ENGINEERING TRIPOS   PART IIB

Wednesday 26 April 2006   2.30 to 4

Module 4F1

CONTROL SYSTEM DESIGN

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Formulae sheet (3 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed
Supplementary pages: Two extra copies of Fig. 2 (Question 2).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Fig. 1 shows the block diagram of a two-degree-of-freedom control system where the plant to be controlled has transfer function $G(s)$.

(i) Explain why the term "two-degree-of-freedom" is used. [10%]

(ii) Assuming that the return-ratio has already been designed, state but do not prove the minimum conditions which apply to the transfer function relating $y(t)$ to $r(t)$ for an internally stable control system. [10%]

(b) A position control system is to be designed for an inertial load in which the plant transfer function is assumed to be $G(s) = 1/s^2$.

(i) In the control scheme of Fig. 1 an initial choice of $H(s) = 1$ is made. If $e(t) = r(t) - y(t)$ show that the Laplace transform of $e(t)$, $E(s)$, must have a zero at the origin when $r(t)$ is a step input and the closed-loop is internally stable. [10%]

(ii) By considering the definition of the Laplace transform, or otherwise, show that

$$\int_0^\infty e(t)\,dt = 0,$$

under the assumptions of part (b)(i). [15%]

(iii) Explain why $y(t)$ must always experience overshoot when $r(t)$ is a step input for this plant when $H(s) = 1$. [10%]

(c) A control system for the plant of part (b) is to be designed in which $y(t)$ does not exhibit overshoot when $r(t)$ is a step input. A desired transfer function $D(s) = 1/(s+1)^2$ relating $y(t)$ to $r(t)$ is selected.

(i) State why $D(s)$ is an admissible transfer function and show by direct calculation that it guarantees the required overshoot property. [20%]

(ii) Complete the design of a two-degree-of-freedom control system with the chosen $D(s)$. [Hint: a useful first step might be to show that a lead compensator is stabilising for $G(s)$.] [25%]

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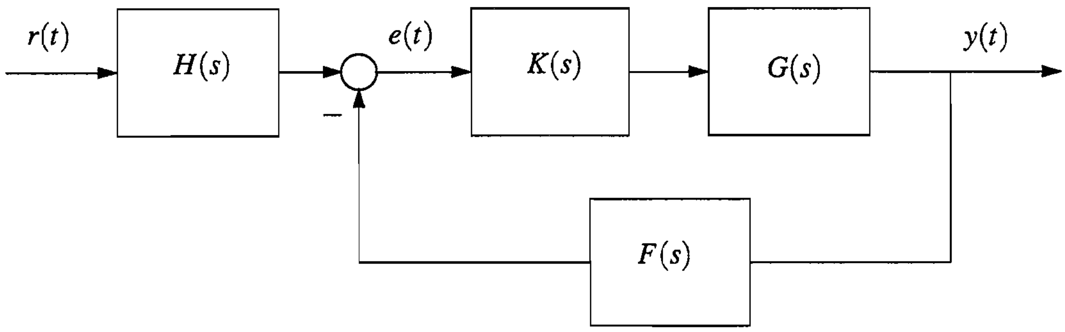

Fig. 1

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2 Fig. 2 is the Bode diagram of a system $G(s)$ for which a feedback compensator $K(s)$ is to be designed. It is known that $G(s)$ has precisely one zero in the right half plane, and it may be assumed that $G(s)$ is a real-rational transfer function.

(a) (i) Sketch on a copy of Fig. 2 the expected phase of $G(j\omega)$ if $G(s)$ were stable and minimum phase. [15%]

(ii) Show that $G(s)$ must have one right half plane pole, estimate the location of the right half plane zero and pole and give the form of the all-pass factor corresponding to the excess phase. [20%]

(iii) Comment on any limitations that this might impose on the achievable crossover frequency. [10%]

(b) If a constant controller $K(s) = k$ is used, determine the number of right half plane poles of the closed-loop system as k varies over positive and negative values. [Hint: use a sketch of the Nyquist diagram.] [20%]

(c) Show that a compensator $K(s)$ having one pole and one zero can be designed to achieve the following specifications:

A: internal stability of the closed-loop,

B: $|G(j\omega)K(j\omega)| = 1$ at $\omega = 10$,

C: a phase margin of at least 40°.

Show on another copy of Fig. 2 the effect of this compensator on the return-ratio transfer function. [35%]

Two copies of Fig. 2 are provided on separate sheets. These should be handed in with your answers.

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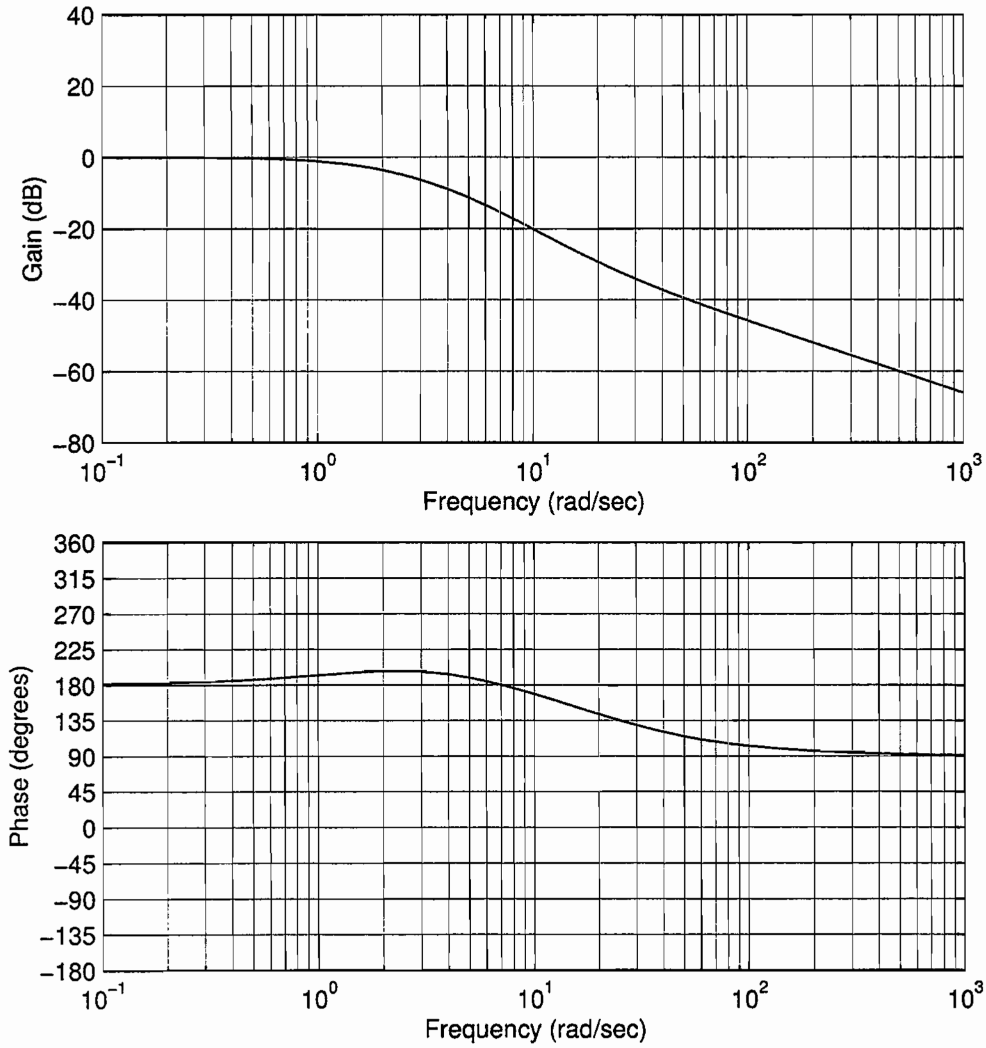

Fig. 2

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3 Consider the plant

$$G(s) = \frac{(s+\alpha)(s^2+0.01s+1)}{(s+1)^4}.$$

Let $S(s)$ and $T(s)$ denote the sensitivity function and the complementary sensitivity function, respectively.

(a) Suppose $\alpha = -1$ and a stabilising controller is required to achieve the following specifications:

A: $|S(j\omega)| < \varepsilon$ for $0 \leq \omega \leq 1$,

B: $|S(j\omega)| < 2$ for $1 \leq \omega \leq 10$,

C: $|T(j\omega)| < 0.01$ for all $\omega \geq 10$.

Find a positive lower bound for ε. [50%]

(b) Suppose $\alpha = 1$ and a stabilising controller is required to achieve the following specifications:

A: $|S(j\omega)| < \varepsilon$ for $0 \leq \omega \leq 1$,

D: $|S(j\omega)| \leq 1$ for all ω.

(i) Show that these specifications are achievable for any $\varepsilon > 0$. [Hint: you may find it useful to work directly with a candidate return ratio $L(s)$.] [30%]

(ii) Comment on any practical problems that might be experienced if ε is taken to be very small. [20%]

END OF PAPER

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Formulae sheet for Module 4F1: Control System Design

To be available during the examination.

1 Terms

For the standard feedback system shown below, the **Return-Ratio Transfer Function** $L(s)$ is given by

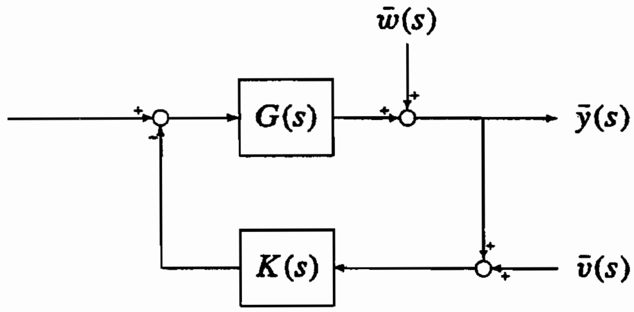
$$L(s) = G(s)K(s),$$

the **Sensitivity Function** $S(s)$ is given by

$$S(s) = \frac{1}{1+G(s)K(s)}$$

and the **Complementary Sensitivity Function** $T(s)$ is given by

$$T(s) = \frac{G(s)K(s)}{1+G(s)K(s)}$$



The closed-loop system is called **Internally Stable** if each of the *four* closed-loop transfer functions

$$\frac{1}{1+G(s)K(s)}, \quad \frac{G(s)K(s)}{1+G(s)K(s)}, \quad \frac{K(s)}{1+G(s)K(s)}, \quad \frac{G(s)}{1+G(s)K(s)}$$

are stable (which is equivalent to $S(s)$ being stable and there being no right half plane pole/zero cancellations between $G(s)$ and $K(s)$).

A transfer function is called **real-rational** if it can be written as the ratio of two polynomials in s, the coefficients of each of which are purely real.

2 Phase-lead compensators

The phase-lead compensator

$$K(s) = \alpha \frac{s+\omega_c/\alpha}{s+\omega_c\alpha}, \quad \alpha > 1$$

achieves its maximum phase advance at $\omega = \omega_c$, and satisfies:

$$|K(j\omega_c)| = 1, \quad \text{and} \quad \angle K(j\omega_c) = 2\arctan\alpha - 90°.$$

3 The Bode Gain/Phase Relationship

If

1. $L(s)$ is a real-rational function of s,
2. $L(s)$ has no poles or zeros in the *open* RHP ($\mathrm{Re}(s) > 0$) and
3. satisfies the normalization condition $L(0) > 0$.

then

$$\boxed{\angle L(j\omega_0) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{d}{dv}\log|L(j\omega_0 e^v)| \log\coth\frac{|v|}{2}\,dv}$$

Note that

$$\log\coth\frac{|v|}{2} = \log\left|\frac{\omega+\omega_0}{\omega-\omega_0}\right|, \text{ where } \omega = \omega_0 e^v.$$

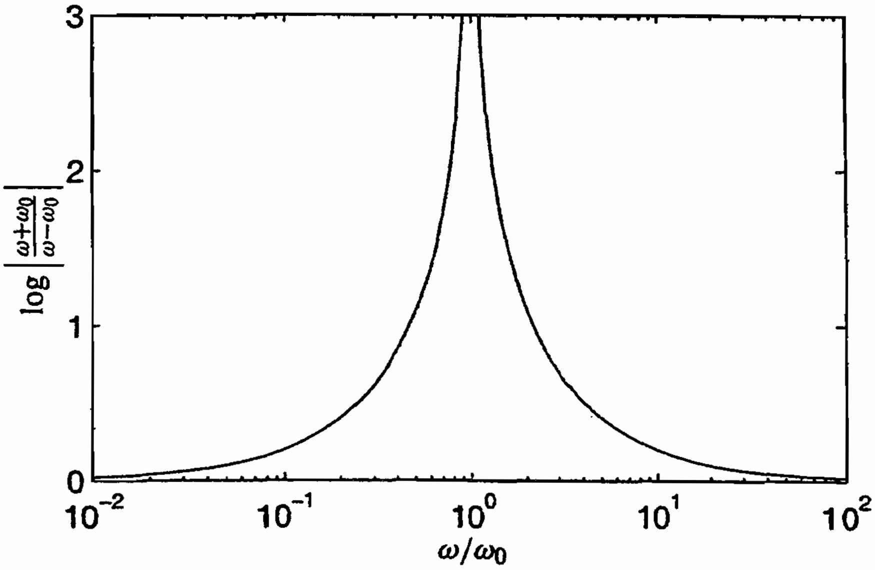


Figure 1:

If the slope of $L(j\omega)$ is approximately constant for a sufficiently wide range of frequencies around $\omega = \omega_0$ we get the *approximate form of the Bode Gain/Phase Relationship*

$$\angle L(j\omega_0) \approx \frac{\pi}{2}\left.\frac{d\log|L(j\omega_0 e^v|)}{dv}\right|_{\omega=\omega_0}.$$

4 The Poisson Integral

If $H(s)$ is a real-rational function of s which has no poles or zeros in $\mathrm{Re}(s) > 0$, then if $s_0 = \sigma_0 + j\omega_0$ with $\sigma_0 > 0$

$$\boxed{\log H(s_0) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sigma_0}{\sigma_0^2 + (\omega - \omega_0)^2} \log H(j\omega)\, d\omega}$$

and

$$\boxed{\log |H(s_0)| = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\cosh v \cos\theta}{\sinh^2 v + \cos^2\theta} \log \left|H\big(j|s_0|e^{v}\big)\right| dv}$$

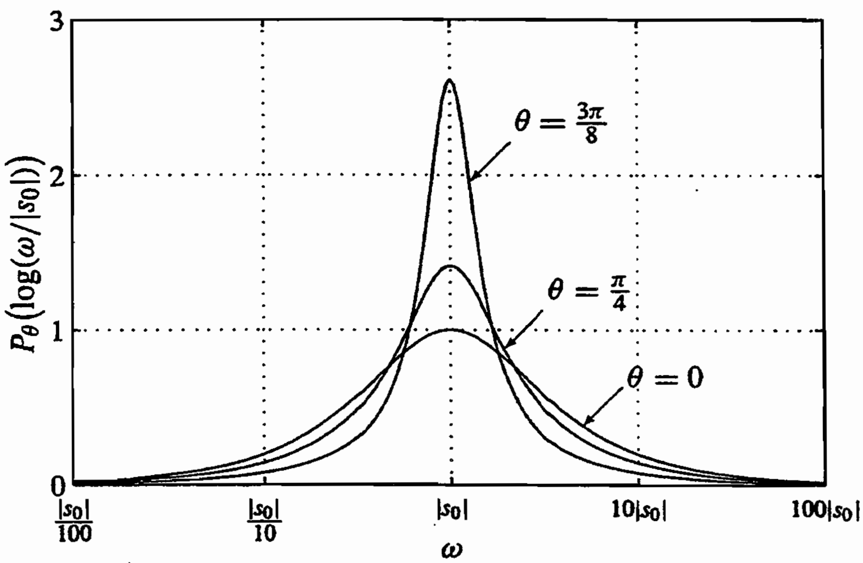
where $v = \log\left(\frac{\omega}{|s_0|}\right)$ and $\theta = \angle(s_0)$. Note that, if s_0 is real, so $\angle s_0 = 0$, then

$$\frac{\cosh v \cos\theta}{\sinh^2 v + \cos^2\theta} = \frac{1}{\cosh v}.$$

We define

$$P_\theta(v) = \frac{\cosh v \cos\theta}{\sinh^2 v + \cos^2\theta}$$

and give graphs of P_θ below.



The indefinite integral is given by

$$\int P_\theta(v)\, dv = \arctan\left(\frac{\sinh v}{\cos\theta}\right)$$

and

$$\frac{1}{\pi}\int_{-\infty}^{\infty} P_\theta(v)\, dv = 1 \quad \text{for all } \theta.$$

G. Vinnicombe
M.C. Smith
November 2002

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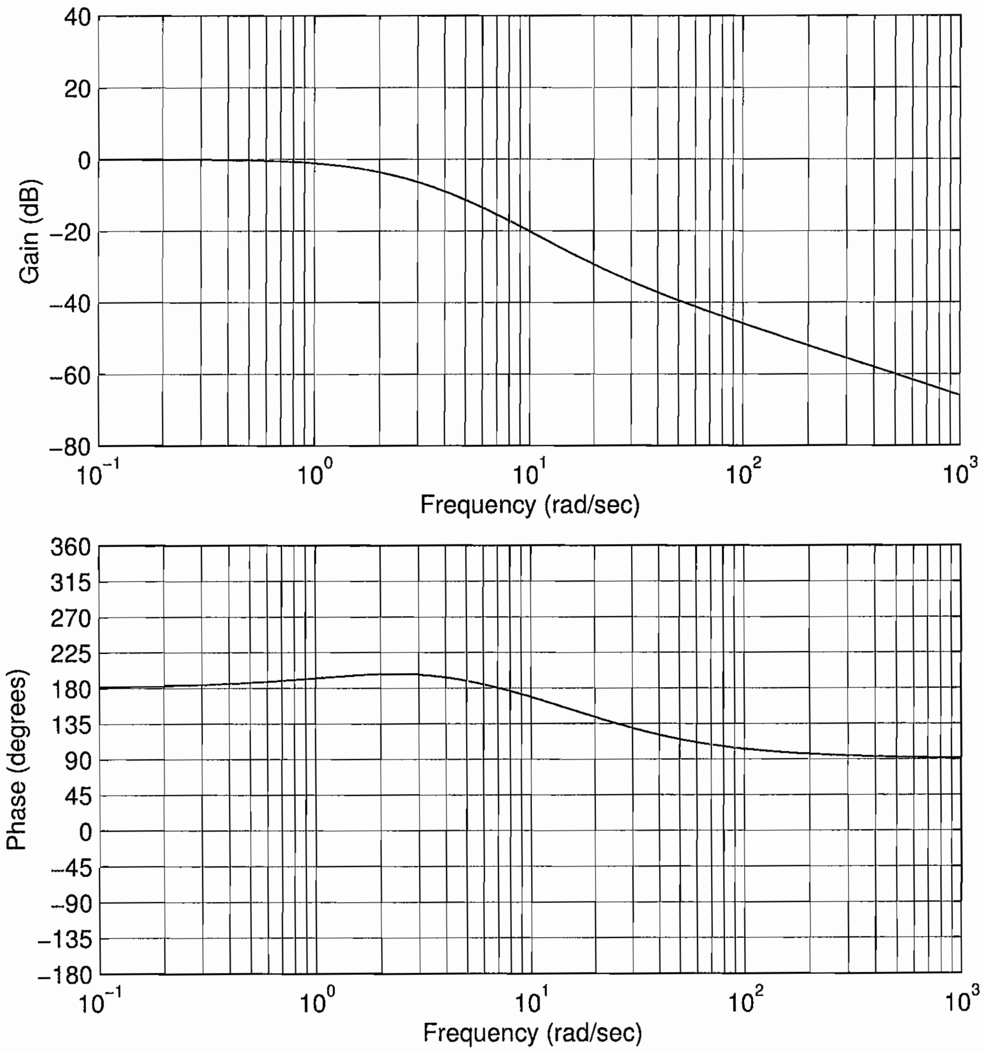
Wednesday 26 April 2006, Module 4F1, Question 2.



Extra copy of Fig. 2: Bode diagram of $G(s)$ for Question 2.

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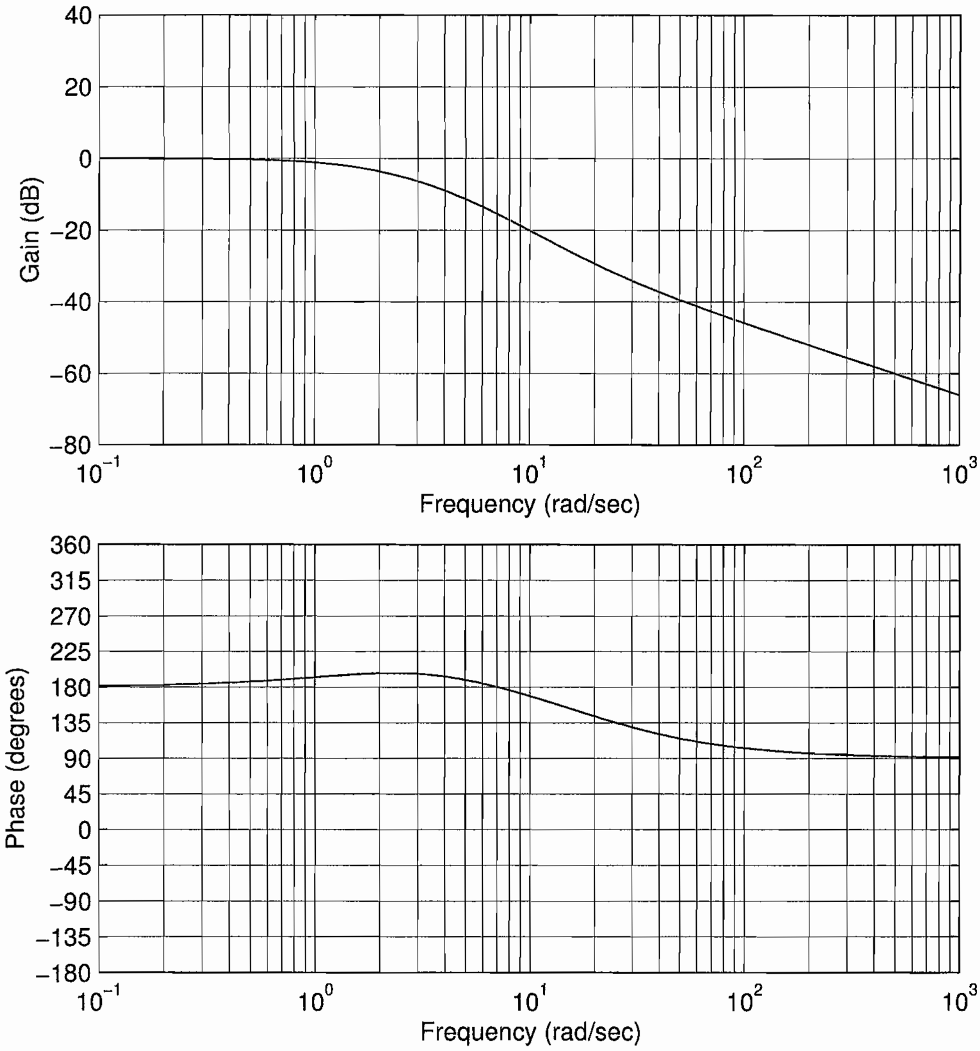
Wednesday 26 April 2006, Module 4F1, Question 2.



Extra copy of Fig. 2: Bode diagram of $G(s)$ for Question 2.