ENGINEERING TRIPOS PART IIB

Monday 24 April 2006 2.30 to 4

Module 4C15

MEMS DESIGN

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Explain qualitatively how the Hertz contact pressure distribution can be combined with that due to a flat punch to investigate the elastic contact between a sphere and a plane surface in the presence of significant adhesion. [30%]

(b) For a flat adhesive punch of radius a, as shown in Fig. 1, the contact pressure $p_a(r)$ is

$$p_a(r) = -\frac{P_a}{2\pi a^2}\left\{1-\left(\frac{r}{a}\right)^2\right\}^{-1/2}$$

where P_a is the adhesive tensile load. The singularity in stress at radius $r = a$ can be described by the Mode I stress intensity factor K_I where

$$K_I = \frac{P_a}{2a\sqrt{\pi a}}$$

For a crack to propagate from the edge of the contact, the critical value of K_I is related to the elastic contact modulus E^* and the work of adhesion w of the solids by

$$\frac{K_I{}^2}{2E^*} = w$$

Show how these relations may be combined, together with the usual Hertzian analysis for elastic contact between a sphere of radius R and a plane surface, to generate the Johnson-Kendall-Roberts relation between the net applied load P and the contact radius a

$$P = \frac{4E^* a^3}{3R} - \sqrt{8\pi a^3 w E^*}$$

[40%]

(c) A displacement-controlled surface force apparatus uses two cylinders each of radius 8 mm crossed at 90°. If E^* equals 37 GPa and w equals 1.3 Jm^{-2}, what is the size of (i) the pull-off force and (ii) the radius of the contact patch, immediately before the surfaces separate? [30%]

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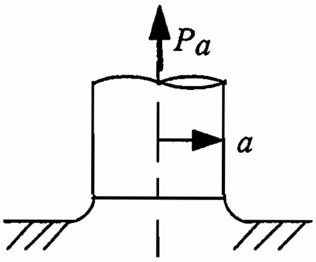

Fig. 1

Hertzian contact between spheres 1 and 2 under a load P

Reduced radius R given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Contact modulus E^* by $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$

Radius of contact circle $a = \left\{\frac{3PR}{4E^*}\right\}^{1/3}$

Maximum contact pressure $p_0 = \frac{3P}{2\pi a^2} = \left\{\frac{6PE^{*2}}{\pi^3 R^2}\right\}^{1/3}$

Mean contact pressure $\bar{p} = \frac{2}{3} p_0$

Approach of distant points $\delta = \frac{a^2}{R} = \left\{\frac{9P^2}{16RE^{*2}}\right\}^{1/3}$

Maximum shear stress is of magnitude $0.31 p_0$ and at depth $0.48a$.

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2 A top view of a 'breathing bar' resonator beam is shown in Fig. 2. The process is based on polysilicon surface micromachining and the thickness H of the structural polysilicon layer is 5 μm. The length L of the beam is 100 μm and the width W of the beam is 10 μm. A single electrode of the same width as the beam is spaced at a gap of 500 nm from one end of the beam as shown. You may assume that polysilicon has a Young's Modulus E of 160 GPa and a density ρ of 2330 kg m^{-3}.

(a) The resonator is operated in an extensional mode. The effective spring constant k and effective mass m for resonant operation in this mode are expressed as

$$k = \frac{\pi^2 HWE}{4L}$$

$$m = \rho HWL$$

Estimate the natural frequency ω of the system. Assuming the Quality factor in air is 10^4, estimate the damping constant b. [20%]

(b) Construct a lumped-element equivalent electrical circuit model for the mechanical system and hence derive the transfer function relating displacement-to-force by the application of Kirchhoff's voltage and current laws to the circuit. [30%]

(c) Estimate the electrostatic force generated as a function of the geometry and the voltage V applied between the electrode and the beam? Determine whether this device will exhibit pull-in behaviour. [20%]

(d) The voltage V is now adjusted such that its AC component v_{ac} has a frequency coinciding with the natural frequency of the structure and is of amplitude equal to 100 mV. The DC component V_{DC} of the applied voltage is 10 V. Estimate the end displacement of the beam. [30%]

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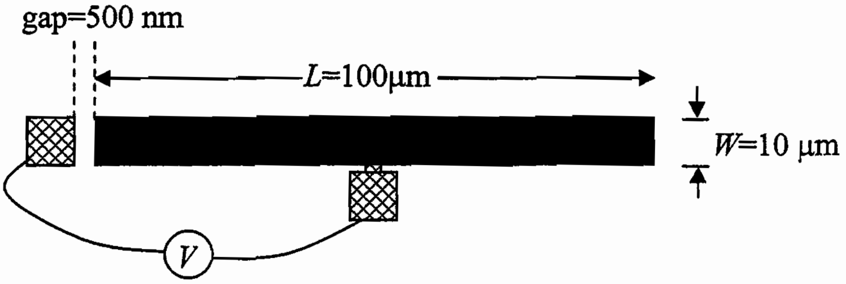

■ movable
▨ fixed

Fig. 2 (not to scale)

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3 A 500 μm x 500 μm square silicon proof mass is supported by a symmetrical crab-leg suspension as shown in Fig. 3. Each beam comprising the suspension has a length L of 200 μm and a width W of 4 μm. The thickness H of the silicon structural layer is 5 μm. The proof mass is actuated by an electrostatic comb drive as shown. The nominal overlap between comb fingers is 20 μm and the gap between comb fingers g is 1 μm. The number N of comb finger gaps is 400. Silicon has a Young's Modulus E of 160 GPa and a density ρ of 2330 kg m^{-3}.

(a) Derive an expression for the electrostatic force generated by a comb drive actuator as a function of voltage V applied between the electrode and the proof mass, the number of electrode gaps N and the geometrical parameters defining the system. [20%]

(b) The spring constants for motion of the proof mass in the in-plane directions are

$$k_x = k_y = \frac{5EH}{2}\left(\frac{W}{L}\right)^3$$

Estimate the static deflection of the proof mass if a voltage of 10 V is applied between the proof mass the fixed comb electrodes. [20%]

(c) Estimate the deflection of the proof mass for an input acceleration of 1 g along either the x- or y-axis (1 g = 9.81 m s^{-2}). [20%]

(d) Explain briefly the trade-off between parallel-plate and comb drive electrode structures for capacitive detection of proof mass motion. [10%]

(e) The device is now configured to operate as a vibratory gyroscope. The proof mass is driven to a displacement amplitude of 5 μm at a frequency of 500 Hz along the x-axis. Estimate the deflection of the proof mass along the y-axis in response to an externally applied rotation rate Ω_z of 1 rad s^{-1} about the z-axis. [20%]

(f) Explain briefly the sensitivity versus bandwidth trade-off for resonant operation of a micromechanical vibratory rate gyroscope. [10%]

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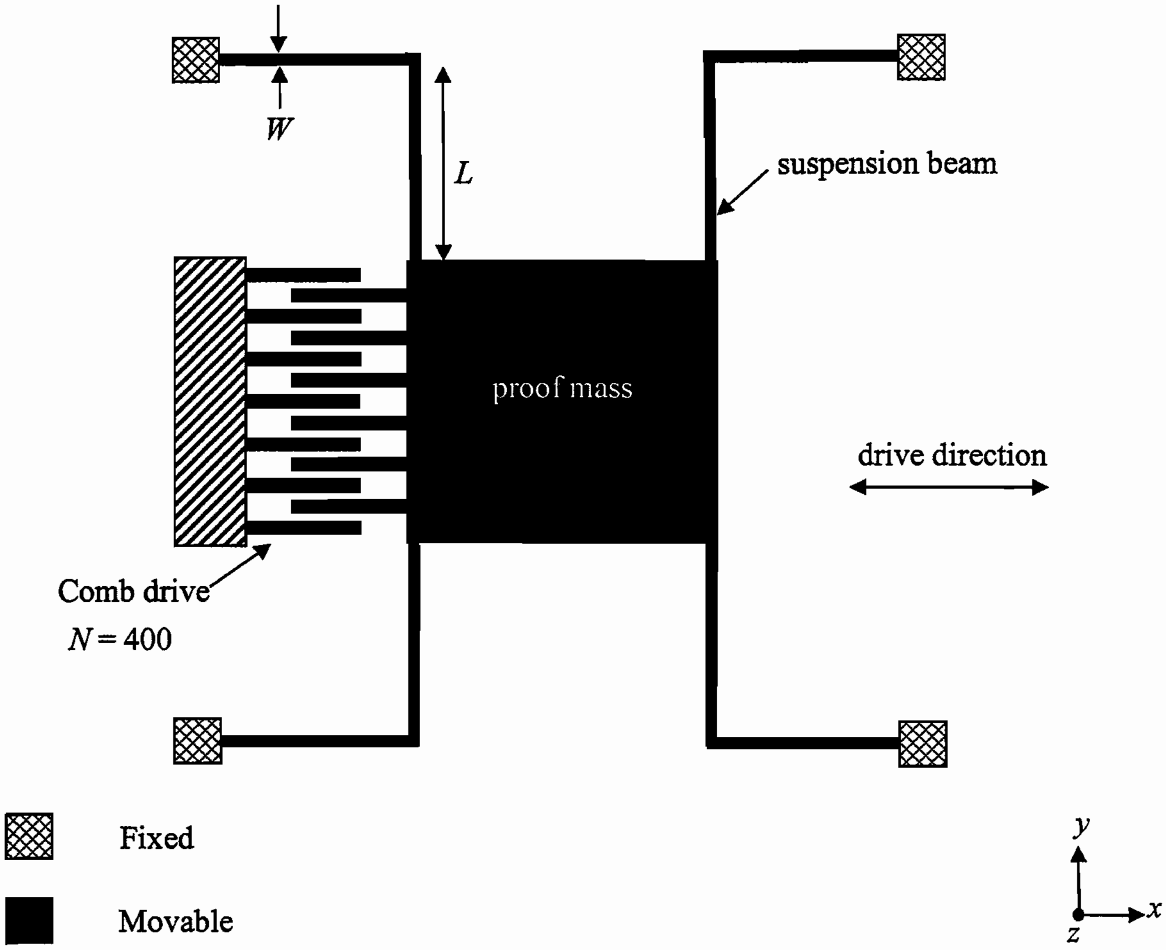

Fig. 3 (not to scale)

4 A silicon cantilever is 500 μm long, 5 μm wide and 2 μm thick as shown in cross-section in Fig. 4. The deflection of the beam is sensed by capacitance and by piezoresistance. The cantilever is bent by a point force acting at its free end. Silicon has a Young's Modulus E of 160 GPa and density ρ of 2330 kg m^{-3}.

(a) Estimate the force at the free end of the cantilever that would result in a vertical downwards tip deflection of 400 nm. You may assume linear elastic bending. [20%]

(b) A piezoresistor is located longitudinally at the base of the cantilever as shown. Estimate the maximum bending stress at the support and hence the fractional change in resistance for a downwards tip deflection of 400 nm if the longitudinal piezoresistive coefficient is -10^{-9} Pa^{-1}. [20%]

(c) An electrode of length 250 μm is attached to the substrate to allow for capacitive sensing as shown in Fig. 4. One end of the electrode is aligned to the free end of the beam as shown. At zero load, the gap between the cantilever and the electrode is 2 μm. Estimate the fractional change in capacitance for a vertical downwards tip deflection of 400 nm. [50%]

(d) Compare and contrast the advantages and disadvantages of piezoresistive sensing and capacitive sensing. [10%]

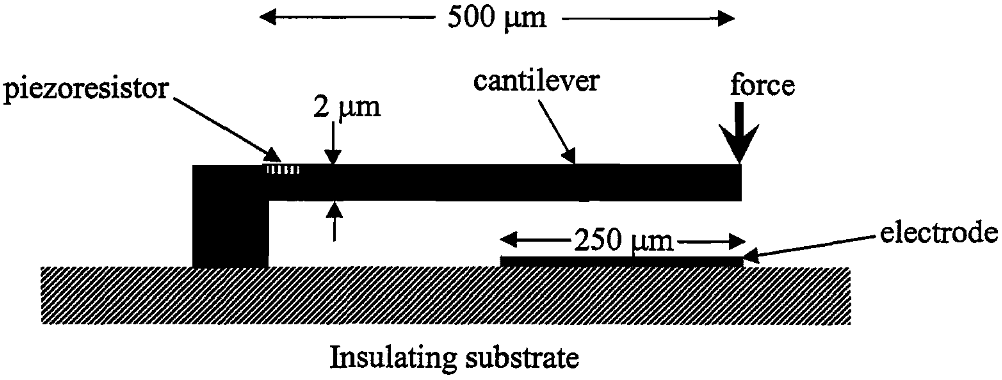

Fig. 4 (not to scale)

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