ENGINEERING TRIPOS PART IIB

Tuesday 25 April 2006 9 to 10.30

Module 4C6

ADVANCED LINEAR VIBRATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 An instrumented impulse hammer is to be used for modal testing. The total mass of the hammer head is m and the stiffness of the hammer tip is k.

(a) Sketch a typical instrumented hammer and identify its principal parts. [10%]

(b) The impulse-force wave form can be idealized by the function

$$f(t) = F_0 \cos(\Omega t) \ , \qquad -\frac{b}{2} \leq t < \frac{b}{2}$$

(i) Sketch the impulse wave form $f(t)$ and use your sketch to explain the significance of the parameter Ω. Explain how Ω is influenced by m and k and express Ω in terms of the duration of the impulse b. [20%]

(ii) The spectrum of the impulse can be expressed in the form

$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \ = B \frac{\cos \frac{\pi\omega}{2\Omega}}{\Omega^2 - \omega^2}$$

By considering the value of the spectrum at $\omega = 0$ (or otherwise) determine the value of B in terms of F_0 and Ω. [40%]

(iii) Sketch the impulse spectrum $F(\omega)$ showing clearly the significance of the parameter Ω and the distribution of zeros. [20%]

(c) With reference to your sketches explain the "rule of thumb" that an impulse does not excite vibration at frequencies (in Hz) above $1/b$. [10%]

2 (a) Describe the major physical mechanisms of damping in structural vibration. Be sure to distinguishing between *material damping* and *boundary damping*. [50%]

(b) For the following systems, what are likely to be the major sources of damping?

(i) a tall building subject to earthquake excitation;

(ii) the blades of a wind turbine used to generate electricity;

(iii) the roof panel of a passenger car. [25%]

Indicate how damping might be added to these systems in order to reduce vibration. [25%]

3 A simple model of a tuning fork held in the hand is shown in Fig. 1. Three equal masses m are joined by two massless leaf springs each of stiffness k, and the base mass is restrained by a spring of stiffness S (representing the hand) which may be assumed to be much less than k. Small vibration of the system is described by the vector of displacements $[x\,y\,z]^T$ as defined in the figure.

(a) Show that the mass and stiffness matrices are respectively

$$M = m\begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \qquad K = \begin{bmatrix} S & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$ [25%]

(b) Use Rayleigh's principle, with assumed normal modes $[0\ 1\ -1]^T$, $[1\ 0\ 0]^T$ and $[2\ -3\ -3\,]^T$, to find approximate natural frequencies of the tuning fork. [30%]

(c) Damping (caused by the hand) can be represented by replacing the spring S by the complex value $S(1+i\eta)$. If $\eta \ll 1$, obtain approximate expressions for the damping factors of the three vibration modes. [30%]

(d) In practice η is *not* small. Noting that the spring S is weak, would your approximate answers from (c) still be valid? Comment on the significance of the results for the behaviour of a tuning fork. [15%]

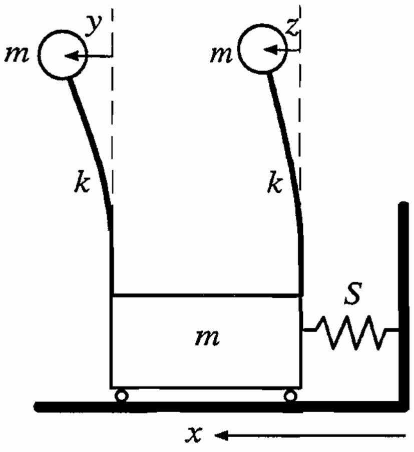

Fig.1

4 *The following results for a Helmholtz resonator may be useful:*

- *a Helmholtz resonator of volume V with a neck of effective length L and cross-sectional area S has resonant frequency*

$$\omega = c\sqrt{\frac{S}{VL}} \qquad \text{where } c \text{ is the speed of sound in air;}$$

- *the end correction for an unflanged circular neck of radius a is 0.6a;*
- *the end correction for a flanged circular neck of radius a is 0.85a.*

(a) Explain briefly how a Helmholtz resonator can be described by an equivalent mass-spring system. Give expressions for the equivalent mass and spring stiffness for the thin-walled resonator shown in Fig. 2(a), which has volume V and a circular hole of radius a. Take the density of air to be ρ and the speed of sound in air to be c. [30%]

(b) The system shown in Fig. 2(b) has two volumes V_1 and V_2 separated by a thin-walled internal baffle. The baffle and the surrounding thin-walled enclosure both contain circular holes of radius a. Give the equivalent mass-spring representation of the system and find an expression for the natural frequencies. [45%]

(c) A rigid plug of negligible volume is used to block the hole in the internal baffle. What effect does this have on the natural frequencies? Does the result agree with the prediction of the interlacing theorem? [25%]

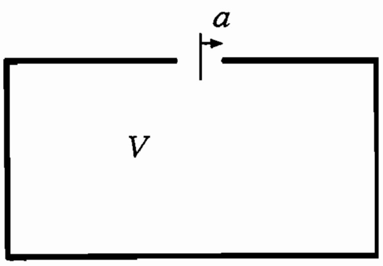

Fig. 2(a)

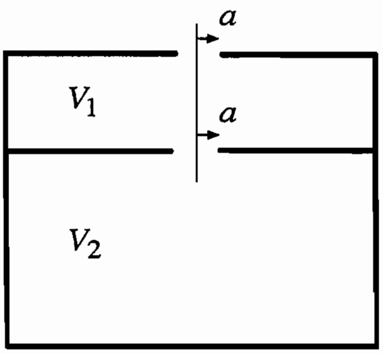

Fig 2(b)

END OF PAPER

Engineering Tripos Part IIB 2006 – 4C6 Advanced Vibration

Answers

1. (b) (i) $b=\pi/\Omega$ (ii) $B=F_o\Omega/\pi$

3. (b) $\omega^2 = \ k/m \ , \ s/3m \ , \ 3k/m$
   (c) $Q = \ 0 \ , \ 1/\eta \ , \ 9k/2s\eta$

4. (a) $m= 1.7\pi a^3\rho$ $k= \rho\pi^2 c^2 a^4/V$
   (b) $\omega^2 m = [2k_1 + k_2 \pm \sqrt{(4k_1^{\,2} + k_2^{\,2})}]/2$

HEMH May 2006