ENGINEERING TRIPOS PART IIB

Monday 24 April 2006 2.30 to 4

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4D6 Data sheets (4 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 An idealisation of a proposed skyscraper of height h = 200 m is presented in Fig. 1a. Under lateral loading the building may be modelled as a cantilever beam rigidly fixed at ground level by the foundations. The flexural rigidity of the tower is provided by a concrete core with the cross section shown in Fig. 1b. Young's modulus of the concrete core is $E = 30$ GPa. The total mass of the tower is 200×10^6 kg.

(a) The mode shape for this tower may be assumed to be of the form;

$$\bar{u}(x) = 1 - \cos\frac{\pi x}{2h}$$

where h is the height of the tower.

Estimate the natural frequency of the tower in the fundamental sway mode. [30%]

(b) Estimate the top storey lateral deflection of the tower due to a gust of wind that has a rectangular pulse of 1.75 s duration and loading intensity of 100 kN m^{-1}. You may use the mode shape suggested in part (a). [40%]

(c) Using the same assumed mode shape obtain an estimate of the top storey deflection of the tower due to a single earthquake pulse which is represented by a constant ground acceleration of 0.1 m s^{-2} for a duration of 1.75 s. [30%]

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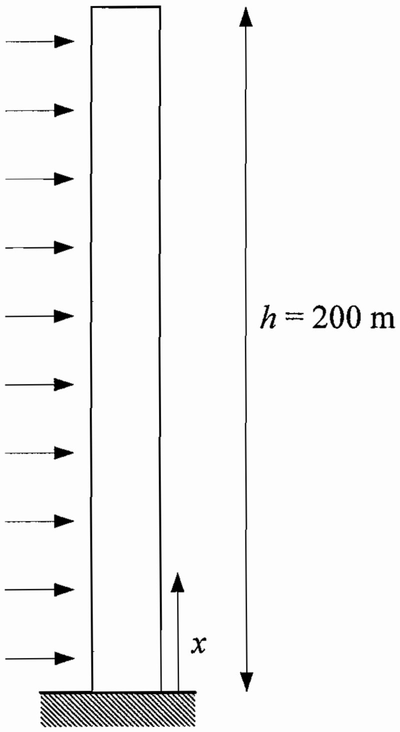

Fig. 1a

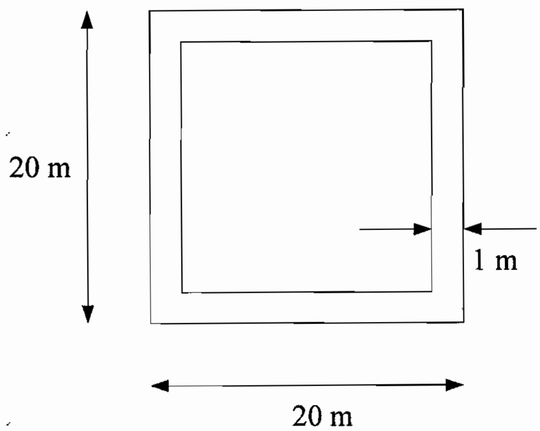

Fig. 1b

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2 A three storey building is modelled as the two-dimensional, rigid-jointed, linear elastic sway frame shown in Fig. 2. This building is to be located on a sloping ground and therefore the column heights of the ground floor are unequal as shown in Fig. 2. The floor masses for each storey are as shown in Fig. 2. The bending stiffness of all the columns is $EI = 25.3 \times 10^6$ N m^2.

(a) Determine the natural frequency of the fundamental vibration mode of this building by considering the following mode shape: [40%]

$$\begin{bmatrix} 1 \\ 0.57 \\ 0.38 \end{bmatrix}$$

(b) The building was to be designed according to the Eurocode 8 design spectrum for acceleration shown in Fig. 3 which is appropriate for the ground conditions at this building site. The peak ground acceleration a_g expected at the building site is 0.65g. Calculate the floor accelerations using this design spectrum. [30%]

(c) Estimate the shear force experienced at the bases of the ground floor columns during an earthquake that generates the peak ground acceleration a_g of 0.65g considered in part (b). [30%]

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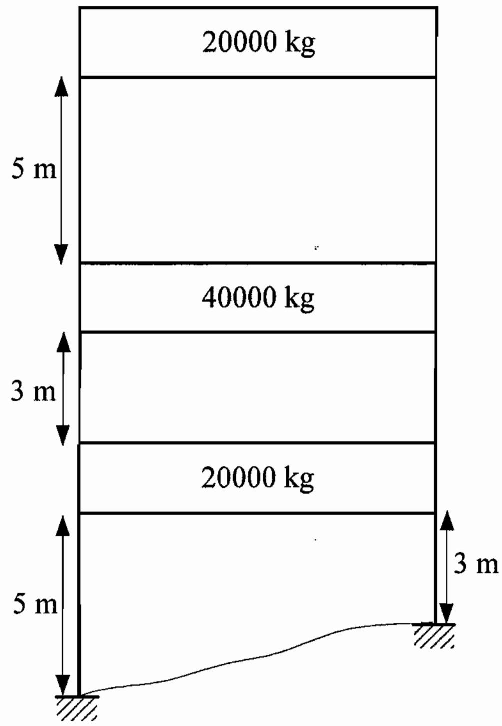

Fig. 2

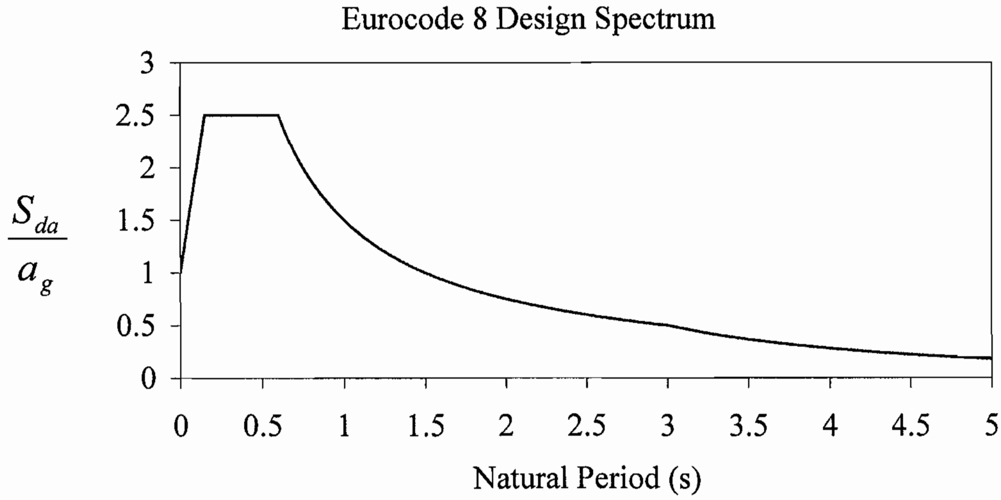

Fig. 3

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3 (a) Explain what you understand by dynamic soil-structure interaction. How can soil liquefaction affect soil-structure interaction during a strong earthquake event ? [20%]

(b) Describe two conditions under which the stiffness of the soil can be expected to degrade. [10%]

(c) A horizontal layer of loose sandy soil at the Port Island site in Japan is 12 m thick and overlies the bed rock as shown in Fig. 4. The water table at this site is expected to be at the ground surface. The unit weight of the saturated soil below the water table is 20 kN m^{-3}. The Poisson's ratio of this soil may be taken as 0.3 and the specific gravity of the sand grains is 2.65. By considering a reference plane 6 m below the ground surface, estimate the small-strain shear modulus of the soil. Using this, estimate the shear wave velocity in this soil layer. [30%]

(d) Assuming that the soil vibrates in its fundamental mode, estimate the natural frequency of the soil layer in part (c). [10%]

(e) A strong earthquake was experienced at this site during the Kobe earthquake of 1995. During this event positive excess pore water pressures of 35 kPa were recorded on the reference plane. Estimate the shear modulus and hence the shear wave velocity under these conditions. Comment on the reliability of the shear wave velocity you have calculated. [30%]

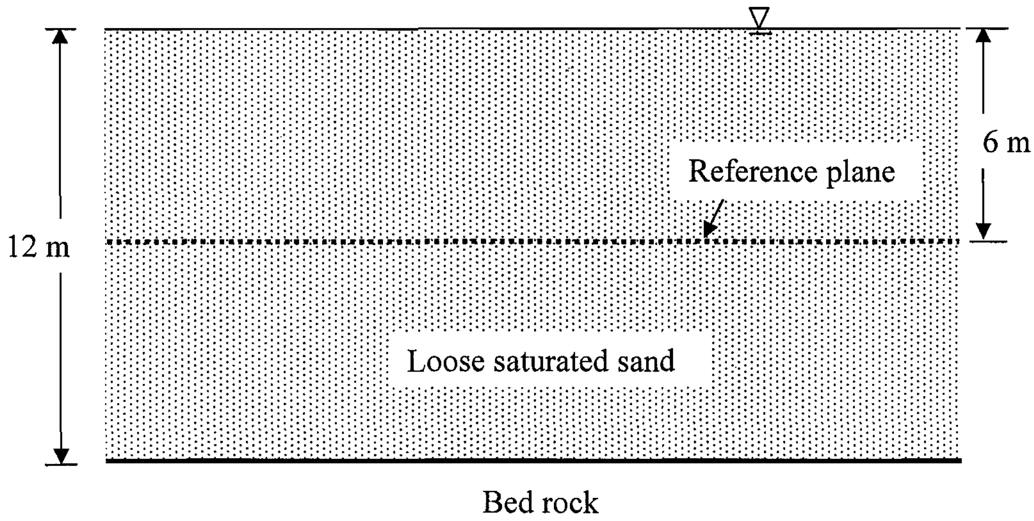


Fig. 4

4 (a) Briefly discuss the generation of unsteady forces on a structure that exists within a fluid that flows, arising from:

(i) a steady flow and a rigid structure; [10%]

(ii) a steady flow and a flexible structure. [30%]

(b) An under-sea walkway is proposed at a tourist resort in water of depth 30 m. It will have circular cross-section and be suspended half way between the sea floor and the surface. Its diameter will be 3 m and its mass per unit length will be 885 kg m^{-1}. Its resonant frequency of vertical motion in air will be 0.5 Hz. The velocity of water in that region never exceeds 3 m s^{-1} in any direction. Are there any problems with the design from a flow-structure interaction perspective?

Some of the following information may be useful: The density of seawater is 1027 kg m^{-3} and its viscosity is 1.2×10^{-3} kg $m^{-1}s^{-1}$. The density of air is 1.2 kg m^{-3} and its viscosity is 1.8×10^{-5} kg $m^{-1}s^{-1}$. The added mass coefficient for a circular cross-section is 1. The Strouhal number of vortex shedding is defined as $S = fD/U$, where f is the frequency of one complete cycle in Hz, D is the diameter of the circular tunnel and U is the speed of flow. For shedding behind a cylinder at a Reynolds number $Re > 1000$ the Strouhal number is approximately 0.2. The gravitational acceleration is $g = 9.81$ m s^{-2}. [60%]

END OF PAPER

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Approximate SDOF model for a beam

for an assumed vibration mode $\bar{u}(x)$, the equivalent parameters are

$$M_{eq} = \int_0^L m\,\bar{u}^2 dx \qquad K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\,\bar{u}dx + \sum_i F_i\,\bar{u}_i$$

Frequency of mode $u(x,t) = U\sin\omega t\,\bar{u}(x)$ $\quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} \quad \omega = 2\pi f$

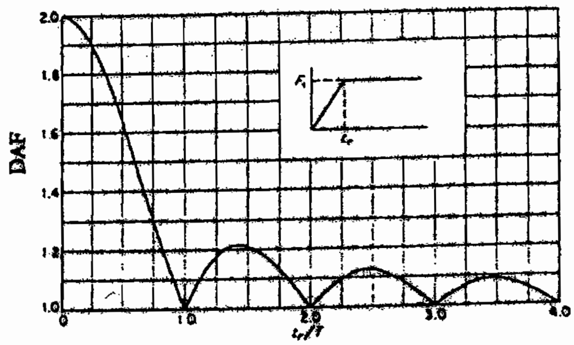
Modal analysis of simply-supported uniform beams

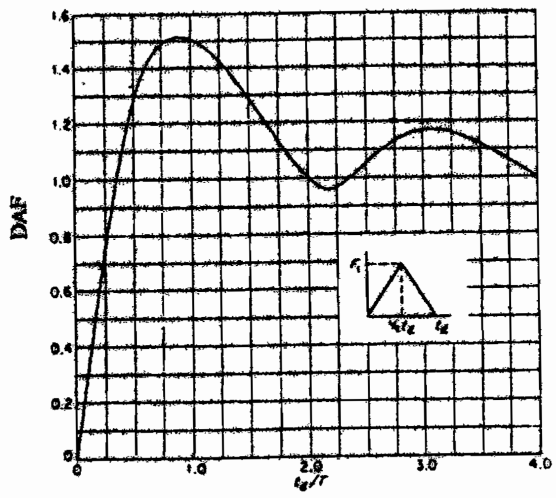
$$u_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

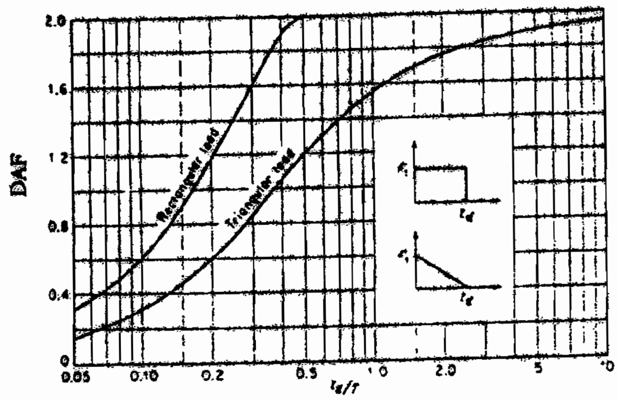
Ground motion participation factor

$$\Gamma = \frac{\int m\bar{u}dx}{\int m\bar{u}^2 dx}$$

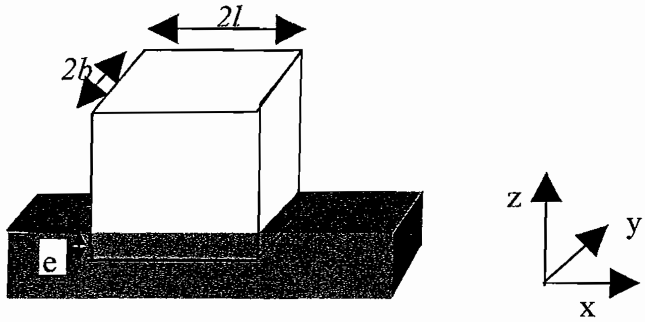
Dynamic amplification factors







Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions *2l* and *2b*, embedded to a depth *e* are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^3}{1-\nu}\left[3.2\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\frac{l}{b}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\left(\frac{l}{b}\right)^{4}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{tor} = Gb^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1+e}$$

where e is the void ratio, S_r is the degree of saturation, G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

Effective mean confining stress

$$p' = \sigma'_v \frac{(1+2K_o)}{3}$$

where σ'_v is the effective vertical stress, K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1-\nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{max} = 100 \frac{(3 - e)^2}{(1 + e)} (p')^{0.5}$$

where p′ is the effective mean confining pressure in **MPa,** e is the void ratio and Gmax is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{max}} = \frac{1}{1 + \gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a \cdot e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{\max}}{G_{\max}}$$

where

$$\tau_{\max} = \left[\left(\frac{1+K_o}{2}\sigma_v' \sin\phi'\right)^2 - \left(\frac{1-K_o}{2}\sigma_v'\right)^2\right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity v_s as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

Natural frequency of a horizontal soil layer f_n is;

$$f_n = \frac{v_s}{4H}$$

where v_s is shear wave velocity and H is the thickness of the soil layer.

SPGM
January, 2006