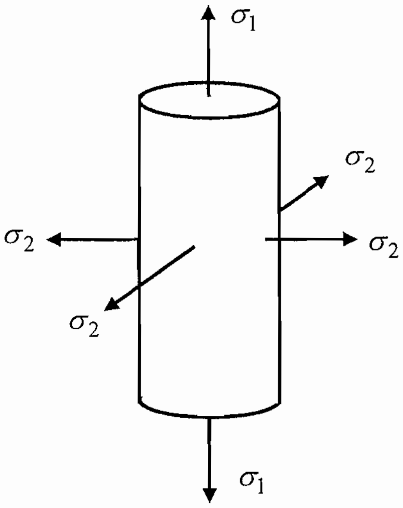


Fig. 1

3 A semi-infinite solid contains a thin brittle interface layer parallel to the surface at a depth a as sketched in Fig. 2. The bulk properties of the solid are identical on either side of the layer. A surface crack extends normal to the surface, with its tip at the interface as shown in Fig. 2. Remote loading may be applied to generate either a mode I stress intensity K_I or a mode II stress intensity K_{II}. The stress components $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ are

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(K_I \cos^2\frac{\theta}{2} - \frac{3}{2} K_{II} \sin\theta \right)$$

$$\sigma_{r\theta} = \frac{1}{2\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(K_I \cos^2\frac{\theta}{2} + K_{II} (3\cos\theta - 1) \right)$$

where the polar coordinates (r, θ) are defined in Fig. 2. It may be assumed that the crack in Fig. 2 propagates in the bulk when the maximum value of $\sigma_{\theta\theta}$ at a distance $r = 0.01a$ reaches a critical value ρ_A. Failure of the interface may be assumed to occur when the stress normal to the interface at a distance $r = 0.01a$ reaches a critical value ρ_B.

(a) For the case of remote mode I loading, derive a criterion to determine whether cracking occurs along the interface or in the bulk, in terms of the ratio ρ_A / ρ_B. [40%]

(b) The crack is now loaded in pure mode II.

(i) Find the direction in which the crack would propagate in the bulk. [20%]

(ii) Derive the criterion to determine whether the crack branches into the interface or into the bulk, in terms of ρ_A / ρ_B. [40%]

(contd.

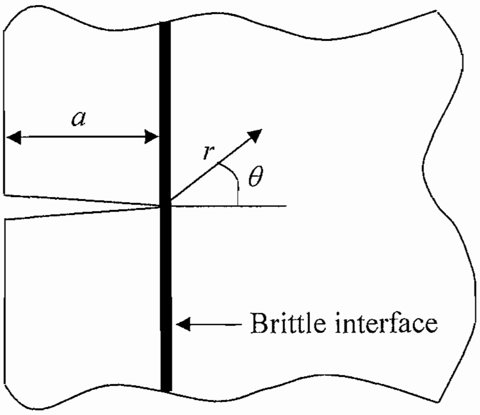


Fig. 2

4 (a) Explain why the critical energy release rate for fracture, G_{IC}, is much greater than twice the surface energy for engineering alloys. [15%]

(b) A flat circular indenter of radius a is pressed against the surface of a brittle semi-infinite solid with a load P, as shown in the cross-section in Fig. 3. A cylindrical crack of length l forms in the solid, initiating at the edge of the indenter.

(i) Show that the compliance C is approximately given by

$$C = \frac{l}{E\pi a^2}$$

where E is the Young's modulus for the solid. [15%]

(ii) Find an expression for the strain energy release rate G for the cylindrical crack under the indenter, and comment on the stability of crack advance. [50%]

(iii) Briefly discuss whether you would expect the cylindrical crack to deviate from a constant radius with increasing depth. [20%]

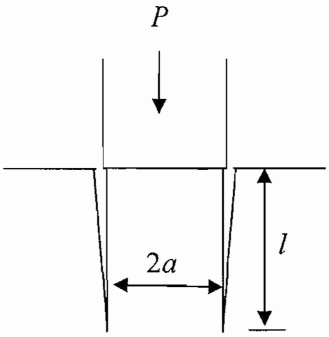


Fig. 3

END OF PAPER

Engineering Tripos Part IIA THIRD YEAR

Paper G4: Mechanics of Solids

ELASTICITY and PLASTICITY FORMULAE

1. Axi-symmetric deformation : discs, tubes and spheres

	Discs and tubes	Spheres
Equilibrium	$\sigma_{\theta\theta} = \frac{\mathrm{d}(r\sigma_{rr})}{\mathrm{d}r} + \rho\omega^2 r^2$	$\sigma_{\theta\theta} = \frac{1}{2r}\frac{\mathrm{d}(r^2\sigma_{rr})}{\mathrm{d}r}$
Lamé's equations (in elasticity)	$\sigma_{rr} = A - \frac{B}{r^2} - \frac{3+\nu}{8}\rho\omega^2 r^2 - \frac{E\alpha}{r^2}\int_c^r rT\mathrm{d}r$	$\sigma_{rr} = A - \frac{B}{r^3}$
	$\sigma_{\theta\theta} = A + \frac{B}{r^2} - \frac{1+3\nu}{8}\rho\omega^2 r^2 + \frac{E\alpha}{r^2}\int_c^r rT\mathrm{d}r - E\alpha T$	$\sigma_{\theta\theta} = A + \frac{B}{2r^3}$

2. Plane stress and plane strain

	Cartesian coordinates	Polar coordinates
Strains	$\varepsilon_{xx} = \frac{\partial u}{\partial x}$	$\varepsilon_{rr} = \frac{\partial u}{\partial r}$
	$\varepsilon_{yy} = \frac{\partial v}{\partial y}$	$\varepsilon_{\theta\theta} = \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta}$
	$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$	$\gamma_{r\theta} = \frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r}$
Compatibility	$\frac{\partial^2\gamma_{xy}}{\partial x\partial y} = \frac{\partial^2\varepsilon_{xx}}{\partial y^2} + \frac{\partial^2\varepsilon_{yy}}{\partial x^2}$	$\frac{\partial}{\partial r}\left\{r\frac{\partial\gamma_{r\theta}}{\partial\theta}\right\} = \frac{\partial}{\partial r}\left\{r^2\frac{\partial\varepsilon_{\theta\theta}}{\partial r}\right\} - r\frac{\partial\varepsilon_{rr}}{\partial r} + \frac{\partial^2\varepsilon_{rr}}{\partial\theta^2}$
or (in elasticity)	$\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right\}(\sigma_{xx} + \sigma_{yy}) = 0$	$\left\{\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right\}(\sigma_{rr} + \sigma_{\theta\theta}) = 0$
Equilibrium	$\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} = 0$	$\frac{\partial}{\partial r}(r\sigma_{rr}) + \frac{\partial\sigma_{r\theta}}{\partial\theta} - \sigma_{\theta\theta} = 0$
	$\frac{\partial\sigma_{yy}}{\partial y} + \frac{\partial\sigma_{xy}}{\partial x} = 0$	$\frac{\partial\sigma_{\theta\theta}}{\partial\theta} + \frac{\partial}{\partial r}(r\sigma_{r\theta}) + \sigma_{r\theta} = 0$
$\nabla^4\phi = 0$ (in elasticity)	$\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right\}\left\{\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right\} = 0$	$\left\{\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right\}$
		$\times \left\{\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}\right\} = 0$
Airy Stress Function	$\sigma_{xx} = \frac{\partial^2\phi}{\partial y^2}$	$\sigma_{rr} = \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}$
	$\sigma_{yy} = \frac{\partial^2\phi}{\partial x^2}$	$\sigma_{\theta\theta} = \frac{\partial^2\phi}{\partial r^2}$
	$\sigma_{xy} = -\frac{\partial^2\phi}{\partial x\partial y}$	$\sigma_{r\theta} = -\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial\phi}{\mathrm{d}\theta}\right\}$

3. Torsion of prismatic bars

Prandtl stress function: $\sigma_{zx}\ (= \tau_x) = \frac{dF}{dy}$, $\sigma_{zy}\ (= \tau_y) = -\frac{dF}{dx}$

Equilibrium: $T = 2\int_A F dA$

Governing equation for elastic torsion: $\nabla^2 F = -2G\beta$ where β is the angle of twist per unit length.

4. Total potential energy of a body

$$\Pi = U - W$$

where $U = \frac{1}{2}\int_V \underset{\sim}{\varepsilon}^T [D] \underset{\sim}{\varepsilon}\, dV$, $W = \underset{\sim}{P}^T \underset{\sim}{u}$ and $[D]$ is the elastic stiffness matrix.

5. Principal stresses and stress invariants

Values of the principal stresses, σ_P, can be obtained from the equation

$$\begin{vmatrix} \sigma_{xx} - \sigma_P & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} - \sigma_P & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} - \sigma_P \end{vmatrix} = 0$$

This is equivalent to a cubic equation whose roots are the values of the 3 principal stresses, i.e. the possible values of σ_P.

Expanding: $\sigma_P^3 - I_1 \sigma_P^2 + I_2 \sigma_P - I_3 = 0$ where $I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$,

$$I_2 = \begin{vmatrix} \sigma_{yy} & \sigma_{yz} \\ \sigma_{yz} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \sigma_{xz} \\ \sigma_{xz} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{vmatrix} \quad \text{and} \quad I_3 = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{vmatrix} .$$

6. Equivalent stress and strain

Equivalent stress $\bar{\sigma} = \sqrt{\frac{1}{2}}\left\{ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right\}^{1/2}$

Equivalent strain increment $d\bar{\varepsilon} = \sqrt{\frac{2}{3}}\left\{ d\varepsilon_1^2 + d\varepsilon_2^2 + d\varepsilon_3^2 \right\}^{1/2}$

7. Yield criteria and flow rules

Tresca

Material yields when maximum value of $|\sigma_1 - \sigma_2|$, $|\sigma_2 - \sigma_3|$ or $|\sigma_3 - \sigma_1| = Y = 2k$, and then,

if σ_3 is the intermediate stress, $d\varepsilon_1 : d\varepsilon_2 : d\varepsilon_3 = \lambda(1 : -1 : 0)$ where $\lambda \neq 0$.

von Mises

Material yields when, $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2 = 6k^2$, and then

$$\frac{d\varepsilon_1}{\sigma'_1} = \frac{d\varepsilon_2}{\sigma'_2} = \frac{d\varepsilon_3}{\sigma'_3} = \frac{d\varepsilon_1 - d\varepsilon_2}{\sigma_1 - \sigma_2} = \frac{d\varepsilon_2 - d\varepsilon_3}{\sigma_2 - \sigma_3} = \frac{d\varepsilon_3 - d\varepsilon_1}{\sigma_3 - \sigma_1} = \lambda = \frac{3}{2}\frac{d\bar{\varepsilon}}{\bar{\sigma}} .$$

ENGINEERING TRIPOS PART IIB
ELECTRICAL AND INFORMATION SCIENCES TRIPOS PART II

B1 DEFORMATION AND FRACTURE

FRACTURE MECHANICS DATASHEET

Crack tip plastic zone sizes

$$\text{diameter, } d_p = \begin{cases} \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2 & \text{Plane stress} \\ \dfrac{1}{3\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2 & \text{Plane strain} \end{cases}$$

Crack opening displacement

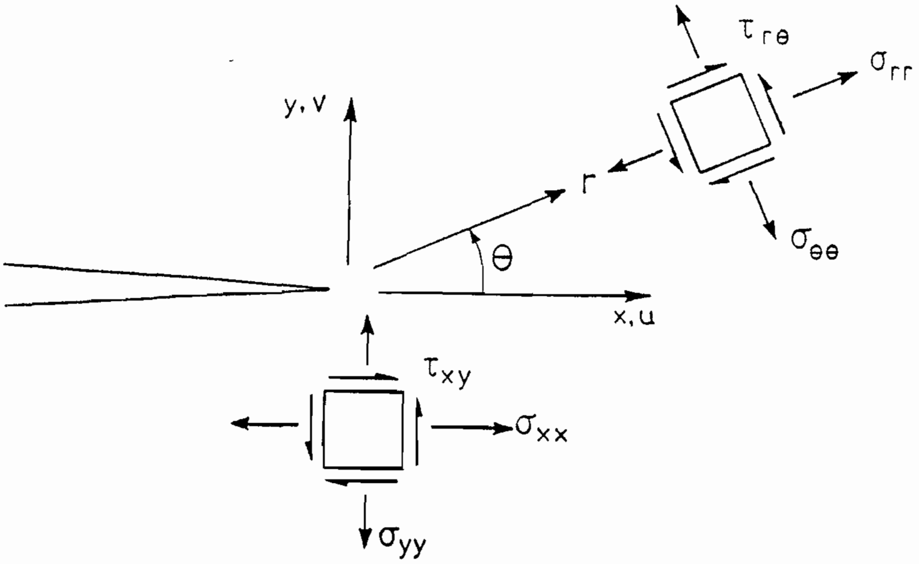
$$\delta = \begin{cases} \dfrac{K_I^2}{\sigma_y E} & \text{Plane stress} \\ \dfrac{1}{2}\dfrac{K_I^2}{\sigma_y E} & \text{Plane strain} \end{cases}$$

Energy release rate

$$G = \begin{cases} \dfrac{1}{E}K_I^2 & \text{Plane stress} \\ \dfrac{1-\nu^2}{E}K_I^2 & \text{Plane strain} \end{cases}$$

Related to compliance C: $G = \dfrac{1}{2}\dfrac{P^2}{B}\dfrac{dC}{da}$

Asymptotic crack tip fields in a linear elastic solid



Mode I

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2}-\frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2}+\frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2}+\frac{1}{4}\sin\frac{3\theta}{2}\right)$$

$$u = \begin{cases} \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{1-\nu}{1+\nu}+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(1-2\nu+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{2}{1+\nu}-\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(2-2\nu-\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Crack tip stress fields (cont'd)

Mode II

$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \sin\frac{\theta}{2} \cos\frac{3\theta}{2}$$

$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2} \left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{rr} = \frac{K_{II}}{\sqrt{2\pi r}} \left(-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{\theta\theta} = -\frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{3}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right)$$

$$u = \begin{cases} \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}} \left(\dfrac{2}{1+\nu} + \cos^2\dfrac{\theta}{2}\right) \sin\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}} \left(2 - 2\nu + \cos^2\dfrac{\theta}{2}\right) \sin\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}} \left(\dfrac{\nu - 1}{1+\nu} + \sin^2\dfrac{\theta}{2}\right) \cos\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}} \left(-1 + 2\nu + \sin^2\dfrac{\theta}{2}\right) \cos\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Mode III

$$\tau_{zx} = -\frac{K_{III}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}$$

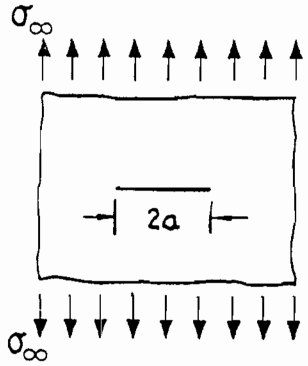
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{xy} = 0$$

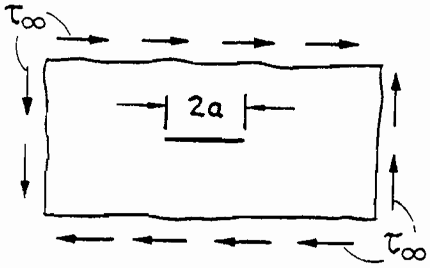
$$w = \frac{K_{III}}{G}\sqrt{\frac{2r}{\pi}} \sin\frac{\theta}{2}$$

$$u = v = 0$$

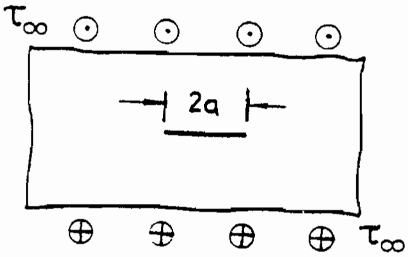
Tables of stress intensity factors



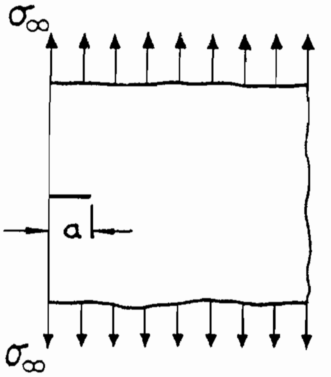
$$K_I = \sigma_\infty \sqrt{\pi a}$$



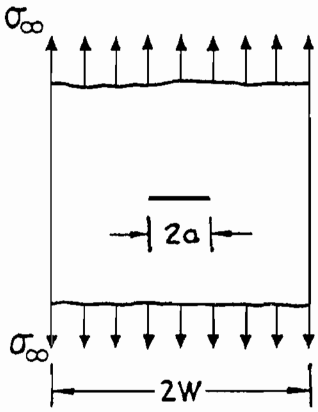
$$K_{II} = \tau_\infty \sqrt{\pi a}$$



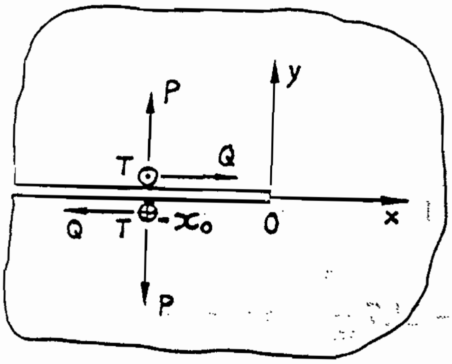
$$K_{III} = \tau_\infty \sqrt{\pi a}$$



$$K_I = 1.12\, \sigma_\infty \sqrt{\pi a}$$



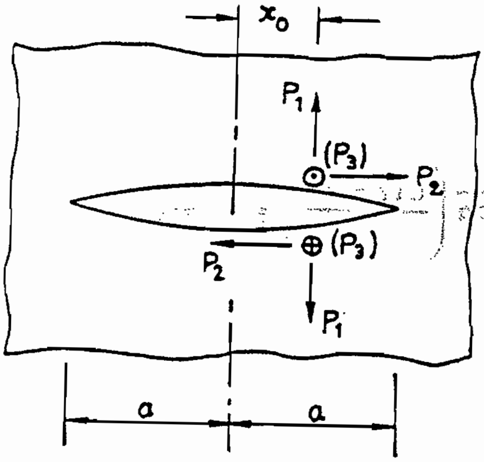
$$K_I = \sigma_\infty \sqrt{\pi a} \left(\frac{1 - a/2W + 0.326 a^2 / W^2}{\sqrt{1 - a/W}} \right)$$



$$K_I = \frac{2P}{\sqrt{2\pi x_o}}$$

$$K_{II} = \frac{2Q}{\sqrt{2\pi x_o}}$$

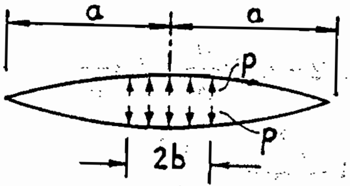
$$K_{III} = \frac{2T}{\sqrt{2\pi x_o}}$$



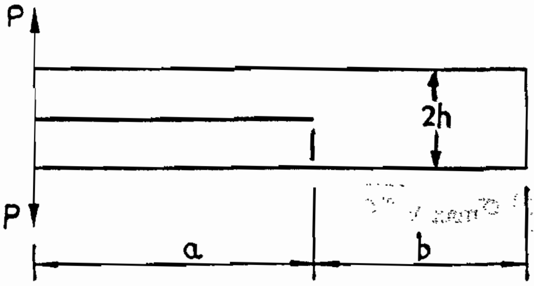
$$K_I = \frac{P_1}{\sqrt{\pi a}}\sqrt{\frac{a+x_o}{a-x_o}}$$

$$K_{II} = \frac{P_2}{\sqrt{\pi a}}\sqrt{\frac{a+x_o}{a-x_o}}$$

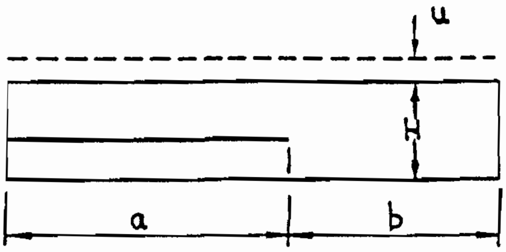
$$K_{III} = \frac{P_3}{\sqrt{\pi a}}\sqrt{\frac{a+x_o}{a-x_o}}$$



$$K_I = \frac{2pb}{\sqrt{\pi a}}\frac{a}{b}\arcsin\frac{b}{a}$$

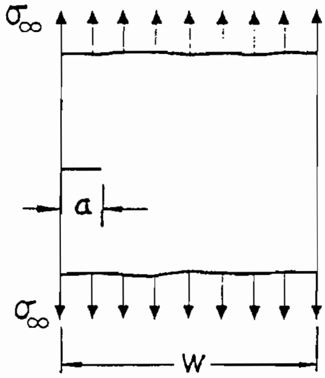


$$K_I = \frac{2\sqrt{3}}{h\sqrt{h}}\frac{Pa}{B} \qquad h \ll a \text{ and } h \ll b$$



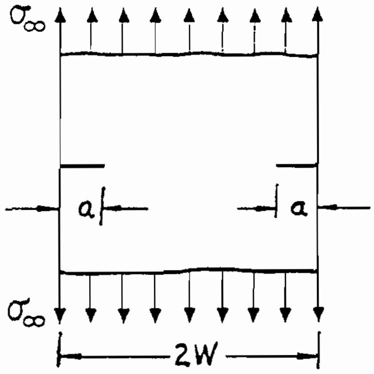
$$K_I = \sqrt{\frac{1}{2\alpha H}}\,Eu \qquad H \ll a \text{ and } H \ll b$$

$$\alpha = \begin{cases} 1-\nu^2 & \text{Plane stress} \\ 1-3\nu^2-2\nu^3 & \text{Plane strain} \end{cases}$$

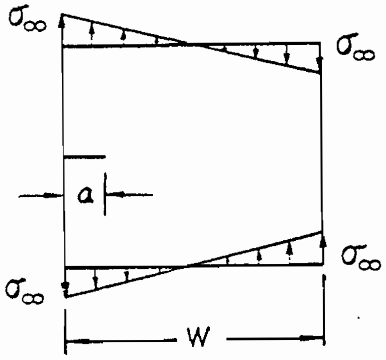


$a / W < 0.7$

$$K_I = \sigma_\infty \sqrt{\pi a}\left(1.12 - 0.23\frac{a}{W} + 10.6\frac{a^2}{W^2} - 21.7\frac{a^3}{W^3} + 30.4\frac{a^4}{W^4}\right)$$

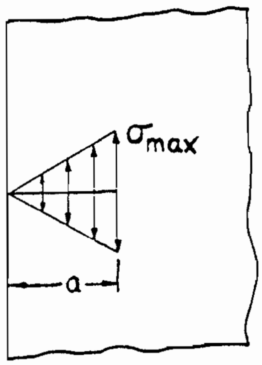


$$K_I = \sigma_\infty \sqrt{\pi a}\left(\frac{1.12 - 0.61a / W + 0.13a^3 / W^3}{\sqrt{1 - a / W}}\right)$$

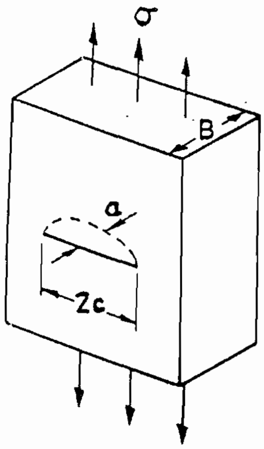


$a / W < 0.7$

$$K_I = \sigma_\infty \sqrt{\pi a}\left(1.12 - 1.39\frac{a}{W} + 7.3\frac{a^2}{W^2} - 13\frac{a^3}{W^3} + 14\frac{a^4}{W^4}\right)$$

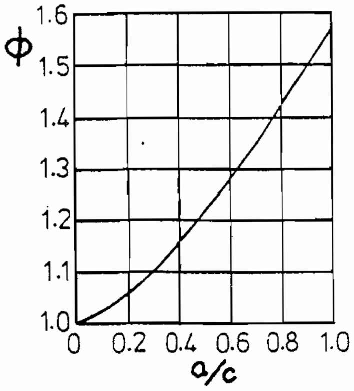


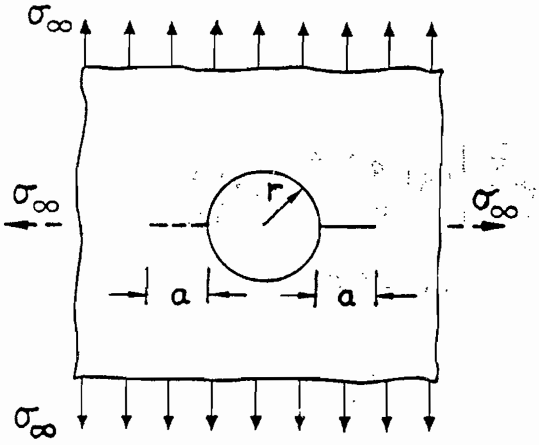
$$K_I = 0.683\ \sigma_{\max} \sqrt{\pi a}$$



$$K_I = \frac{1.12}{\Phi}\sigma\sqrt{\pi a}$$

$$\Phi = \int_0^{\pi/2}\left(1 - \frac{c^2 - a^2}{c^2}\sin^2\theta\right)^{\frac{1}{2}} d\theta$$



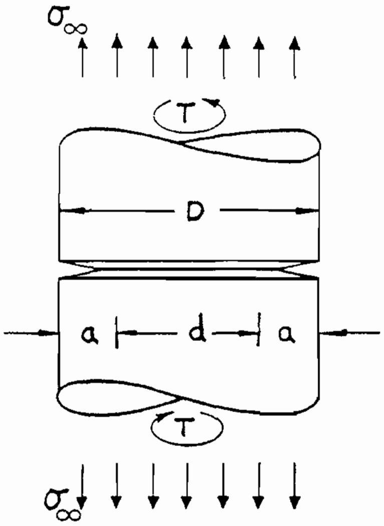


$$K_I = \sigma_\infty\sqrt{\pi a}\; F\left(\frac{a}{r}\right)$$

value of $F\,(a\,/\,r\,)^{\dagger}$

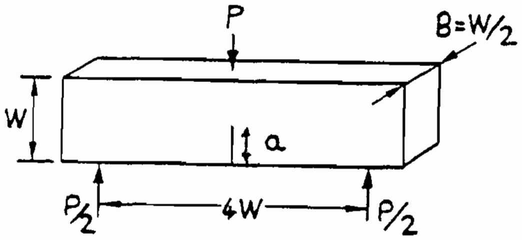
	One crack		Two cracks	
$\frac{a}{r}$	U	B	U	B
0.00	3.36	2.24	3.36	2.24
0.10	2.73	1.98	2.73	1.98
0.20	2.30	1.82	2.41	1.83
0.30	2.04	1.67	2.15	1.70
0.40	1.86	1.58	1.96	1.61
0.50	1.73	1.49	1.83	1.57
0.60	1.64	1.42	1.71	1.52
0.80	1.47	1.32	1.58	1.43
1.0	1.37	1.22	1.45	1.38
1.5	1.18	1.06	1.29	1.26
2.0	1.06	1.01	1.21	1.20
3.0	0.94	0.93	1.14	1.13
5.0	0.81	0.81	1.07	1.06
10.0	0.75	0.75	1.03	1.03
∞	0.707	0.707	1.00	1.00

$^{\dagger}U$ = uniaxial σ_∞ B = biaxial σ_∞.

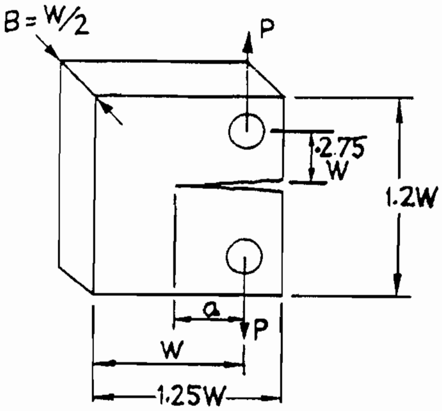


$$K_I = \sigma_\infty \sqrt{\pi a}\left(\frac{D}{d} + \frac{1}{2} + \frac{3}{8}\frac{d}{D} - 0.36\frac{d^2}{D^2} + 0.73\frac{d^3}{D^3}\right)\frac{1}{2}\sqrt{\frac{D}{d}}$$

$$K_{III} = \frac{16T}{\pi D^3}\sqrt{\pi a}\left(\frac{D^2}{d^2} + \frac{1}{2}\frac{D}{d} + \frac{3}{8} + \frac{5}{16}\frac{d}{D} + \frac{35}{128}\frac{d^2}{D^2} + 0.21\frac{d^3}{D^3}\right)\frac{3}{8}\sqrt{\frac{D}{d}}$$



$$K_I = \frac{4P}{B}\sqrt{\frac{\pi}{W}}\left\{1.6\left(\frac{a}{W}\right)^{1/2} - 2.6\left(\frac{a}{W}\right)^{3/2} + 12.3\left(\frac{a}{W}\right)^{5/2} - 21.2\left(\frac{a}{W}\right)^{7/2} + 21.8\left(\frac{a}{W}\right)^{9/2}\right\}$$



$$K_I = \frac{P}{B}\sqrt{\frac{\pi}{W}}\left\{16.7\left(\frac{a}{W}\right)^{1/2} - 104.7\left(\frac{a}{W}\right)^{3/2} + 369.9\left(\frac{a}{W}\right)^{5/2} - 573.8\left(\frac{a}{W}\right)^{7/2} + 360.5\left(\frac{a}{W}\right)^{9/2}\right\}$$

NAF/TJL
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