ENGINEERING TRIPOS PART IIB

Friday 28 April 2006 2.30 to 4

Module 4C8

APPLICATIONS OF DYNAMICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4C8 datasheet (3 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

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1 A model of a tyre contact patch has length $2l$ and width $2h$. The contact pressure p is uniform over the area of the patch. The tyre is rolled on a flat horizontal surface at yaw angle δ to the direction of motion. In order to improve the 'brush' model, it is suggested that different values are used for the static coefficient of friction μ_s and the dynamic coefficient of friction μ_d, where $\mu_s > \mu_d$. The wheel is running freely (i.e. with no longitudinal creep) and the lateral brush stiffness per unit area is K.

(a) Show that the yaw angle when microslip first occurs at the rear of the contact patch is given by:

$$\delta_{crit} = \frac{\mu_s p}{2lK}$$

[10%]

(b) When the yaw angle increases above this value a region of microslip appears, in which the coefficient of friction is μ_d. Show that the lateral force Y generated by the tyre is given by:

$$Y = 4\mu_d plh + \frac{p^2 h}{K\delta}\mu_s\left(\mu_s - 2\mu_d\right)$$

[50%]

(c) Show that if $\mu_d < \mu_s/2$ the lateral force decreases with increasing yaw angle for $\delta > \delta_{crit}$. Sketch graphs of Y against δ for $\mu_d < \mu_s/2$, $\mu_d = \mu_s/2$ and $\mu_d > \mu_s/2$.

[40%]

2 (a) Explain why both lateral and longitudinal creep forces are important in curving of railway bogies at constant forward speed. Compare/contrast this situation with the tyre forces acting during constant-speed cornering of automobiles.

[30%]

(b) The castered wheel of a supermarket trolley is idealised as shown in plan view in Fig. 1. A wheel of mass m with *negligible diametral moment of inertia*, is free to rotate about a horizontal axis through its mass centre B. The bearings allow lateral displacement y of the wheel along the axle, with linear stiffness k. (The wheel does not yaw with respect to the axle.) The lateral creep coefficient of the tyre is C.

The axle is attached to a rigid yoke AB, of length a which is free to rotate about a vertical axis at A, with yaw angle θ. The yoke has moment of inertia I about A. Point A moves in a straight line at constant speed U, and the floor is smooth and level.

(i) Derive equations for small lateral oscillations of the system in the horizontal plane (neglecting gyroscopic effects.)

[35%]

(ii) Use the Rbuth-Hurwitz criterion to determine the conditions for which the lateral motion of the system will be stable.

[35%]

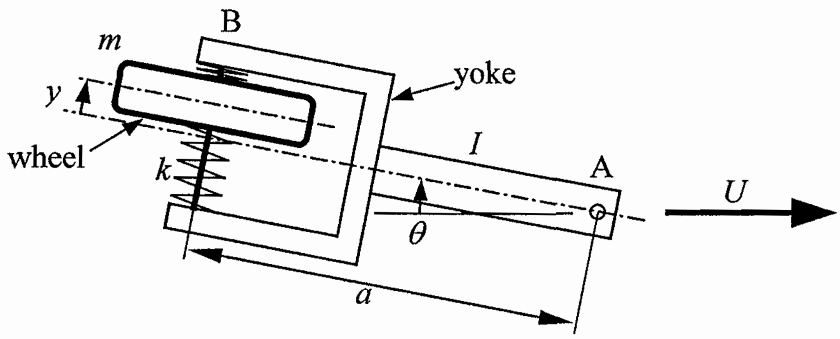

Fig. 1

3 Figure 2a shows a pitch-plane model of a vehicle comprising a sprung mass m with bounce (vertical) and pitch degrees of freedom, and two unsprung masses m_{u1} and m_{u2} each with a vertical degree of freedom. Travel at speed V from left to right along a rough road results in vertical inputs at the front and rear wheels.

(a) Explain 'wheelbase filtering', that is, the mechanism by which the speed V and wheelbase $L=a+b$ of a vehicle influence the excitation of the pitch and bounce modes of vibration associated with the sprung mass. [30%]

Figure 2b shows the magnitudes of frequency response functions relating the road input (referenced to the vertical velocity input $\dot{z}_{r1}$ at the front axle) to the sprung mass bounce and pitch accelerations. The dashed lines correspond to a vehicle speed $V = 10$ m s^{-1} and the solid lines correspond to a vehicle speed $V = 30$ m s^{-1}. The other parameter values are:

$m = 800$ kg	$a = 1.35$ m
$I = 1350$ kg m^2	$b = 1.55$ m
$k_1 = 23$ kN m^{-1}	$c_1 = 2.3$ kN s m^{-1}
$k_2 = 27$ kN m^{-1}	$c_2 = 2.7$ kN s m^{-1}
$m_1 = m_2 = 45$ kg	$k_{t1} = k_{t2} = 200$ kN m^{-1}

(b) By simplifying the model to have two degrees of freedom and no damping, and by assuming that the two natural modes of vibration associated with the sprung mass are pure bounce and pure pitch about the centre of mass, estimate the natural frequencies of these two modes. (You do not have to use all of the parameter values given, and you do not have to perform an eigenvalue/eigenvector calculation.) [20%]

(c) With the aid of your answers to (a) and (b), explain the shape of the frequency response functions shown in Fig. 2b, in the range 0 Hz to 5 Hz. Comment on the shape of the frequency response functions at frequencies above 5 Hz. [30%]

(d) State some of the issues a suspension engineer might have to consider when using this model to optimise the suspension stiffness and damping of a passenger car. [20%]

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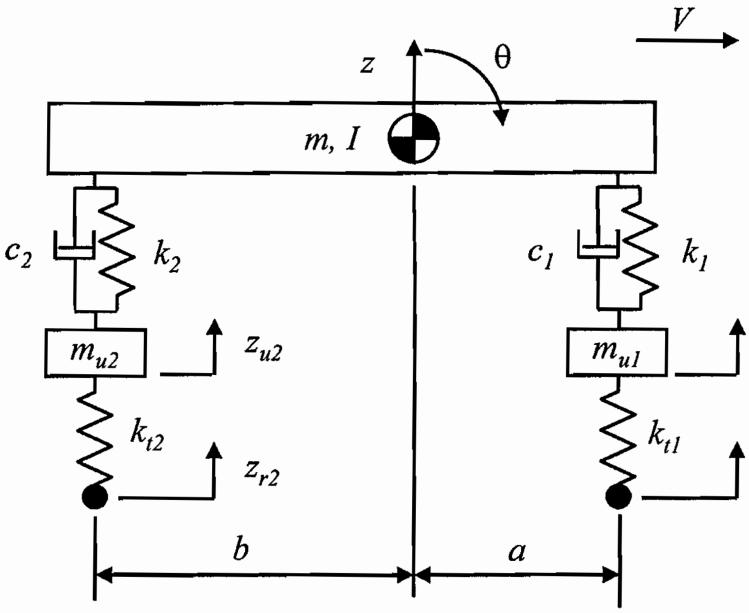

Fig. 2a

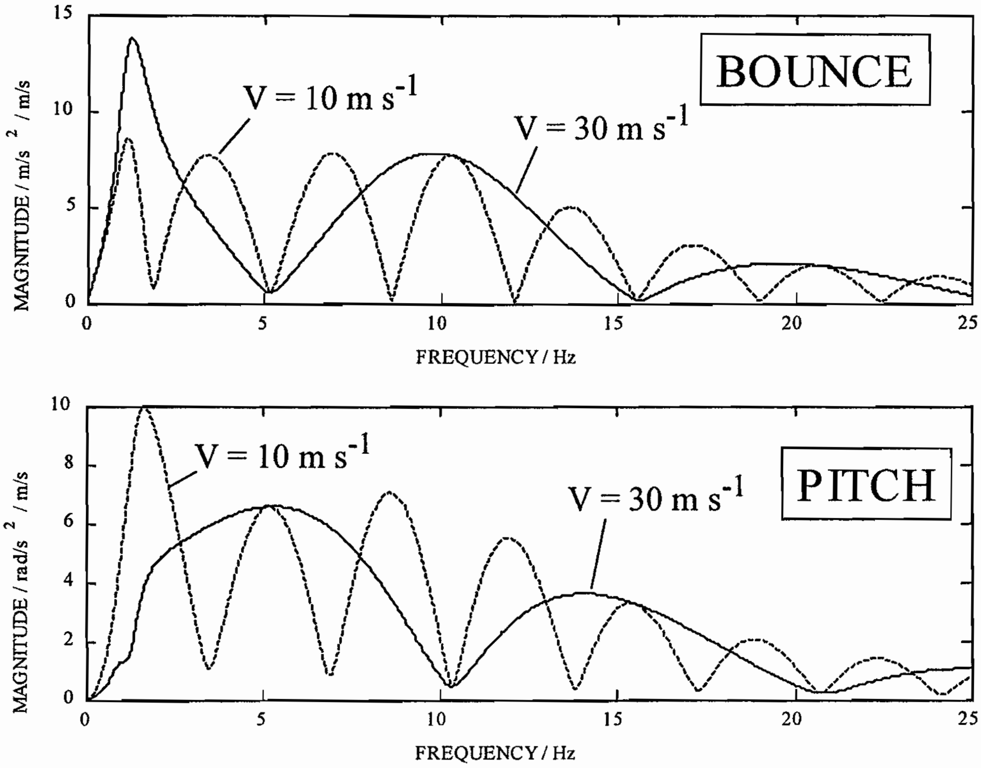

Fig. 2b

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4 Figure 3a shows a simplified version of the six degrees-of-freedom (6 DOF) model of lateral-roll vibration shown in Fig. 3b. The simpler model has mass m and roll inertia I about mass centre G. The height of the centre of mass is h. The mass has lateral and roll degrees of freedom y and θ. The suspension roll stiffness and roll damping are τ and b. The lateral tyre behaviour is represented by a series spring and damper, k_{lat} and c_{lat}. Roll displacement input from the road is ϕ.

(a) Show that the damping coefficient of the lateral tyre model is given by:

$$c_{\text{lat}} = \frac{C}{V},$$

where V is the speed of the vehicle and C is the tyre cornering stiffness. [20%]

(b) For the case of very low vehicle speed, show that one of the equations of motion of the simple vehicle model in Fig. 3a is given by:

$$I\ddot{\theta} + b\dot{\theta} + \tau\theta - mh\ddot{y} = \tau\phi + b\dot{\phi}$$

Find a second equation of motion.

[20%]

(c) Show that the undamped natural frequencies of vibration of the simple vehicle model in Fig. 3a are given by the solutions to:

$$\omega^4 mI - \omega^2\left(m\tau + k_{\text{lat}}I + mh^2 k_{\text{lat}}\right) + k_{\text{lat}}\tau = 0.$$ [20%]

(d) Using the parameters of the 6 DOF model, estimate a suitable value for the suspension roll stiffness τ. Use the result in (c) to estimate the resonant frequencies in the roll response of the sprung mass of the 6 DOF model traversing a randomly rough road at very low speed.

[20%]

(e) Comment on how the roll response of the sprung mass in part (d) would be different if the vehicle were travelling at very high speed.

[20%]

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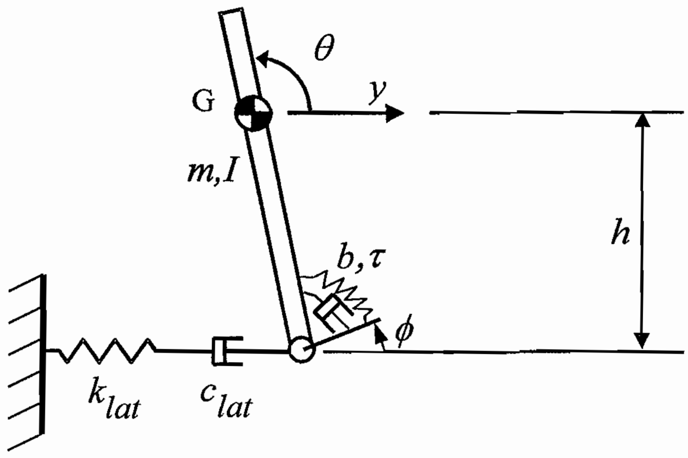

Fig. 3a

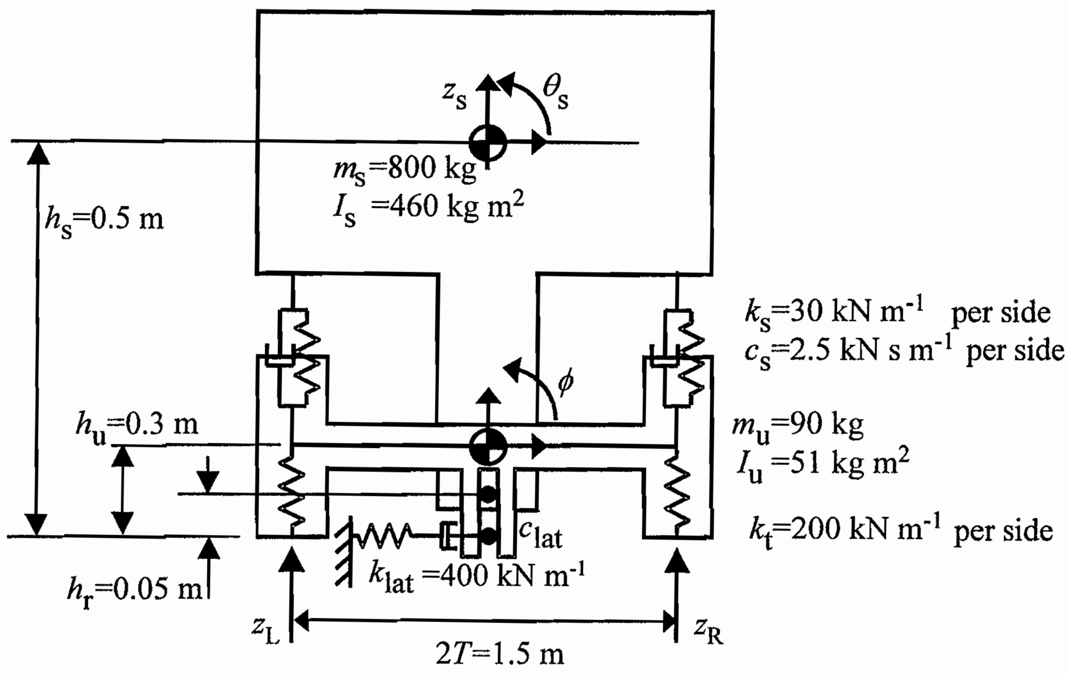

Fig. 3b

END OF PAPER

Answers

2. (i) $$\begin{pmatrix} m & ma \\ 0 & I \end{pmatrix}\begin{pmatrix} \ddot{y} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} \frac{C}{U} & \frac{Ca}{U} \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{y} \\ \dot{\theta} \end{pmatrix} + \begin{pmatrix} k & C \\ -ak & a^2k \end{pmatrix}\begin{pmatrix} y \\ \theta \end{pmatrix} = 0;$$ (ii) k>C/a

3. (b) Bounce: 1.19 Hz; Pitch: 1.33 Hz

4. (b) $m\ddot{y} + k_{lat}y = -k_{lat}h\theta$;
(d) $\tau = 29.35$ kNm/rad; 1.05 Hz and 4.32 Hz.