1. (a) The decay reaction is

$$ {}^{210}Po \rightarrow {}^{206}Pb + {}^{4}He $$

$$ \Delta u = 209.98287 - 205.97447 - 4.00260 $$

$$ = 5.8 \times 10^{-3} $$

$\nwarrow$ datasheet p2

$$ 1\,u \equiv 931.5 \text{ MeV} \quad \text{(datasheet p 1)} $$

$\therefore$ energy of α particle is $\underline{\underline{5.403 \text{ MeV}}}$

Alpha particles have a limited range in air and will not penetrate skin (which is dead) so they only represent a health hazard if the first tissue encountered is live. Thus the source needs to be inhaled or ingested.

[20%]

(b) Number of ^{210}Po atoms in 1 g

$$ N = N_A \frac{m}{M} = 6.022 \times 10^{23} \times \frac{10^{-3}}{0.210} = 2.868 \times 10^{21} $$

$$ \lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{138.4 \times 24 \times 3600} = 5.797 \times 10^{-8} \text{ s}^{-1} $$

$$ \therefore A = \lambda N = 5.797 \times 10^{-8} \times 2.868 \times 10^{21} $$

$$ = \underline{\underline{1.663 \times 10^{14} \text{ Bq}}} $$

[10%]

(c) 30 days is a significant fraction of the half-life so assuming activity is constant is a poor approximation.

Activity varies as $A_0 e^{-\lambda t}$

$\therefore$ The total number of decays in 30 days

$$ = \int A\,dt \qquad \overset{\shortparallel}{2.592 \times 10^{6}} \text{ s} $$

$$ = \int_0^T A_0 e^{-\lambda t} \quad = \frac{A_0}{\lambda}\left[1 - e^{-\lambda T}\right] $$

1. (c) cont.

∴ # decays

from (b)

$$= \frac{1.663\times10^{14}}{5.797\times10^{-8}}\left(1 - \exp\{-5.797\times10^{-8}\times 2.592\times10^{6}\}\right)$$

$$= 4.002\times10^{20}$$

J/MeV

from (a)

∴ Energy deposited $= 4.002\times10^{20}\times 5.403\times 1.602\times10^{-1}$

$= 346.4\times10^{6}$ J

mass of man

∴ Absorbed dose $= 346.4\times10^{6} \div 70 = 4.949$ MGy

∴ Equivalent dose $= 20\times 4.949\times10^{6}$

weighting factor

$= \underline{\underline{98.98 \text{ MSv}}}$

This assumes that all the energy of the emitted alphas is deposited in tissue etc. There may be some self-shielding, but the form of the ^{210}Po is not specified so this effect cannot be estimated. [30%]

(d) The dose calculated in (c) is 10^{7} times larger than that needed to kill a man. So the hypothesis is certainly credible. A lethal dose would be received from much less than 1g or even if the ^{210}Po did not stay in the body for a month. The dose would be very localised rather than spread over the 70 kg of the whole organism, but an ingested or inhaled sample would pass close to several major organs. [15%]

(e) Non-stochastic effects are those in which the severity of the effects varies with the size of the dose and the effects occur within a very short time of exposure.

1. (e) cont.

Stochastic effects:

- the statistical risk of an effect (usually some form of cancer) is proportional to the dose but the severity of the risk is not
- estimation of this increased risk is difficult because cancers have long and variable latent periods and radiation-induced cancers cannot be distinguished from those due to other causes
- the fact that over 40% of the population in the western world die of cancer makes separate analysis of the nuclear risk almost impossible
- the only data available comes from the survivors of the atom bombs dropped on Japan and the records of workers exposed in known high radiation risk occupations; all this data is for relatively high exposures so extrapolation to the dose levels resulting from "everyday" exposures is difficult and uncertain
- genetic damage (radiation damage to reproductive cells causing mutations of the genes and possible hereditary defects) is another stochastic effect, albeit a debatable one - there is no conclusive evidence that hereditary damage has ever occurred in humans as a result of ionising radiation

[25%]

2.(a) $$\frac{dn}{dt} = -\nabla.\underline{j} + (\eta-1)\Sigma_a\,\phi + S$$

where $\underline{j} = -D\nabla\phi$

Source-free $\Rightarrow$ $S = 0$

Steady-state $\Rightarrow$ $\frac{dn}{dt} = 0$

Homogeneous $\Rightarrow$ D does not vary spatially

$$\therefore\ -\nabla.\underline{j} = -\nabla.(-D\nabla\phi) = D\nabla^2\phi$$

Hence $D\nabla^2\phi + (\eta-1)\Sigma_a\,\phi = 0$

For cylindrical geometry with rotational symmetry

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$$

$$\therefore\ \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{\partial^2\phi}{\partial z^2} + (\eta-1)\frac{\Sigma_a\phi}{D} = 0$$

Define $B_m^2 = (\eta-1)\frac{\Sigma_a}{D}$

$$\therefore\ \underline{\underline{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{\partial^2\phi}{\partial z^2} + B_m^2 = 0}}$$

[25%]

(b) Let $\phi = F(r)Z(z)$

$$\therefore\ \frac{Z}{r}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right) + F\frac{\partial^2 Z}{\partial z^2} + B_m^2 FZ = 0$$

$$\therefore\ \frac{1}{rF}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right) + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + B_m^2 = 0$$

$$\therefore\ \frac{1}{rF}\frac{d}{dr}\left(r\frac{dF}{dr}\right) + \alpha^2 = 0 \quad \text{and} \quad \frac{1}{Z}\frac{d^2Z}{dz^2} + \beta^2 = 0$$

where $\alpha^2 + \beta^2 = B_m^2$

Solution of the equation in z is

$$Z(z) = L\cos(\beta z) + M\sin(\beta z)$$

2. (b) cont.

By symmetry arguments $M = 0$

The equation in r is Bessel's Equation of 0^{th} order, so solution is

$$F(r) = P J_0(\alpha r) + Q Y_0(\alpha r)$$

As flux must be finite at $r = 0$ and $Y_0(0) = -\infty$

$Q = 0$

Boundary conditions

$\phi = 0$ at $z = H/2$ (neglecting extrapolation distance)

$\therefore \cos \frac{\beta H}{2} = 0 \Rightarrow \beta = \pi/H$

$\phi = 0$ at $r = R$ (same assumption)

$\therefore J_0(\alpha R) = 0 \Rightarrow \alpha = 2{\cdot}405/R$

$$\therefore B_g^2 = \alpha^2 + \beta^2 = \underline{\underline{\left(\frac{2{\cdot}405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2}}$$

[35%]

(c) $V = \pi R^2 H$

As $B_g^2 = B_m^2$ for criticality

$$\left(\frac{2{\cdot}405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = B_m^2$$

$$\therefore R^2 = \frac{2{\cdot}405^2}{B_m^2 - (\pi/H)^2}$$

$$\therefore V = \frac{\pi\, 2{\cdot}405^2 H}{B_m^2 - (\pi/H)^2}$$

Volume is minimised when $\frac{dV}{dH} = 0$

$$\frac{dV}{dH} = \pi\, 2{\cdot}405^2 \left\{ \frac{B_m^2 - (\pi/H)^2 + H\left(-\frac{2\pi^2}{H^3}\right)}{(B_m^2 - (\pi/H)^2)^2} \right\}$$

2. (c) cont.

∴ $\frac{dV}{dH} = 0$ when $B_m^2 - \frac{3\pi^2}{H^2} = 0$

∴ $H = \frac{\sqrt{3}\,\pi}{B_m}$

∴ $R^2 = \frac{2.405^2}{B_m^2 - \pi^2 . \frac{B_m^2}{3\pi^2}}$ ⇒ $R = \sqrt{\frac{3}{2}}\;\frac{2.405}{B_m}$

∴ $\frac{H}{R} = \frac{\pi\sqrt{2}}{2.405} = \underline{\underline{1.847}}$

[20%]

(d) $\eta = \frac{\nu\sigma_f}{\sigma_a} = \frac{\sum_i \nu_i p_i \sigma_{fi}}{\sum_i p_i(\sigma_{ci} + \sigma_{fi})}$

∴ Using data on pp 3&5 of 4A1 datasheet

$\eta = \frac{2.43 \times 0.025 \times 580}{0.025(107 + 580) + 0.975 \times 2.75} = 1.775$

[denominator = 19.86 barns]

Number of uranium atoms per unit volume

$N_u = \frac{\rho}{A} \times 0.05$ A = mass of uranium atom

$\rho = 18900$ kg m^{-3} (from p3)

$= \frac{18900 \times 0.05}{238 \times 1.661 \times 10^{-27}}$

$= 2.390 \times 10^{27}$ m^{-3}

∴ $\Sigma_a = N_u \sigma_a = 2.39 \times 10^{27} \times 19.86 \times 10^{-28} = 4.747$ m^{-1}

D for graphite = 0.0094 m (from p 7)

∴ $B_m^2 = (\eta - 1)\frac{\Sigma_a}{D} = \frac{0.775 \times 4.747}{0.0094} = 391.4$ m^{-2}

∴ $B_m = 19.8$ m^{-1}

∴ $H = \sqrt{3}\,\pi / B_m = 0.275$ m

$R = H / 1.847 = 0.149$ m

$V = \pi R^2 H = \underline{\underline{0.0192 \text{ m}^3}}$

[20%]

3. (a) The sinusoidal term reflects the variation in coolant temperature along the channel. This depends on the total amount of heat transferred from the fuel and thus on the integral of the cosinusoidal power distribution.

The cosinusoidal term reflects the temperature difference between the coolant and the location in question. This depends on the total thermal resistance and the local power density which varies cosinusoidally. [15%]

(b) $$\Theta = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

Θ is a maximum when $\frac{d\Theta}{dx} = 0$

$$\therefore \frac{\pi}{2L'}\cos\left(\frac{\pi x}{2L'}\right) - \frac{\pi Q}{2L'}\sin\left(\frac{\pi x}{2L'}\right) = 0$$

$$\therefore \underline{\underline{x = \frac{2L'}{\pi}\tan^{-1}\frac{1}{Q}}}$$

Let $\tan^{-1}\frac{1}{Q} = \phi$

Maximum value of $\Theta = \sin\phi + Q\cos\phi$

$$\therefore \frac{\Theta_{max}}{\cos\phi} = \tan\phi + Q = \frac{1}{Q} + Q = \frac{1+Q^2}{Q}$$

$$\therefore \frac{\Theta_{max}^2}{\cos^2\phi} = \frac{(1+Q^2)^2}{Q^2}$$

Now $\sin^2\phi + \cos^2\phi = 1$

$$\therefore \frac{1}{\cos^2\phi} = 1 + \tan^2\phi = 1 + \frac{1}{Q^2}$$

$$\therefore \Theta_{max}^2\left(1 + \frac{1}{Q^2}\right) = \frac{(1+Q^2)^2}{Q^2}$$

3. (b) cont.

$$\therefore \Theta_{max}^2 \left(\frac{1+Q^2}{Q^2} \right) = \frac{(1+Q^2)^2}{Q^2}$$

$$\therefore \underline{\underline{\Theta_{max}^2 = 1 + Q^2}}$$

[30%]

(c) (i) To find the coolant outlet temperature (the maximum coolant temperature) use overall energy balance

$$P = \dot{m}c_p (T_{co} - T_{ci})$$

$$\therefore T_{co} = T_{ci} + P/(\dot{m}c_p)$$

$$= 325 + 5\times10^6/18\times10^3 = \underline{\underline{602.8\,°C}}$$

(ii) For the cladding outer surface

$$\frac{1}{U} = \frac{1}{h} \Rightarrow U = h = 9 \text{ kW m}^{-2}\text{K}^{-1}$$

$$Q = \frac{\pi \dot{m}c_p}{UA} \frac{L}{L'}$$

$L = 3$ m

$L' = 4$ m

$$A = 4\pi r_o L = 4\pi(\underset{\uparrow \text{ pellet radius}}{7.5} + \underset{\uparrow \text{ cladding thickness}}{1})\times10^{-3} \times 3 = 0.320 \text{ m}^2$$

$\dot{m}c_p$ <u>per fuel pin</u> $= 18 \div 36 = 0.5$ kW K^{-1}

$$\therefore Q = \frac{\pi \times 0.5\times10^3 \times 3}{U\times 0.320\times 4} = \frac{3682}{U}$$

So in this case $Q = \dfrac{3682}{9\times10^3} = 0.409$

$$\therefore \Theta_{max} = (1+Q^2)^{1/2} = 1.080$$

$$\Theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right)$$

$$T_{c1/2} = \frac{1}{2}(325 + 602.8) = 463.9\,°C$$

3. (c) (ii) cont.

$$\therefore \quad T = T_{c\frac{1}{2}} + \theta \frac{(T_{co} - T_{c\frac{1}{2}})}{\sin(\pi L/2L')}$$

$$= 463{\cdot}9 + 1{\cdot}08 \times \frac{(602{\cdot}8 - 463{\cdot}9)}{\sin(3\pi/8)}$$

$$= \underline{\underline{626{\cdot}3\,^\circ C}}$$

(iii) For fuel centre-line

$$\frac{1}{U} = \frac{1}{h} + \frac{t_c}{\lambda_c} + \frac{r_o}{2\lambda_f}$$

$$= \frac{1}{9 \times 10^3} + \frac{10^{-3}}{15} + \frac{8{\cdot}5 \times 10^{-3}}{2 \times 2{\cdot}7}$$

$$\therefore \quad U = 570{\cdot}8 \quad W\,m^{-2}\,K^{-1}$$

$$\therefore \quad Q = 3682/570{\cdot}8 = 6{\cdot}451$$

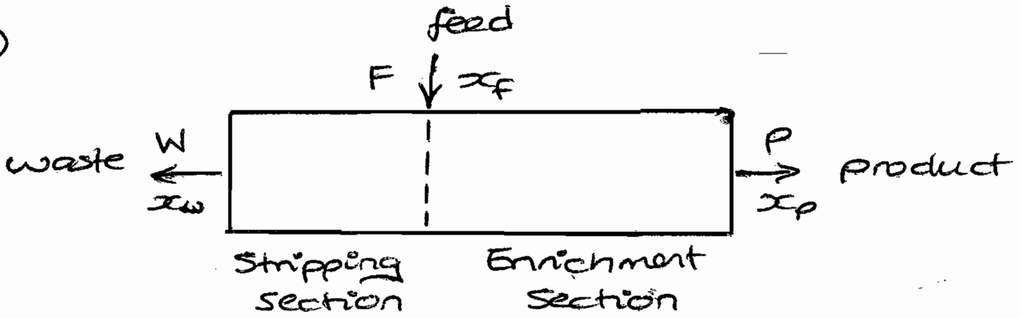
$$\therefore \quad \theta_{max} = (1 + Q^2)^{1/2} = 6{\cdot}528$$

$$\therefore \quad T = 463{\cdot}9 + 6{\cdot}528 \times \frac{(602{\cdot}8 - 463{\cdot}9)}{\sin(3\pi/8)}$$

$$= \underline{\underline{1445{\cdot}3\,^\circ C}}$$

[55%]

4. (a)



$$\text{Cost} = C_f F + C_s \left[P\upsilon(x_p) + W\upsilon(x_w) - F\upsilon(x_f) \right]$$

Overall mass balance $\quad P + W = F \qquad (1)$

U_{235} mass balance $\quad x_p P + x_w W = x_f F$

$x_w \times (1) \quad x_w P + x_w W = x_w F$

subtracting $\quad P(x_p - x_w) = (x_f - x_w) F$

$$\therefore \; F = \frac{x_p - x_w}{x_f - x_w} P$$

$$\text{Similarly} \quad W = \frac{x_p - x_f}{x_f - x_w} P$$

$$\therefore \text{Cost} = C_f \frac{(x_p - x_w)}{(x_f - x_w)} P + C_s \left[P\upsilon(x_p) + \frac{(x_p - x_f)}{(x_f - x_w)} P\upsilon(x_w) - \frac{(x_p - x_w)}{(x_f - x_w)} P\upsilon(x_f) \right]$$

$$\therefore \frac{\text{Cost}}{P} = C_f \frac{(x_p - x_w)}{(x_f - x_w)} + C_s \left[\upsilon(x_p) + \frac{(x_p - x_f)}{(x_f - x_w)} \upsilon(x_w) - \frac{(x_p - x_w)}{(x_p - x_w)} \upsilon(x_f) \right]$$

[30%]

(b) If the cost of the feed is negligible there is no economic benefit in running a stripping section. Therefore $x_w \approx x_f$.

If the change in x is small, $\upsilon(x)$ can be

4. (b) cont.

approximated by a linear function near x_f using the first two terms of a Taylor series

$$v(x_f + \delta x) = v(x_f) + \delta x \left.\frac{dv}{dx}\right|_{x_f}$$

Hence if $x_f + \delta x = x_w$

$$\delta x = x_w - x_f$$

and $v(x_w) = v(x_f) + (x_w - x_f) \left.\frac{dv}{dx}\right|_{x_f}$

Thus if $c_f \approx 0$

$$\frac{\text{Cost}}{P} = c_s \left[v(x_p) + \frac{(x_p - x_f)}{(x_f - x_w)} \left\{ v(x_f) - (x_f - x_w) \left.\frac{dv}{dx}\right|_{x_f} \right\} - \frac{(x_p - x_w)}{(x_f - x_w)} v(x_f) \right]$$

$$= c_s \left[v(x_p) + v(x_f) \left\{ \frac{x_p - x_f}{x_f - x_w} - \frac{x_p - x_w}{x_f - x_w} \right\} - (x_p - x_f) \left.\frac{dv}{dx}\right|_{x_f} \right]$$

$$= \underline{\underline{c_s \left[v(x_p) - v(x_f) - (x_p - x_f) \left.\frac{dv}{dx}\right|_{x_f} \right]}}$$

[40%]

(c) From (a) $\dfrac{F}{P} = \dfrac{x_p - x_w}{x_f - x_w}$

and $\dfrac{W}{P} = \dfrac{x_p - x_f}{x_f - x_w}$

For $x_f = 0.00715$, $x_p = 0.032$, $x_w = 0.0021$ $\Big\}$ $F/P = 5.92$, $W/P = 4.92$

$$\therefore \frac{x_w W}{x_p P} = \frac{0.0021}{0.032} \times 4.92$$

$$= \underline{\underline{0.323}}$$

[10%]

4 (d) For $\left.\begin{array}{l} x_f = 0.00715 \\ x_p = 0.032 \\ x_w = 0.0236 \end{array}\right\} \quad \begin{array}{l} F/P = 6.188 \\ W/P = 5.188 \end{array}$

If the plant configuration cannot be changed the amount of separative work must remain the same

$$\therefore \quad P_o\left[v(x_p) + \frac{W_o}{P_o} v(x_{wo}) - \frac{F_o}{P_o} v(x_f)\right] =$$

$$P_1\left[v(x_p) + \frac{W_1}{P_1} v(x_{w1}) - \frac{F_1}{P_1} v(x_f)\right]$$

where subscript 0 = original case
1 = new case

For small x $\quad v(x) \approx -\ln x$ $\quad$ (datasheet p7)

$P_1 / P_o = 1.065$

$$\therefore \quad \left[-\ln(0.032) + 4.92(-\ln(0.0021)) - 5.92(-\ln(0.00715))\right]$$

$$= 1.065\left[-\ln(0.032) + 5.188(-\ln(0.00236)) - 6.188(-\ln(0.00715))\right]$$

$$\therefore \quad 4.529 = 4.528$$

near enough the same [20%]