ENGINEERING TRIPOS PART IIB

Wednesday 26 April 2006 9 to 10.30

Module 4F9

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

This paper consists of three sections.

Answer not more than **one** *question from each section.*

Answers to questions in each section should be tied together and handed in separately.

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A *Medical Image Acquisition*

*Answer not more than **one** question from this section*

1 (a) Describe what is being represented in each of the following medical images. Comment on the properties of the material being scanned that influence the image produced.

(i) A planar X-ray.

(ii) A slice through an X-ray CT reconstruction.

(iii) A slice through a PET reconstruction.

(iv) An ultrasound B-scan. [40%]

(b) Why is it sometimes useful to combine a PET scan with a CT scan? [10%]

(c) Figure 1 shows two rectangular blocks of fat and a rectangular block of liver suspended in a water bath at room temperature. An ultrasound machine with a 5 MHz linear array probe is used to scan the contents of the water bath. The B-scan image produced by the ultrasound machine shows part of both blocks of fat, a cross-section of the block of liver, and part of the base of the water bath. Sketch this B-scan image and describe how it differs from an accurate representation of the geometry of the objects scanned. [50%]

You may take the properties of fat, liver and water to be as follows.

	sound speed	**specific acoustic impedance**	**attenuation factor**
fat	1450	1.38	0.63
liver	1604	1.79	0.7
water	1498	1.5	0.0022
units	$\mathrm{m\,s^{-1}}$	$10^6\,\mathrm{kg\,m^{-2}\,s^{-1}}$	$\mathrm{dB\,cm^{-1}\,MHz^{-1}}$

(cont.

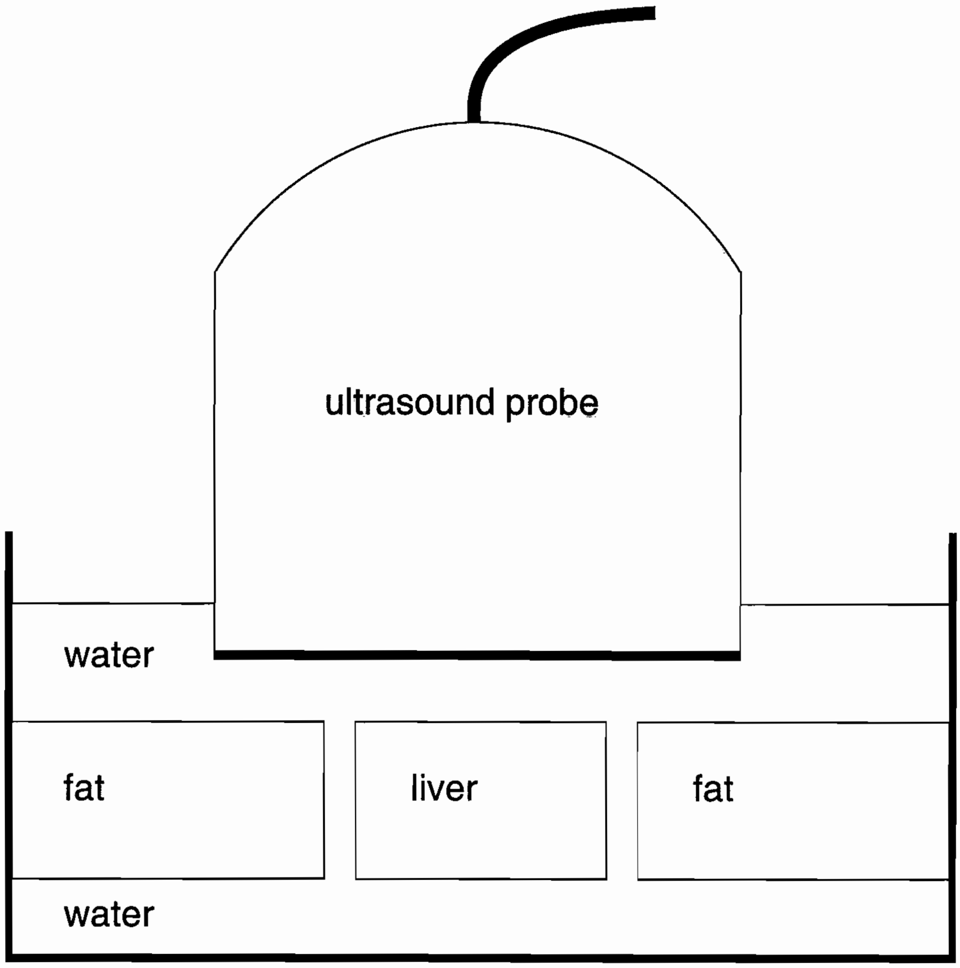

Fig. 1

(TURN OVER

2 (a) Draw a diagram of a photomultiplier tube and describe how it works. [35%]

(b) What characteristics of a photomultiplier tube make it suitable for use in nuclear medicine? [10%]

(c) A gamma camera is to be constructed with a collimator resolution of 2 mm at a distance 10 cm from the face of the collimator, and a collimator resolution of 5 mm at a distance 40 cm from the face of the collimator. Calculate the length and diameter of the holes required to form the collimator. [25%]

(d) A gamma camera has a collimator built in the shape of a circular disk. The disk contains 73 holes arranged symmetrically in four concentric circles (plus the central hole), as shown in Fig. 2. The holes are all of diameter 1 mm and depth 40 mm.

A point source emits radiation isotropically. It is located in line with the centre of the collimator and a distance 1 m away from it. Estimate the proportion of the radiation that is incident on the face of the collimator that reaches the detectors at the back of the holes. [30%]

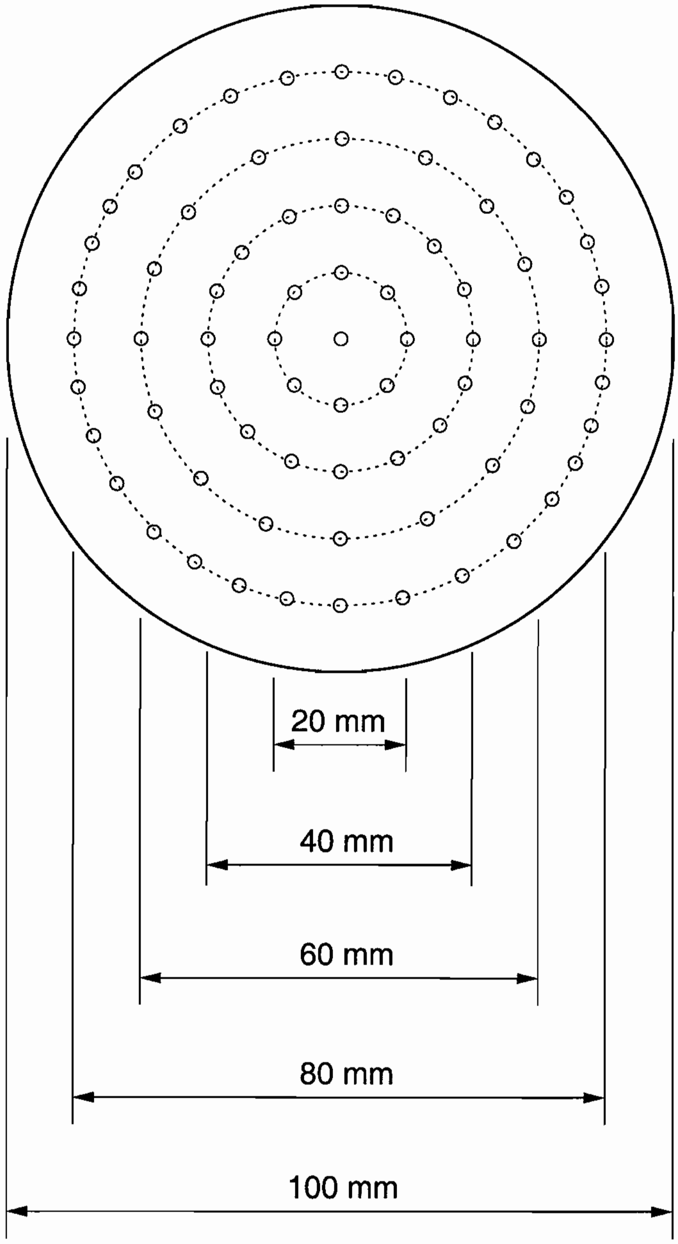

Fig. 2

(TURN OVER

SECTION B *Curves, Surfaces and Interpolation*

Answer not more than ***one*** *question from this section*

3 (a) Explain how the *marching cubes* algorithm can be used to extract a triangulated isosurface from a regular voxel array. [30%]

(b) How many different triangulation cases are there for each voxel? To how many can this be reduced using symmetry? [10%]

(c) Give an example of one case for which there are alternative triangulations. Explain, with the aid of sketches, why the choice is not arbitrary and the consequences of using the wrong triangulation. [20%]

(d) A surface, such as might be created with marching cubes, comprises N triangles with vertices $\mathbf{a}_i$, $\mathbf{b}_i$ and $\mathbf{c}_i$, $i \in \{1 \ldots N\}$. Derive an expression for the volume enclosed by the surface. What properties must the surface exhibit for this volume to be meaningful? [40%]

4 (a) Compare and contrast Catmull-Rom splines and B-splines. How can individual spline segments be joined together to make more complex curves? [20%]

(b) The basis matrices for the Catmull-Rom spline (M_{CR}) and the B-spline (M_B) are as follows:

$$M_{CR} = \frac{1}{2}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \quad M_B = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

For each type of spline, calculate the location, and the first and second parametric derivatives, at the $t = 1$ end of a segment with control points **a**, **b**, **c** and **d**. [20%]

(c) To terminate the curve in a more natural way, a designer wishes to add automatically an additional point after the last user-defined point. Figure 3 shows two possible locations for the additional point **d**, given the last three user-defined points **a**, **b** and **c**. In (a), **d** is coincident with **b**, whereas in (b), **b**, **c** and **d** are collinear and **d** is the same distance from **c** as **b** is from **c**.

(i) For both types of spline and both possible locations for **d**, calculate in terms of **a**, **b** and **c** (not **d**) the location, and the first and second parametric derivatives, at the $t = 1$ end of the curve. Sketch these curves. [40%]

(ii) Discuss the advantages of each option for each type of spline. [20%]

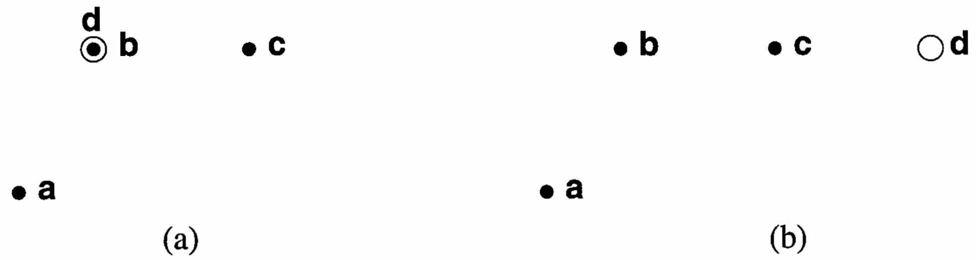

Fig. 3

SECTION C *3D Graphical Rendering*

Answer not more than **one** *question from this section*

5 (a) The basic Phong reflection model can be written as

$$I_\lambda = c_\lambda(I_a k_a + I_p k_d \mathbf{L}.\mathbf{N}) + I_p k_s (\mathbf{R}.\mathbf{V})^n$$

Explain the meanings of the various terms. Show that

$$\mathbf{R} = 2(\mathbf{L}.\mathbf{N})\mathbf{N} - \mathbf{L}$$ [40%]

(b) A sphere has unit radius and is positioned with its centre at the origin. The light source is at infinity in the direction $[1 \;\; -1 \;\; 1]^T$ and the viewer is at infinity in the direction $[0 \;\; 1 \;\; 0]^T$. Using the basic Phong reflection model given in (a), with $I_a = I_p = k_s = c_b = 1$, $k_a = k_d = 0.5$, $c_r = c_g = 0$ and $n = 10$, calculate the intensity of light reflected from the point $[0 \;\; 1/2 \;\; \sqrt{3}/2]^T$ on the surface of the sphere. [20%]

(c) Why might it be advantageous to replace **R**.**V** with **N**.**H**, where **H** is half way between **L** and **V**? [15%]

(d) In reality, illumination I and surface reflectivity k are continuous functions of wavelength λ. In its treatment of colour, in what way is the Phong model an approximation? What are the likely consequences of this approximation? How could the model's treatment of colour be improved? [25%]

6 (a) Why do surface rendering pipelines usually clip polygons to a view volume? Write down inequalities which define the view volume in homogeneous 3D screen coordinates. [20%]

(b) The vertices of two triangles A and B are defined in homogeneous 3D screen coordinates as follows:

$$A : \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix} \qquad B : \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 2 \end{bmatrix}$$

(i) State whether each vertex lies inside or outside the view volume. Why is this inside/outside test usually performed in homogeneous 3D screen coordinates? [20%]

(ii) In world coordinates, which of A and B has the larger area? How many pixels will each occupy in the final rendering? Justify your answers. [20%]

(iii) Work out the vertices of A after clipping. Express your answer in non-homogeneous 3D screen coordinates. [20%]

(c) Most surface rendering pipelines decompose arbitrary input polygons into triangles. List the benefits of triangle decomposition. Where in the pipeline does the decomposition usually take place? Why not right at the start of the pipeline? [20%]

END OF PAPER