ENGINEERING TRIPOS PART IIB

Monday 24 April 2006 2.30 to 4

Module 4F6

SIGNAL DETECTION AND ESTIMATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Using Bayesian methods, prove the *Neyman-Fisher Factorisation* theorem. [20%]

The *scalar exponential family* of probability density functions for a random variable x may be written as

$$p(x|\theta) = \exp(A(\theta)B(x) + C(x) + D(\theta))$$

If data $x(n)$ for $n = 0, 1, 2, ..., N-1$ are observed which are *iid* and whose probability density function belongs to this family, show that a *sufficient statistic*, $T(x)$, for the parameter θ is given by

$$T(x) = \sum_{n=0}^{N-1} B(x(n))$$

[40%]

Show that both the Gaussian and the exponential probability density functions belong to the *scalar exponential family* and that the sufficient statistics in both cases are given by

$$T(x) = \sum_{n=0}^{N-1} x(n)$$

. [40%]

2 Outline a proof of the *Cramer-Rao Lower Bound* inequality and show that

$$E\left(\left(\frac{\partial \ln p(x|\theta)}{\partial \theta}\right)^2\right) = -E\left(\frac{\partial^2 \ln p(x|\theta)}{\partial \theta^2}\right)$$

using the standard notation. [35%]

Calculate the Cramer-Rao lower bounds for the estimation of the slope and intercept of a straight line model fitted to data, $d(n)$, given by

$$d(n) = A + Bn + w(n)$$

where $n = 0, 1, 2, 3, ..., N-1$ and $w(n)$ is white Gaussian noise of variance σ^2.

[30%]

Show that for $N \geq 3$ it is easier to estimate B than it is to estimate A. [35%]

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3 Explain the *Neyman-Pearson decision rule* and explain how the threshold of the detector is related to the slope of the *Receiver Operator Characteristic* (ROC) curve. [40%]

It is required to detect a signal given by

$$s(n) = A + Bn$$

where $n = 0, 1, 2, ..., N-1$, in additive white Gaussian noise of variance σ^2 and where A and B are known.

Show that the data may be written in the form of a *General Linear Model* and determine the *Neyman-Pearson detector* for this problem. [60%]

4 Derive an expression for the *Maximum A-Posteriori* (MAP) likelihood ratio test to decide between two alternative hypotheses H_0 and H_1 based on a vector y of N observations. [30%]

Obtain an expression for the average error probability of the test. [20%]

In a particular communication system, the signal vector

$$s_0 = \left[s_0(1) \quad s_0(2) \quad \ldots \quad s_0(N)\right]^T$$

is used to represent binary 0 and the signal vector

$$s_1 = \left[s_1(1) \quad s_1(2) \quad \ldots \quad s_1(N)\right]^T$$

is used to represent binary 1. The transmission system introduces additive zero-mean white Gaussian noise with variance σ^2 and the binary symbols 0 and 1 are equiprobable.

Derive an expression for the MAP detector based on a measurement vector y at the receiver input and show that the form of the detector is that of comparing the vector product $y^T(s_1 - s_0)$ with a threshold value. [30%]

Discuss briefly how the detector could be extended to deal with coloured channel noise with a covariance matrix C and say what is meant by *pre-whitening*. [20%]

END OF PAPER