ENGINEERING TRIPOS PART IIA
ENGINEERING TRIPOS PART IIB

~~Thursday 27 April 2006 9 to 10.30~~

Module 4C4

DESIGN METHODS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Module 4C4 Data Book (6 pages)

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 With increasing concern about identity fraud, people are being encouraged to dispose securely of all documents that contain personal information about bank accounts etc. The office equipment company you work for has decided to design a strip cut shredder for the domestic market. A strip cut shredder cuts documents into parallel thin strips. You have been given responsibility for the conceptual design of the new shredder.

(a) Abstract the task to at least four levels and prepare an appropriate solution-neutral problem statement for your task. [10%]

(b) List ten requirements for your new shredder, identifying them as demands and wishes. [10%]

(c) Establish the overall function for your shredder. Identify 10 sub-functions and arrange these into a *product* function structure. [30%]

(d) Figure 1 shows some data for three existing strip cut shredders. Establish evaluation criteria, weight them and prepare an evaluation chart to identify the one that most closely matches your proposed requirements and is therefore likely to be your main competitor. Comment on the results of the evaluation. [40%]

(e) Summarise briefly the selling features your design will aim to include to give it a distinct market advantage over and above the shredder you have identified as your main competitor. [10%]

(cont.

	Shredder 1	Shredder 2	Shredder 3
Price in £	45	70	10
Overall size in cm	36x33x18	46x38x29	32x29x16
A4 sheet feed capacity	5	12	5
Bin capacity in litres	11	26	9
Speed of shredding	Fast	Very fast	Medium
Cope with staples (S) and paper clips (P)	S	S, P	No
Durability	Good	Very good	Medium

Fig. 1

(TURN OVER

2 With changing weather patterns, people living near to rivers, lakes and the sea are at increased risk from local flooding. The company you work for has decided to enter the market for temporary domestic flood protection systems. In the event of a risk from flooding, the aim is to keep a house watertight against a maximum water level of 750 mm measured from the base of the house.

Your company decides to market a system that involves building a permanent brick wall around parts of the house with access points that have to be closed. During your research you come across a similar system that was recently installed by a competitor. Figure 2 shows schematically the approach adopted for closing the access points of that system.

(a) List the main functions to be fulfilled by the closure system, prepare a process function structure and produce a fault-tree for the closure method shown in Fig. 2. [50%]

(b) Draw an embodiment sketch or sketches of an improved system that you might market, explaining briefly the main features that make your closure better than the one shown in Fig. 2. Make reference, where possible, to appropriate embodiment design guidelines. [50%]

(cont.

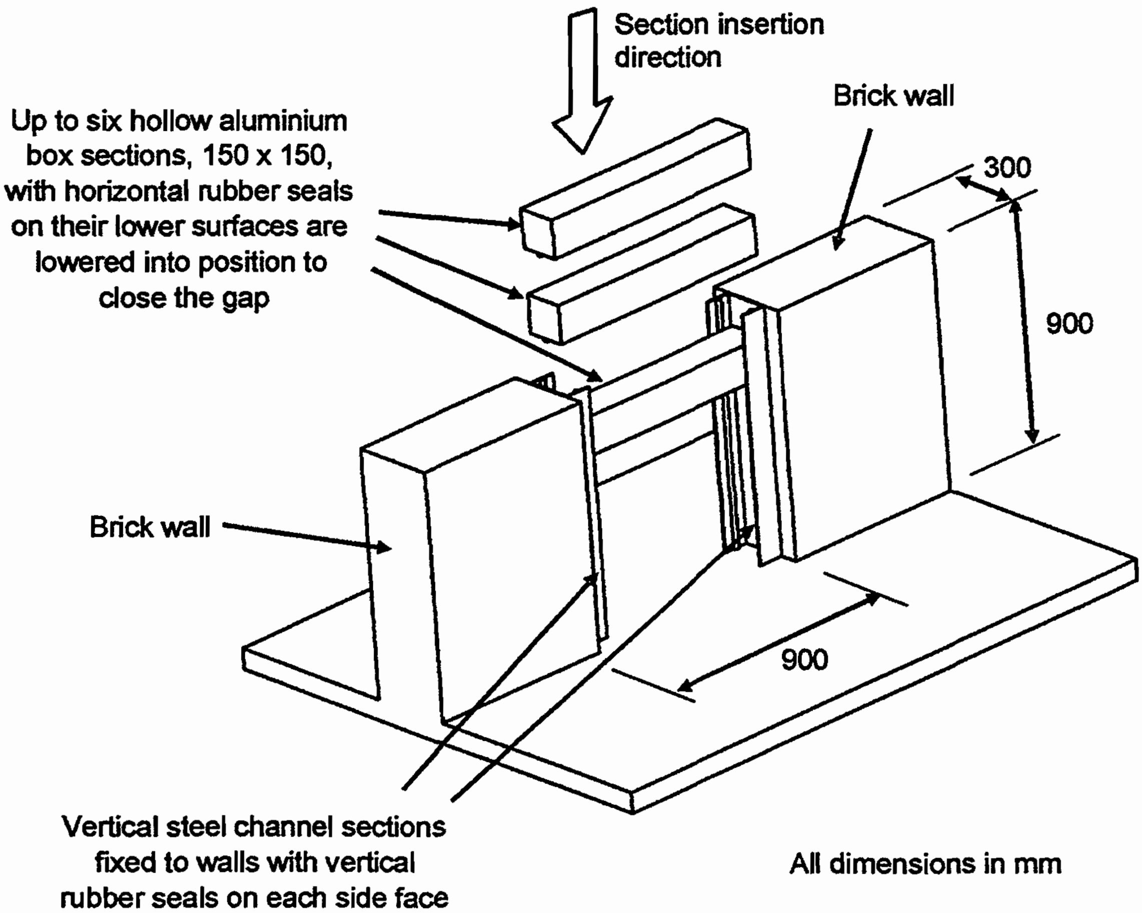

Fig. 2

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3 An architect wishes to design a row of flat-roofed terraced houses so as to minimise heat loss. The terrace, shown diagrammatically in Fig. 3, will consist of five equal sized houses, each of which must have an internal volume of at least 600 m^3. The rate of heat loss through the roof of each house will be four times greater than the heat loss through an external wall. You may assume that heat flux $q = hA\Delta T$, where h is the heat transfer coefficient, A is the area and ΔT is the temperature difference between the internal and external environments, and that no heat is lost to the ground.

(a) Given that the width and depth of each of the five houses is w and d respectively and the height from the ground to the roof is z, make a formal statement of the optimisation problem. [30%]

(b) (i) The height of the roof z is fixed at 6 m. If the houses are of the minimum required volume, show that the problem can be simplified to one involving only the width w. [10%]

(ii) Starting with an initial range of w from 2 m to 10 m conduct two interval reductions using the Golden Section Method. Determine the new interval in which w must lie for minimum heat loss from the row of houses. [30%]

(iii) Using First Order Conditions, find the optimum dimensions for the houses. [10%]

(c) The architect prefers to consider the height z as a continuous variable. However, planning constraints specify a maximum height z_{max}. Suggest an alternative objective function that may be used to turn this constrained problem into an unconstrained one. [20%]

(cont.

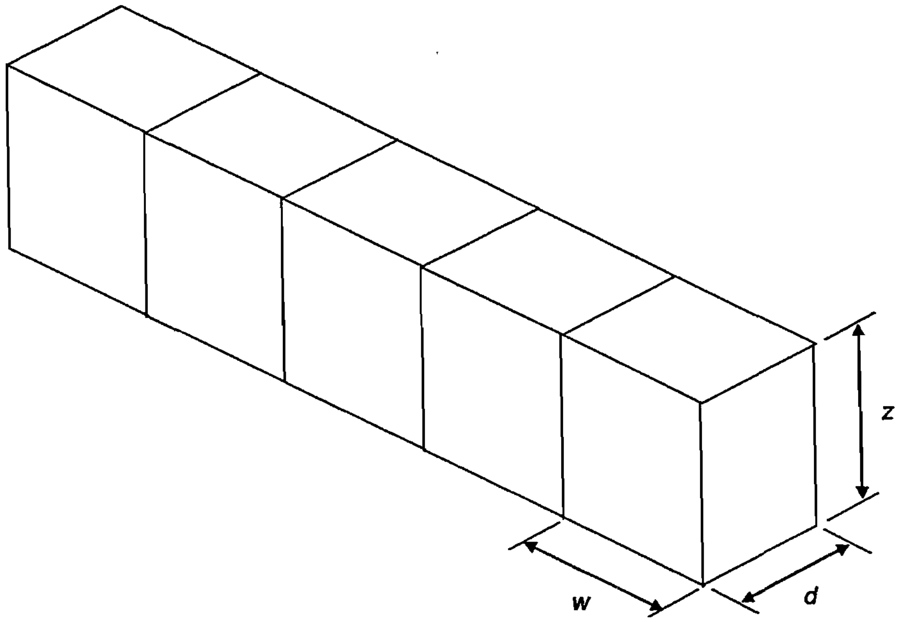

Fig. 3

(TURN OVER

4 Figure 4 shows a design for a bicycle seatpost clamp with toleranced dimensions. By tightening a bolt to close the gap g the diameter of the clamp is reduced and the clamp grips the seatpost. If tightening the bolt causes the gap to fully close before a required diametrical interference of 0.3 mm is achieved, the seatpost is likely to slip.

(a) Show that the expression for the diametrical interference i when the gap is fully closed, in terms of the post and clamp diameters d_p and d_c, and the initial clamp gap g, is

$$i = d_p - d_c + g/\pi$$ [10%]

(b) Hence find the safety factors for the achievable diametrical interference compared with the requirement of 0.3 mm for nominal and for worst-case dimensions. [20%]

(c) Find the percentage of clamp assemblies that it will not be possible to tighten to the required diametrical interference. Assume that each dimension is a normally distributed random variable and that the tolerance ranges given represent six standard deviations. [40%]

(d) How could the design be improved to eliminate the failure mode described in part (c)? [10%]

(e) What other failure modes should the designer consider? [20%]

(cont.

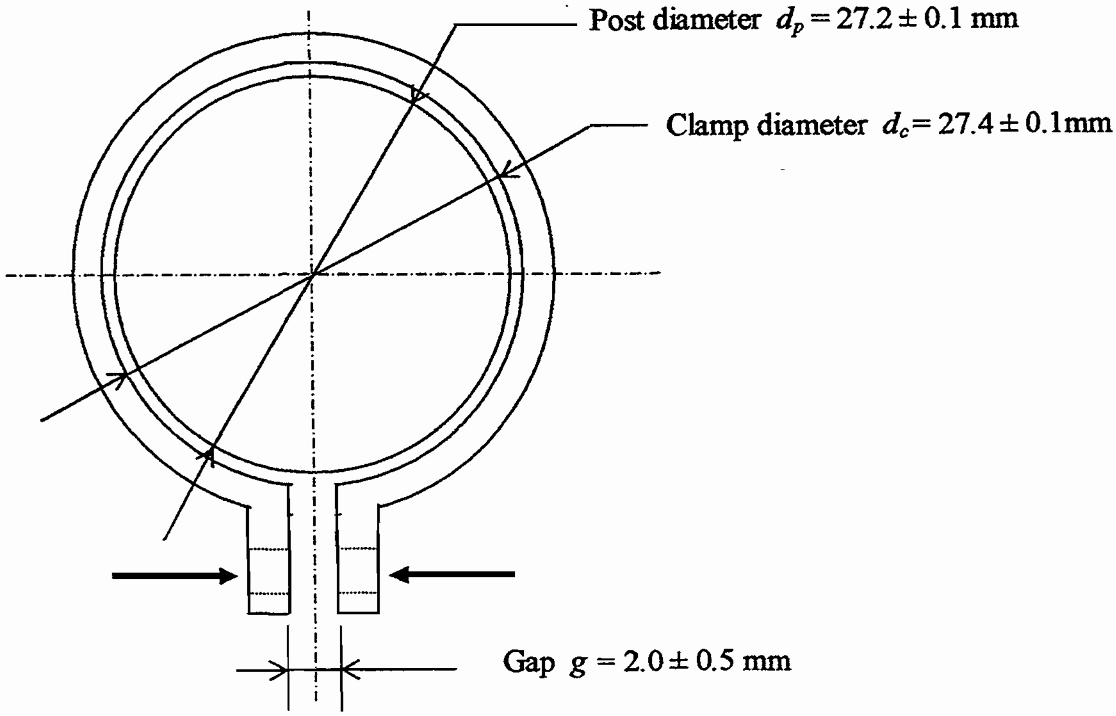

Fig. 4

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1995
(Revised 2001)
(Revised 2002)
(Revised 2003)

MODULE 4C4

DATA BOOK

1.0 OPTIMIZATION

DATA SHEET

1.1 Series

Taylor Series

For a function of one variable:

$$f(x_k+\delta)=f(x_k)+\delta f'(x_k)+\frac{1}{2}\delta^2 f''(x_k)+\ldots \quad \text{where} \quad x_{k+1}=x_k+\delta$$

For a function of several variables:

$$f(\underline{x}_k+\underline{\delta x})=f(\underline{x}_k)+\{\nabla f(\underline{x}_k)\}^t\underline{\delta x}+\frac{1}{2}\underline{\delta x}^t\mathbf{H}(\underline{x}_k)\underline{\delta x}+\ldots \quad \text{where} \quad \underline{x}_{k+1}=\underline{x}_k+\underline{\delta x}$$

where $\{\nabla f(\underline{x}_k)\}^t$ is the Grad of the function at $\underline{x}_k$:

$$\begin{bmatrix} \frac{\partial f(\underline{x}_k)}{\partial x_1} & \frac{\partial f(\underline{x}_k)}{\partial x_2} & \cdots & \frac{\partial f(\underline{x}_k)}{\partial x_n} \end{bmatrix}$$

and $\mathbf{H}(\underline{x}_k)$ is the Hessian of the function at $(\underline{x}_k)$:

$$\begin{bmatrix} \frac{\partial^2 f(\underline{x}_k)}{\partial x_1^2} & \frac{\partial^2 f(\underline{x}_k)}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f(\underline{x}_k)}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f(\underline{x}_k)}{\partial x_2 \partial x_1} & & & \\ \vdots & & & \\ \frac{\partial^2 f(\underline{x}_k)}{\partial x_n \partial x_1} & \frac{\partial^2 f(\underline{x}_k)}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f(\underline{x}_k)}{\partial x_n^2} \end{bmatrix}$$

Note: 1. $\nabla f(\underline{x}_k)$ is defined as a column vector.
2. The Hessian is symmetric.
3. If $f(\underline{x})$ is a quadratic function the elements of the Hessian are constants and the series has only three terms.

1.2 Line searches

Golden Section Ratio $= \frac{\sqrt{5}-1}{2} \approx 0.6180$

Newton's Method (1D)

When derivatives are available: $x_{k+1} = x_k - \{f'(x_k)\}/\{f''(x_k)\}$

When derivatives are unavailable:

$$x_4 = \frac{1}{2}\frac{(x_2^2 - x_3^2)f(x_1) + (x_3^2 - x_1^2)f(x_2) + (x_1^2 - x_2^2)f(x_3)}{(x_2 - x_3)f(x_1) + (x_3 - x_1)f(x_2) + (x_1 - x_2)f(x_3)}$$

1.3 Multidimensional searches

Conjugate Gradient Method

To find the minimum of the function

$f(\underline{x}) = f(\underline{x}_0) + \nabla f(\underline{x}_0)^t \partial\underline{x} + \frac{1}{2}\partial\underline{x}^t H \partial\underline{x}$, where $\partial\underline{x} = \underline{x} - \underline{x}_0$ and $\underline{x}$ has n dimensions:

First move is in direction $\underline{s}_0$ from $\underline{x}_0$ where:

$$\underline{s}_0 = -\nabla f(\underline{x}_0)$$

Then $\quad \underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{s}_k$

where $\quad \alpha_k = \frac{-\underline{s}_k^t \nabla f(\underline{x}_k)}{\underline{s}_k^t H \underline{s}_k}$ (which minimises $f(\underline{x})$ along the defined line)

Then $\quad \underline{s}_{k+1} = -\nabla f(\underline{x}_{k+1}) + \beta_k \underline{s}_k$

where $\quad \beta_k = \frac{\nabla f(\underline{x}_{k+1})^t H \underline{s}_k}{\underline{s}_k^t H \underline{s}_k}$

For a quadratic function, the method converges at $\underline{x}_n$.

Fletcher-Reeves Method

To find the minimum of the function $f(\underline{x})$ where $\underline{x}$ has n dimensions:

First move is in direction $\underline{s}_0$ from $\underline{x}_0$ where:

$$\underline{s}_0 = -\nabla f(\underline{x}_0)$$

Then $\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{s}_k$ such that $f(\underline{x})$ is minimised along the defined line.

Then $\underline{s}_{k+1} = -\nabla f(\underline{x}_{k+1}) + \beta_k \underline{s}_k$

where $\beta_k = \dfrac{(\nabla f(\underline{x}_{k+1}))^2}{(\nabla f(\underline{x}_k))^2}$

For quadratic functions, the method will converge at $\underline{x}_n$. For higher order functions, the method should be restarted when $\underline{x}_n$ is reached.

1.4 Constrained Minimisation

Penalty and Barrier functions

The most common Penalty function is:

$$q(\mu,\underline{x}) = f(\underline{x}) + \frac{1}{\mu}\sum_{i=1}^{p}\left(\max[0, g_{i(\underline{x})}]\right)^2$$

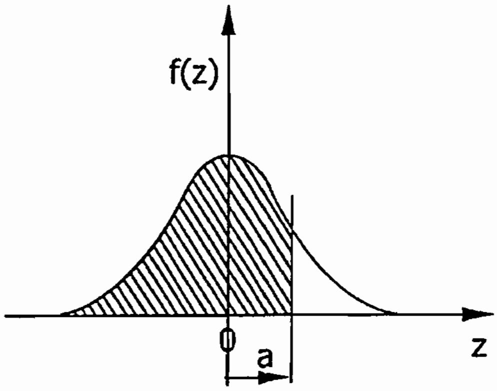
where $f(\underline{x})$ is subject to the constraints $g_1(\underline{x}) \le 0, \ldots, g_p(\underline{x}) \le 0$

A typical Barrier function for the same problem is:

$$q(\mu,\underline{x}) = f(\underline{x}) - \mu\sum_{i=1}^{p} g_i(\underline{x})^{-1}$$

2.0 STATISTICS DATA SHEET

2.1 Standardised normal probability density function



$$P(z<a)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{-\frac{z^2}{2}}\,dz$$

$$z=\frac{x-\mu}{\sigma}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9349	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9723	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABULATED VALUES

2.2 Combining distributed variables

For the function

$$y = f(x_1, x_2, \ldots x_n)$$

where x_1, x_2 etc. are <u>independent</u> and defined by their respective distributions:

	y	μ_y	σ_y^2
1	$x + a$	$\mu_x + a$	σ_x^2
2	ax	$a\mu_x$	$a^2\sigma_x^2$
3	$a_1x_1 + a_2x_2$	$a_1\mu_1 + a_2\mu_2$	$a_1^2\sigma_1^2 + a_2^2\sigma_2^2$
4	x_1x_2	$\mu_1\mu_2$	$\mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2$
5	x_1/x_2	μ_1/μ_2	$\frac{1}{\mu_2^4}\left(\mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2\right)$

Where: μ = mean; σ = standard deviation; a = constant.

Engineering Tripos Part IIA and Part IIB 2006

Module 4C4 – Design Methods

Numerical Answers

3 (b) (ii) w must lie in the range: $3.89 < w < 5.78$ m

(iii) $w = 4.47$ m ; $d = 22.37$ m ; $z = 6.00$ m

4 (b) Nominal safety factor = 1.46

Worst case safety factor = 0.26

(c) Percentage of clamp assemblies = 2.8%