ENGINEERING TRIPOS PART IIB

Monday 1 May 2006 9 to 10.30

Module 4F7

DIGITAL FILTERS AND SPECTRUM ESTIMATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 In the standard adaptive filtering problem we have an input signal $U = \{u(n)\}_{n=0}^{\infty}$, a reference signal $D = \{d(n)\}_{n=0}^{\infty}$, and a Finite Impulse Response (FIR) filter $H = \{h(m)\}_{m=0}^{M}$. This filter is applied to the input signal to produce an output signal $Y = \{y(n)\}_{n=0}^{\infty}$, i.e.

$$y(n) = [Hu](n) = \sum_{m=0}^{M} h(m)u(n-m).$$

The aim is to design H such that the output signal Y is as "close" as possible to D.

(a) Describe how the general adaptive filtering scenario above may be formulated as a Wiener filtering problem. As part of this description you should define the error criterion used and explain (without detailed calculations) the steps involved in deriving a solution. [40%]

(b) For each of the following two cases, describe how the problem may be formulated as an adaptive filtering problem, defining (with the aid of diagrams) appropriate reference signals, input signals and error signals. Detail any assumptions or approximations involved.

(i) Let $X = \{x(n)\}_{n=0}^{\infty}$ be a sequence of independent and identically distributed random symbols such that

$$\Pr\{x(n) = 1\} = \Pr\{x(n) = -1\} = 0.5.$$

These symbols are transmitted through a communication channel H_{channel} which distorts the transmitted symbols to produce

$$z(n) = [H_{\text{channel}}x](n).$$

The aim is to design a FIR filter to recover the transmitted symbols as accurately as possible. [30%]

(ii) In an echo cancellation problem for telephony, the local (near) user has a hands-free unit comprising a microphone and loudspeaker. The voice of the far speaker, coming out of the loudspeaker, is reflected by the room back to the microphone (the echo), and from there transmitted back to the far speaker. The echo is annoying to the far speaker and the aim is thus to cancel the echo. [30%]

2 (a) Consider two random variables x and y. It is desired to find a linear estimator of x given y, i.e. $\widehat{x} = ay$ where scalar a is to be determined.

By defining an appropriate inner product and norm for random variables, express the Wiener solution to this problem in vector space form (i.e. in terms of the inner products and norms you have defined). State also the orthogonality property that is satisfied by this solution. [30%]

(b) We now have a collection of random variables $\{y_1, \ldots, y_t\}$ and it is desired to find a linear estimator of x, of the form

$$\widehat{x} = \sum_{i=1}^{t} a_i y_i$$

If t is increased by 1, we would like to update the solution for $\{y_1, \ldots, y_t\}$ to $\{y_1, \ldots, y_t, y_{t+1}\}$ using a recursive update. Describe the Gram-Schmidt procedure for solving this problem. [40%]

(c) The standard formulation for the solution to the problem in part (b) is to find $\mathbf{a}_t$ by minimising

$$E\left\{\left(x - \sum_{i=1}^{t} a_i y_i\right)^2\right\} = E\left\{\left(x - \mathbf{a}_t^{\mathrm{T}} \mathbf{y}_t\right)^2\right\}$$

where $\mathbf{a}_t^{\mathrm{T}} = [a_1, \ldots, a_t]$ and $\mathbf{y}_t^{\mathrm{T}} = [y_1, \ldots, y_t]$. Derive an explicit solution to this problem. [30%]

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3 (a) Suppose that a single frequency signal having normalised frequency Ω and amplitude a is buried in independent, uncorrelated, complex Gaussian noise with variance σ_e^2, i.e.

$$x_n = a\exp(jn\Omega) + e_n = a g_n(\Omega) + e_n$$

We observe a sequence of N data points from this process, $\mathbf{x} = \{x_0, x_1, ..., x_{N-1}\}$. Show that the maximum likelihood solution for amplitude and frequency values can be obtained by minimising the total squared error: [25%]

$$J(a,\Omega) = \sum_{n=0}^{N-1} |x_n - a\exp(jn\Omega)|^2$$

(b) Hence show that the maximum likelihood solution for a is given by

$$a^{ML} = \frac{\mathbf{g}(\Omega)^H\mathbf{x}}{\mathbf{g}(\Omega)^H\mathbf{g}(\Omega)}$$

where $\mathbf{g}(\Omega) = [g_0(\Omega), g_1(\Omega), ..., g_{N-1}(\Omega)]^T$. [40%]

(c) Finally, show that the maximum likelihood solution for Ω is found by minimising

$$J(a^{ML},\Omega) = \mathbf{x}^H\mathbf{x} - \frac{|\mathbf{g}(\Omega)^H\mathbf{x}|^2}{\mathbf{g}(\Omega)^H\mathbf{g}(\Omega)}$$

Relate this final result to the standard periodogram power spectrum estimator, and comment on this relationship. [35%]

(Note that $\mathbf{g}(\Omega)^H$ denotes the Hermitian transpose of $\mathbf{g}(\Omega)$.)

4 From measurements of a stationary random process $\{x_n\}$, the autocorrelation sequence is estimated as

$$\hat{R}_{XX}[0] = 1,\ \hat{R}_{XX}[1] = 0.9,\ \hat{R}_{XX}[2] = 0.9^2, \ldots \hat{R}_{XX}[l] = 0.9^l, \ldots$$

(a) Determine the first order ($P = 1$) autoregressive model which corresponds to this estimated autocorrelation sequence and show that the corresponding estimated power spectrum is: [30%]

$$\hat{S}_X(e^{j\Omega}) = \frac{1 - 0.9^2}{|1 - 0.9e^{-j\Omega}|^2}$$

(b) Briefly describe the correlogram method for power spectrum estimation from estimated autocorrelation data. [10%]

Obtain and simplify an expression for the correlogram estimate of the power spectrum from the same data as above, based on $2L + 1$ estimated autocorrelation values (i.e. using correlation lags $-L$ to $+L$). [30%]

(c) What relationship would you expect between the two power spectrum estimates obtained in parts (a) and (b)? You should consider both the case of medium and very large values of L. [30%]

END OF PAPER