ENGINEERING TRIPOS PART IIB

Monday 8 May 2006 2.30 to 4

Module 4D7

CONCRETE AND MASONRY STRUCTURES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: (i) Concrete and Masonry Structures: Formula and Data Sheet (4 pages).
(ii) The Cumulative Normal Distribution Function (1 page).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) A simply-supported, precast, reinforced concrete beam spanning 4 m is loaded at mid-span by a point load P with mean value 10 kN and coefficient of variation 0.2. The flexural strength of the beam is assumed to have a mean value of 40 kNm and coefficient of variation of 0.18. Self-weight of the beam can be ignored. Assume the partial safety factors on material and load at ULS are $\gamma_m = 1.5$ and $\gamma_f = 1.4$ respectively.

Both load and resistance are initially assumed to be normally distributed.

(i) Determine the *characteristic* load effect and *design* load effect. [10%]

(ii) Determine the *characteristic* bending strength and *design* bending strength of the beam. [10%]

(iii) Would this beam be considered safe in flexure? Justify your answer. [5%]

(iv) Determine the reliability index, β and hence the probability of failure in bending of this beam. [20%]

(b) The loading P applied to this beam during its design life is now thought to be better represented by a probability density function which is uniform from 6 to 14 kN and zero elsewhere. Find the characteristic value of the load effect on the beam. [15%]

(c) The beam is to be redesigned such that a population of identical beams would have a flexural strength with a probability density distribution which is uniform from R to 1.2R and zero elsewhere.

What is the probability of failure, under the loading in part (b), for such a beam which is designed to have a characteristic strength equal to the characteristic load effect? [40%]

2 (a) List the main mechanisms of deterioration in reinforced concrete structures. [5%]

(b) Material degradation is a major concern for owners of concrete structures. Give one significant example of structural failure caused by long term deterioration. Discuss the mode of failure and the preventative measures that could and/or should have been taken to reduce the risk of such failure. [15%]

(c) Natural hazards are the most common cause of bridge collapse. List the natural hazards a bridge engineer might need to consider in design. What specific details might a designer incorporate to reduce the chance of disproportionate collapse arising from extreme loading situations? [10%]

(d) List the key factors which contribute to good quality, durable concrete and explain the significance of each. [15%]

(e) A bridge was constructed in 1960 and assumed to be initially free of chloride contamination. The design specified the steel cover to be 35 mm.

Five years after construction a core was taken into an exposed face of a bridge abutment and phenolphthalein indicator solution applied. The outer 12 mm region of concrete in the core was clear and the remaining region turned a deep pink colour.

In addition concrete dust samples were taken from this same region of the abutment at a depth of 10 mm from the surface and tested in the lab at regular intervals after construction. Two years after construction these tests showed that the chloride concentration per unit weight of cement was 0.2% ; after eight years it was 0.6%. The diffusion coefficient D for the concrete is assumed constant.

(i) Estimate the age at which corrosion of the steel will first be initiated. [40%]

(ii) What factors might affect the time to initiation and rate of corrosion? [15%]

(TURN OVER

3 A reinforced concrete beam has the uniform rectangular cross-section shown in Fig. 1, with two steel bars of diameter 25 mm near each of the shorter faces. The elastic modulus of the reinforcement is 200 GPa, and the concrete has an effective modulus 25 GPa.

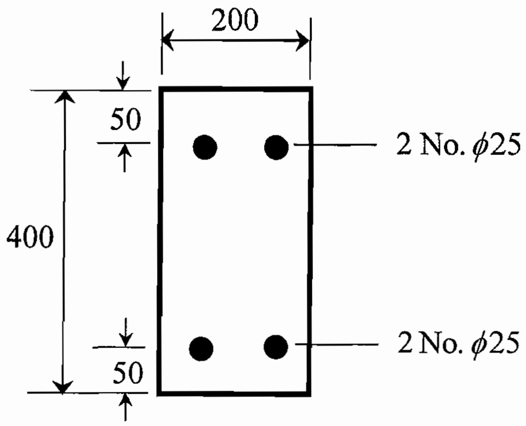


Fig. 1

(a) Calculate the second moment of area, with all materials transformed to concrete, about the major axis of bending, if the concrete is effective in both tension and compression. At what value of the applied moment would the concrete first crack, if the modulus of rupture is 4 MPa? [20%]

(b) Calculate the reduced second moment of area, if the concrete is completely ineffective in tension but linear-elastic in compression. What are the salient stresses in the materials if the beam is subjected to a bending moment of 80 kNm? Estimate the maximum crack width under this moment, assuming high-bond bars. Assume ϕ is the average bar diameter and ρ_r is the effective steel ratio $A_s / A_{c.eff}$ in the tension zone where $A_{c.eff}$ is the area of concrete of depth 2.5 times the distance from the tension face to the steel centroid. [40%]

(c) A beam with this cross-section is fixed-ended with a span of 12m between supports and carries uniformly distributed load which is slowly increased from zero. At what load would cracking first occur? It is suggested that after cracking has occurred near the supports but not at midspan, the beam may be regarded as having the cracked second moment of area within 1 m of each support, and be uncracked elsewhere. Calculate the ratio of support to midspan moment magnitudes. Comment on your result. [40%]

4 Figure 2 shows a cross-section of a Tee-beam in reinforced concrete, with concrete of design cube strength 25 MPa, longitudinal steel of design yield strength 400 MPa, and stirrups of design yield strengh 240 MPa.

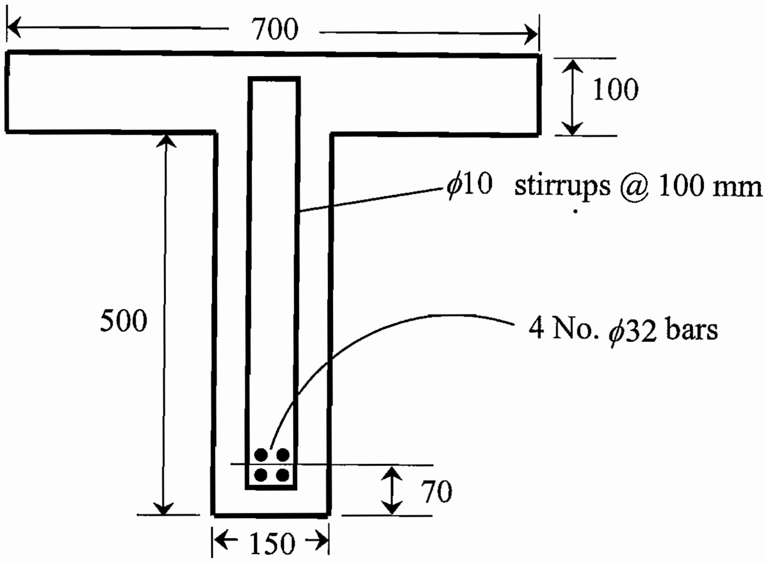


Fig. 2

(a) Assuming the resultant force on the compression zone acts at the centre of the flange, estimate the ultimate sagging moment of resistance. [10%]

(b) Taking the basic shear strength τ_{Rd} to be 0.5 MPa, estimate the ultimate shear strength V_{Rd} of the beam, using the standard design method. Discuss briefly what width should be used in calculating ρ_l. [25%]

(c) Use the truss analogy, with depth 480 mm between top and bottom chords and a series of compression struts at angle θ to the longitudinal axis, to make an alternative estimate of the ultimate shear strength of the beam. Take the effective strength of the concrete in the struts to be 0.5 times the cube strength. Comment on the relative magnitudes of the two estimates of shear strength, and briefly discuss the difference in rationale behind the two approaches to estimating shear strength. [25%]

(d) Near a simple support at the end of the beam, the reinforcement is curtailed such that only one of the ϕ32 bars is effectively anchored beyond the support. What reduction, if any, in the shear strength of the beam is predicted by each approach? [40%]

5 A reinforced concrete column for a motorway bridge has the square cross-section shown in Fig. 3 with suitable stirrups (not shown). The column is to be designed against impact from vehicles: the minimum axial compression in the column at this ultimate limit state is 2 MN.

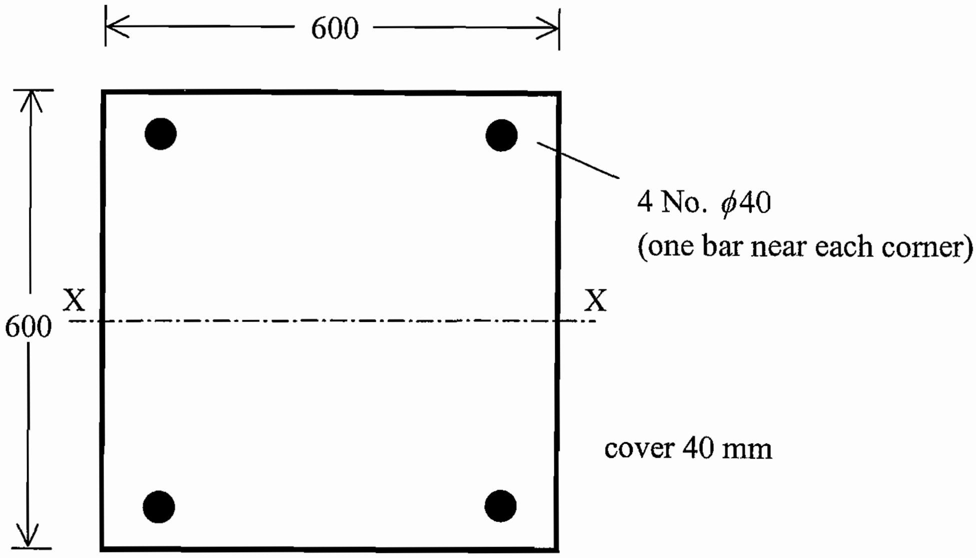


Fig. 3

The concrete has design cube strength 30 MPa, and the reinforcing bars have design yield strength 400 MPa in tension and compression.

(a) Assuming that all steel yields at failure, calculate the ultimate moment that can be developed about the XX-axis through the column centre in the presence of the minimum axial compression. If the assumption that all steel has yielded is invalid, amend the value of this moment given that the maximum compressive strain in concrete at failure is 0.0035 and the yield strain of the steel is 0.002. [20%]

(b) To increase its strength, the column is to be fully wrapped in CFRP fabric, bonded together and to the concrete surface, with longitudinal and transverse fibres.

Initially, any transverse fibres, and any longitudinal fibres in compression, may be disregarded. The CFRP bandage forms a continuous sheet 3 mm thick, with effective Young's modulus 120 GPa and design ultimate strength 1400 MPa in tension, behaving linearly up to failure.

(cont.

Considering bending failure with a neutral axis parallel to the XX-axis, and a concrete compression zone with depth 185 mm and maximum strain 0.0035, evaluate what *extra* axial force and bending moment about the column centre is contributed by the CFRP at ULS. [60%]

(c) Discuss briefly:

(i) whether it is sensible to neglect the longitudinal fibres in compression and the transverse fibres;

(ii) how to construct an interaction diagram for total force and moment on a strengthened column. [20%]

END OF PAPER

Engineering Tripos Part IIB FOURTH YEAR

Module 4D7 : Concrete and masonry structures

Formula and Data Sheet

The purpose of this sheet is to list certain relevant formulae (mostly from Eurocode 2) that are so complex that students may not remember them in full detail. Symbols used in the formulae have their usual meanings, and only minimal definitions are given here. The sheet also gives some typical numerical data.

Material variability and partial safety factors

The word 'characteristic' usually refers to a 1 in 20 standard. At SLS, usually $\gamma_m = 1.0$ on all material strengths, $\gamma_f = 1.0$ on all loads.

At ULS, usually γ_m is 1.15 for steel, 1.5 for concrete; and γ_f is 1.4 for permanent loads, 1.6 for live loads (possibly reduced for combinations of rarely-occurring loads).

The difference between two normally-distributed variables is itself normally distributed, with mean equal to the difference of means, and variance the sum of the squares of the standard deviations.

Cement paste

The density of cement particles is approx. 3.15 times that of water. On hydration, the solid products have volume approx. 1.54 times that of the hydrated cement, with a fixed gel porosity approx. 0.6 times the hydrated cement volume. This gives capillary porosity about

$\left[3.15\frac{W}{C} - 1.14h\right] \Big/ \left[1 + 3.15\frac{W}{C}\right]$ for hydration degree h : and gel/space ratio (gel volume / gel + capillaries) $2.14h \Big/ \left[h + 3.15\ {}^{W}\!/_{C} + a\right]$

Mechanical properties of concrete

Cracking strain typically 150×10^{-6}, strain at peak stress in uniaxial compression typically 0.002. Lateral confinement typically adds about 4 times the confining stress to the unconfined uniaxial strength, as well as improving ductility. In plane stress, the peak strength under biaxial compression is typically 20% greater than the uniaxial strength.

Durability considerations

Present value of some future good : $S_i \,/\, (1 + r)^i$ for stepped, or $S_i \,/\exp\,(r_c t_i)$ for continuous discounting.

Water penetration : cumulative volume uniaxial inflow / unit area is sorptivity times square root of time. On sharp-wet-front theory penetration depth is $\left\{2k\left(H + h_c\right)/\Delta n\right\}^{1/2} t^{1/2}$.

Uniaxial diffusion into homogeneous material : $\dfrac{\partial c}{\partial t} = D\dfrac{\partial^2 c}{\partial x^2}$

solution $c = c_o\left(1 - \text{erf}(z)\right),\ z = x/2\sqrt{Dt}$

Table of erf (z) :

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
erf (z)	0	0.11	0.22	0.33	0.43	0.52	0.60	0.68	
z	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	∞
erf(z)	0.74	0.80	0.84	0.88	0.91	0.93	0.95	0.97	1.00

Passivation for pH > 12 and Cl^- < 0.4% by weight cement.

Corrosion unlikely for corrosion current < 0.2 $\mu A/cm^2$, resistivity > 100 k Ω cm, half-cell potential > –200 mV (but probable for < –350 mV).

SLS : cracking

Steel $A_s > k_c \; k \; f_{ct.ef} \; A_{ct}/f_{yk}$ in tension zone, to produce multiple cracks.

Then, limitation to about 0.3 mm under quasi-permanent loads depending on exposure.

Maximum (characteristic) width $w_k = \beta . s_{rm} . \varepsilon_{sm}$ (β usually 1.7)

where spacing $s_{rm} = 50\text{mm} + 0.25 k_1 k_2 \phi/\rho_r$

with k_1 0.8 for high bond, 1.6 for plain bars; k_2 1.0 for tension
0.5 for bending.

SLS : deflection

Interpolated curvature $\kappa = (1-\xi)\,\kappa_{un} + \xi\,\kappa_{cr}$

$$\text{where} \quad \xi = 1 - \beta_1 \beta_2 \left(\frac{\sigma_{sr}}{\sigma_s}\right)^2$$

β_1 is 1.0 for high bond, 0.5 for plain bars

β_2 is 1.0 for short-term, 0.5 for sustained load

σ_{sr} is steel stress, for cracked section, but using loads which first cause cracking at the section considered. σ_s is current steel stress, calculated for cracked section.

ULS : moment and axial force

It is usual to assume failure at a cross-section to occur when the extreme-fibre compressive strain in the concrete reaches a limiting value, often $\varepsilon_{cm} = 0.0035$. The yield strain of steel ε_y of course depends on strength, as roughly f_y/E.

Initial calculations often use uniform stress of 0.6 f_{cd} on the compression zone at failure.

With these assumptions, for a singly-reinforced under-reinforced rectangular beam

$$M_u = A_s f_y d(1 - 0.5x/d); \quad x/d = \frac{A_s f_y}{0.6 f_{cd} bd};$$

over-reinforcement for $x/d > 0.5$.

For Tee beams, effective flange width b in compression is of order

$b_w + l_o/5 \leq b_{actual}$, where l_o is span between zero-moment points.

For long columns, extra deflection prior to material failure is of order

$e_2 = \frac{l_o^2}{\pi^2} \kappa_m$ where κ_m is curvature at mid-height at failure and l_o is effective length. Eurocode multiplies by further factor K, which is 1 for

$\frac{l_o}{r} > 35$, and $\frac{l_o}{20r} - 0.75$ for $15 \leq l_o/r \leq 35$,

r being radius of gyration of gross concrete section.

Shear in reinforced concrete

For unreinforced webs at ULS, shear strength in Code is

$$V_{Rd1} = b_w d \left\{ \tau_{Rd} \; k \left(1.2 + 40\rho_1\right) + 0.15 \; N/A_c \right\}$$

where ρ_1 is A_s/bd for tension steel, τ_{Rd} is tabulated function of f_{cd}, and $k = 1.6 - d$ (metres) ≥ 1 (and is 1 for more than 50% steel curtailment).

In 'standard' design method, for $V_{sd} > V_{Rd1}$

$V_{Rd} = V_{Rd1} + V_{Rd3} < V_{Rd2}$ (tabulated in Eurocode)

Stirrup term V_{Rd3} follows from truss analogy with 45° struts and "web" depth 90% of effective;

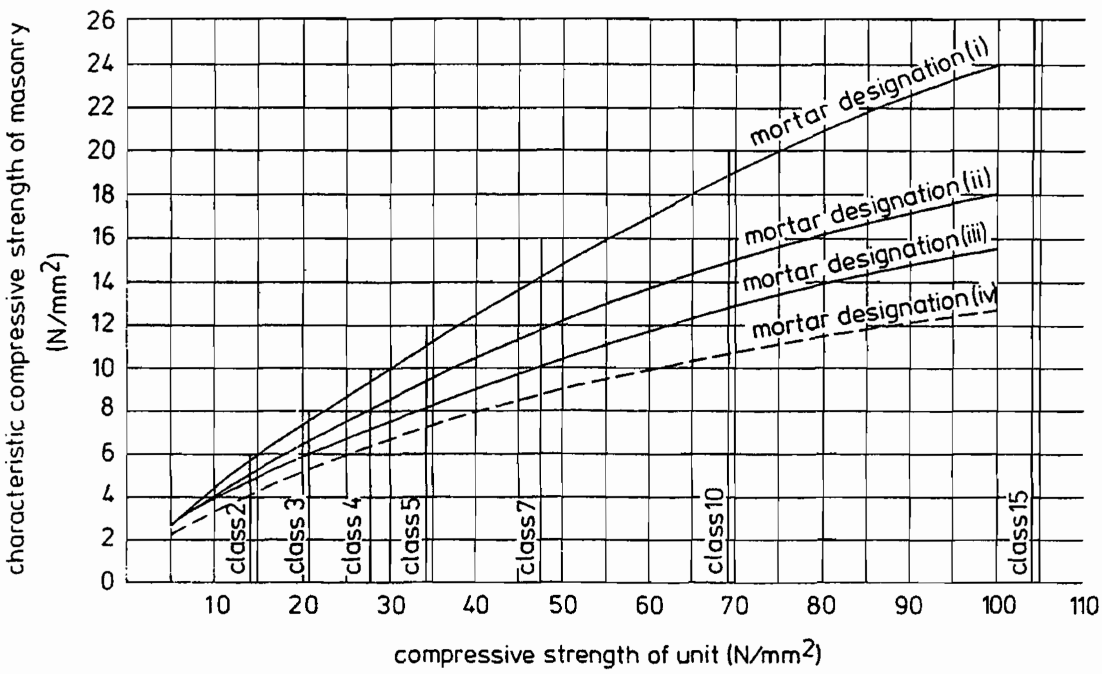
$$V_{Rd3} = A_{sw} \; f_{wyd} \; (0.9d)/s.$$

Torsion at ULS

Based on truss analogy with variable strut angle, for a thin-walled box section; shear flow

$$q = f_{yd} \left\{ \left(A_w/s\right)\left(\Sigma A_\ell/u\right) \right\}^{1/2} \; ; \; \sigma \leq \; \nu \; f_{cd} \, .$$

Masonry walls in compression



interpolation for classes of loadbearing bricks not shown on the graph may be used for average crushing strengths intermediate between those given on the graph, as described in clause 10 of BS 3921: 1985 and clause 7 of BS 187: 1978.

Figure 5.6(a) Characteristic compressive strength, f_k, of brick masonry (see Table 5.4)

Note. Mortar designations in the figure above range from (i) a strong mix of cement and comparatively little sand with 28 day site compressive cube strength of around 11 MPa, through (ii) and (iii) with strengths around 4.5 and 2.5 MPa respectively, to (iv) soft mortars e.g. of cement, lime and plentiful sand or cement, plasticizer and plentiful sand, with strength around 1.0 MPa.

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}\,dx \quad \text{FOR} \quad 0{\cdot}00 \le u \le 4{\cdot}99.$$

u	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	$\cdot9^{2}0097$	$\cdot9^{2}0358$	$\cdot9^{2}0613$	$\cdot9^{2}0863$	$\cdot9^{2}1106$	$\cdot9^{2}1344$	$\cdot9^{2}1576$
2·4	$\cdot9^{2}1802$	$\cdot9^{2}2024$	$\cdot9^{2}2240$	$\cdot9^{2}2451$	$\cdot9^{2}2656$	$\cdot9^{2}2857$	$\cdot9^{2}3053$	$\cdot9^{2}3244$	$\cdot9^{2}3431$	$\cdot9^{2}3613$
2·5	$\cdot9^{2}3790$	$\cdot9^{2}3963$	$\cdot9^{2}4132$	$\cdot9^{2}4297$	$\cdot9^{2}4457$	$\cdot9^{2}4614$	$\cdot9^{2}4766$	$\cdot9^{2}4915$	$\cdot9^{2}5060$	$\cdot9^{2}5201$
2·6	$\cdot9^{2}5339$	$\cdot9^{2}5473$	$\cdot9^{2}5604$	$\cdot9^{2}5731$	$\cdot9^{2}5855$	$\cdot9^{2}5975$	$\cdot9^{2}6093$	$\cdot9^{2}6207$	$\cdot9^{2}6319$	$\cdot9^{2}6427$
2·7	$\cdot9^{2}6533$	$\cdot9^{2}6636$	$\cdot9^{2}6736$	$\cdot9^{2}6833$	$\cdot9^{2}6928$	$\cdot9^{2}7020$	$\cdot9^{2}7110$	$\cdot9^{2}7197$	$\cdot9^{2}7282$	$\cdot9^{2}7365$
2·8	$\cdot9^{2}7445$	$\cdot9^{2}7523$	$\cdot9^{2}7599$	$\cdot9^{2}7673$	$\cdot9^{2}7744$	$\cdot9^{2}7814$	$\cdot9^{2}7882$	$\cdot9^{2}7948$	$\cdot9^{2}8012$	$\cdot9^{2}8074$
2·9	$\cdot9^{2}8134$	$\cdot9^{2}8193$	$\cdot9^{2}8250$	$\cdot9^{2}8305$	$\cdot9^{2}8359$	$\cdot9^{2}8411$	$\cdot9^{2}8462$	$\cdot9^{2}8511$	$\cdot9^{2}8559$	$\cdot9^{2}8605$
3·0	$\cdot9^{2}8650$	$\cdot9^{2}8694$	$\cdot9^{2}8736$	$\cdot9^{2}8777$	$\cdot9^{2}8817$	$\cdot9^{2}8856$	$\cdot9^{2}8893$	$\cdot9^{2}8930$	$\cdot9^{2}8965$	$\cdot9^{2}8999$
3·1	$\cdot9^{3}0324$	$\cdot9^{3}0646$	$\cdot9^{3}0957$	$\cdot9^{3}1260$	$\cdot9^{3}1553$	$\cdot9^{3}1836$	$\cdot9^{3}2112$	$\cdot9^{3}2378$	$\cdot9^{3}2636$	$\cdot9^{3}2886$
3·2	$\cdot9^{3}3129$	$\cdot9^{3}3363$	$\cdot9^{3}3590$	$\cdot9^{3}3810$	$\cdot9^{3}4024$	$\cdot9^{3}4230$	$\cdot9^{3}4429$	$\cdot9^{3}4623$	$\cdot9^{3}4810$	$\cdot9^{3}4991$
3·3	$\cdot9^{3}5166$	$\cdot9^{3}5335$	$\cdot9^{3}5499$	$\cdot9^{3}5658$	$\cdot9^{3}5811$	$\cdot9^{3}5959$	$\cdot9^{3}6103$	$\cdot9^{3}6242$	$\cdot9^{3}6376$	$\cdot9^{3}6505$
3·4	$\cdot9^{3}6631$	$\cdot9^{3}6752$	$\cdot9^{3}6869$	$\cdot9^{3}6982$	$\cdot9^{3}7091$	$\cdot9^{3}7197$	$\cdot9^{3}7299$	$\cdot9^{3}7398$	$\cdot9^{3}7493$	$\cdot9^{3}7585$
3·5	$\cdot9^{3}7674$	$\cdot9^{3}7759$	$\cdot9^{3}7842$	$\cdot9^{3}7922$	$\cdot9^{3}7999$	$\cdot9^{3}8074$	$\cdot9^{3}8146$	$\cdot9^{3}8215$	$\cdot9^{3}8282$	$\cdot9^{3}8347$
3·6	$\cdot9^{3}8409$	$\cdot9^{3}8469$	$\cdot9^{3}8527$	$\cdot9^{3}8583$	$\cdot9^{3}8637$	$\cdot9^{3}8689$	$\cdot9^{3}8739$	$\cdot9^{3}8787$	$\cdot9^{3}8834$	$\cdot9^{3}8879$
3·7	$\cdot9^{3}8922$	$\cdot9^{3}8964$	$\cdot9^{4}0039$	$\cdot9^{4}0426$	$\cdot9^{4}0799$	$\cdot9^{4}1158$	$\cdot9^{4}1504$	$\cdot9^{4}1838$	$\cdot9^{4}2159$	$\cdot9^{4}2468$
3·8	$\cdot9^{4}2765$	$\cdot9^{4}3052$	$\cdot9^{4}3327$	$\cdot9^{4}3593$	$\cdot9^{4}3848$	$\cdot9^{4}4094$	$\cdot9^{4}4331$	$\cdot9^{4}4558$	$\cdot9^{4}4777$	$\cdot9^{4}4988$
3·9	$\cdot9^{4}5190$	$\cdot9^{4}5385$	$\cdot9^{4}5573$	$\cdot9^{4}5753$	$\cdot9^{4}5926$	$\cdot9^{4}6092$	$\cdot9^{4}6253$	$\cdot9^{4}6406$	$\cdot9^{4}6554$	$\cdot9^{4}6696$
4·0	$\cdot9^{4}6833$	$\cdot9^{4}6964$	$\cdot9^{4}7090$	$\cdot9^{4}7211$	$\cdot9^{4}7327$	$\cdot9^{4}7439$	$\cdot9^{4}7546$	$\cdot9^{4}7649$	$\cdot9^{4}7748$	$\cdot9^{4}7843$
4·1	$\cdot9^{4}7934$	$\cdot9^{4}8022$	$\cdot9^{4}8106$	$\cdot9^{4}8186$	$\cdot9^{4}8263$	$\cdot9^{4}8338$	$\cdot9^{4}8409$	$\cdot9^{4}8477$	$\cdot9^{4}8542$	$\cdot9^{4}8605$
4·2	$\cdot9^{4}8665$	$\cdot9^{4}8723$	$\cdot9^{4}8778$	$\cdot9^{4}8832$	$\cdot9^{4}8882$	$\cdot9^{4}8931$	$\cdot9^{4}8978$	$\cdot9^{5}0226$	$\cdot9^{5}0655$	$\cdot9^{5}1066$
4·3	$\cdot9^{5}1460$	$\cdot9^{5}1837$	$\cdot9^{5}2199$	$\cdot9^{5}2545$	$\cdot9^{5}2876$	$\cdot9^{5}3193$	$\cdot9^{5}3497$	$\cdot9^{5}3788$	$\cdot9^{5}4066$	$\cdot9^{5}4332$
4·4	$\cdot9^{5}4587$	$\cdot9^{5}4831$	$\cdot9^{5}5065$	$\cdot9^{5}5288$	$\cdot9^{5}5502$	$\cdot9^{5}5706$	$\cdot9^{5}5902$	$\cdot9^{5}6089$	$\cdot9^{5}6268$	$\cdot9^{5}6439$
4·5	$\cdot9^{5}6602$	$\cdot9^{5}6759$	$\cdot9^{5}6908$	$\cdot9^{5}7051$	$\cdot9^{5}7187$	$\cdot9^{5}7318$	$\cdot9^{5}7442$	$\cdot9^{5}7561$	$\cdot9^{5}7675$	$\cdot9^{5}7784$
4·6	$\cdot9^{5}7888$	$\cdot9^{5}7987$	$\cdot9^{5}8081$	$\cdot9^{5}8172$	$\cdot9^{5}8258$	$\cdot9^{5}8340$	$\cdot9^{5}8419$	$\cdot9^{5}8494$	$\cdot9^{5}8566$	$\cdot9^{5}8634$
4·7	$\cdot9^{5}8699$	$\cdot9^{5}8761$	$\cdot9^{5}8821$	$\cdot9^{5}8877$	$\cdot9^{5}8931$	$\cdot9^{5}8983$	$\cdot9^{6}0320$	$\cdot9^{6}0789$	$\cdot9^{6}1235$	$\cdot9^{6}1661$
4·8	$\cdot9^{6}2067$	$\cdot9^{6}2453$	$\cdot9^{6}2822$	$\cdot9^{6}3173$	$\cdot9^{6}3508$	$\cdot9^{6}3827$	$\cdot9^{6}4131$	$\cdot9^{6}4420$	$\cdot9^{6}4696$	$\cdot9^{6}4958$
4·9	$\cdot9^{6}5208$	$\cdot9^{6}5446$	$\cdot9^{6}5673$	$\cdot9^{6}5889$	$\cdot9^{6}6094$	$\cdot9^{6}6289$	$\cdot9^{6}6475$	$\cdot9^{6}6652$	$\cdot9^{6}6821$	$\cdot9^{6}6981$

Example: $\Phi(3{\cdot}57) = \cdot9^{3}8215 = 0{\cdot}9998215.$