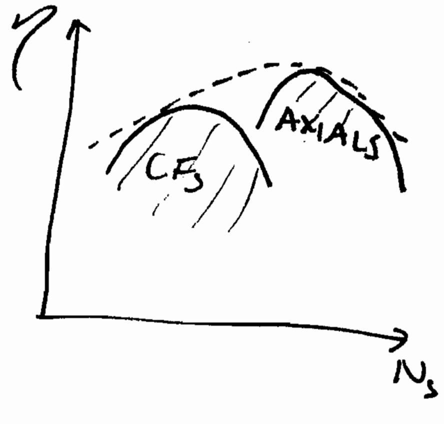
4A3: TURBOMACHINERY I
MAY 2007

Radial turbomachinery

Advantages: Higher pressure ratio per stage
Low cost
Reduced complexity (single cast compone
More compact.

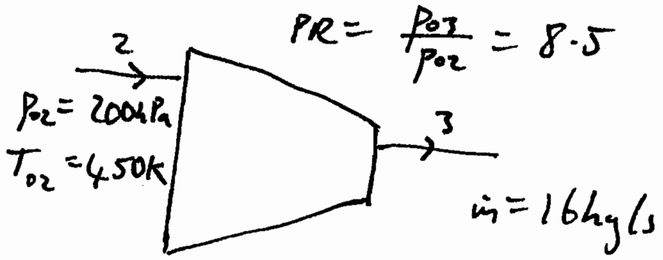
Disadvantages: High losses (particularly in diffuser/scr
leading to lower peak efficiency.
Higher frontal area – leads to larger engine diameter for aero-engine applica
Can be high weight.

Modern jet engines tend to require high mass flow rates and maximum efficiency. Thus, axial turbomachin with high specific speed and high performance are used with multiple stages needed to reach high pressure ratios.



b) i)

$PR = \frac{p_{03}}{p_{02}} = 8.5$



$$\eta_{is} = \frac{PR^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_{03}}{T_{02}} - 1} = \frac{PR^{\frac{\gamma-1}{\gamma}} - 1}{PR^{\frac{\gamma-1}{\gamma\eta_p}} - 1}$$

$$\therefore \eta_{is} = \frac{8.5^{0.4/1.4} - 1}{8.5^{\frac{0.4}{0.91 \cdot 1.4}} - 1} = 0.880$$

$$\underline{\underline{\eta_{is} = 88\%}}$$

1. b) contd.

i) $$\frac{T_{03}}{T_{02}} - 1 = \frac{PR^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{is}}$$

$$\Rightarrow \quad T_{03} = \left\{ \frac{PR^{\frac{\gamma-1}{\gamma}} - 1}{\eta_{is}} + 1 \right\} T_{02} = \left\{ \frac{8.5^{\frac{0.4}{1.4}} - 1}{0.88} + 1 \right\} 450$$

$$\underline{T_{03} = 881\,K}$$

$$N_{stage} \geq \frac{\Delta h_0}{\psi_{max} U^2} = \frac{c_p (T_{03} - T_{02})}{0.4\ U^2} = \frac{1005\,(881 - 450)}{0.4 \cdot 377^2}$$

$$= \underline{7.61}$$

$$U = \frac{15000}{60} \times 2\pi \times 0.24 = \underline{377\,m/s}$$

$$N_{stage} > 7.61 \quad \text{for} \quad \psi_{max} = 0.4$$

$\therefore$ $\underline{\underline{\text{No. of stages} = 8}}$, $\quad \psi = \dfrac{1005\,(881-450)}{8 \cdot 377^2}$

$$= \underline{0.381}$$

ii) $M_{IN} = 0.52 \quad \therefore \quad \dfrac{V_{x,IN}}{\sqrt{c_p T_{02}}} = 0.3203 \quad$ (Data book)

$$\Rightarrow V_{x,IN} = 0.3203 \cdot \sqrt{1005 \cdot 450} = 215.4$$

$$\phi = \frac{V_{x,IN}}{U} = \frac{215.4}{377} = \underline{\underline{0.571}}$$

1. b) contd.

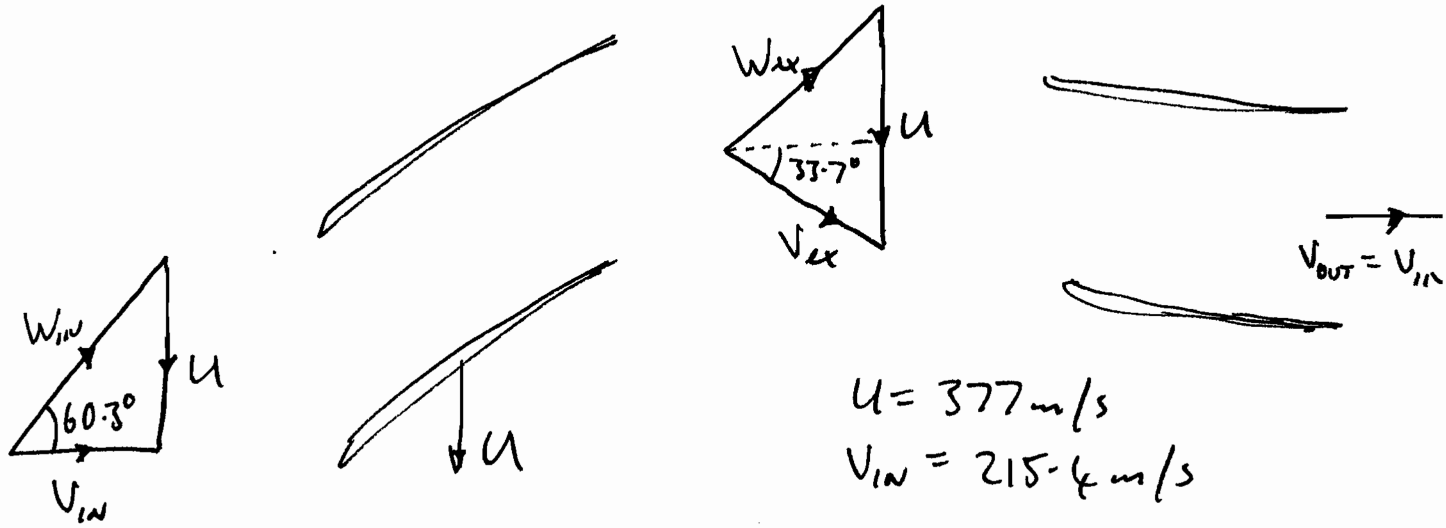
ii) Velocity $\triangle$s:

$$\alpha_{IN} = 0^\circ \qquad \psi = 0.381 = \frac{\Delta V_{\theta,rotor}}{U}$$

$$\phi = \frac{V_x}{U} = 0.571$$

$$\Rightarrow \alpha_{IN,rel} = -\tan^{-1}\left(\frac{1}{0.571}\right) = -60.3^\circ \qquad \tan(\alpha_{ex,rel}) = \left(\frac{0.381-1}{0.571}\right)$$

$$\alpha_{ex} = \tan^{-1}\left(\frac{0.381U}{0.571U}\right) = 33.7^\circ \qquad \alpha_{ex,rel} = -47.3^\circ$$



$U = 377$ m/s
$V_{IN} = 215.4$ m/s

At exit from compressor, $\phi = 0.571$ (repeating stages)

$$\therefore \frac{V_{x3}}{\sqrt{c_p T_{03}}} = \frac{215.4}{\sqrt{1005.881}} = 0.229$$

$$\Rightarrow M_{x3} \simeq 0.37 \text{ (data book)}$$

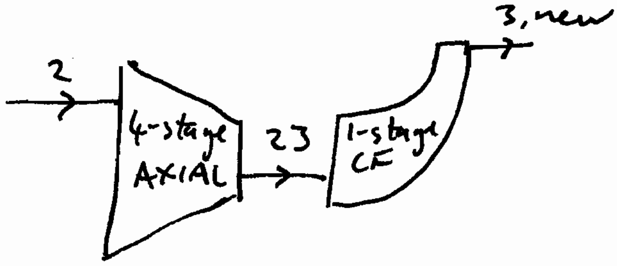
[Velocities stay the same through compressor, so Mach numbers reduce]

$$\frac{\dot{m}\sqrt{c_p T_{03}}}{A_3 p_{03}} = Q(0.37) = 0.7553$$

$$\therefore A_3 = \frac{16 . \sqrt{1005.881}}{0.7553 . 8.5 . 200} = 0.01172 = 2\pi \bar{r} h_3$$

$$\therefore h_3 = \frac{0.01172}{2\pi\, 0.24} = \underline{7.8 \text{ mm}}$$

1. c)



$$\frac{p_{03,new}}{p_{02}} = 8.5$$

For previous design,

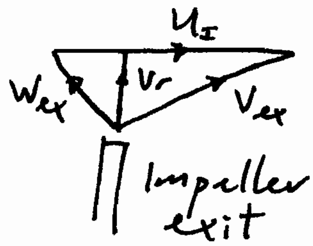
$$h_{03} - h_{023} = 4\psi_{av}\bar{U}^2 \qquad \text{[Location 23 is unchanged}$$

$$\Rightarrow \quad T_{023} = T_{03} - \frac{4\psi_{av}\bar{U}^2}{c_p} = 881 - \frac{4.0.381.377^2}{1005}$$

$$= \underline{665.5K}$$

For CF compressor

$$\Delta h_0 = \Delta(U_I V_\theta) \text{ — for impeller}$$



$V_{\theta IN} = 0$ — no swirl at inlet

$V_{\theta ex} = \sigma U_I$

$$\Rightarrow \Delta h_0 = c_p(T_{03,new} - T_{023}) = \sigma U_I^2 = \sigma\bar{U}^2\left(\frac{r_I}{\bar{r}}\right)^2$$

mean radius in axial compressor

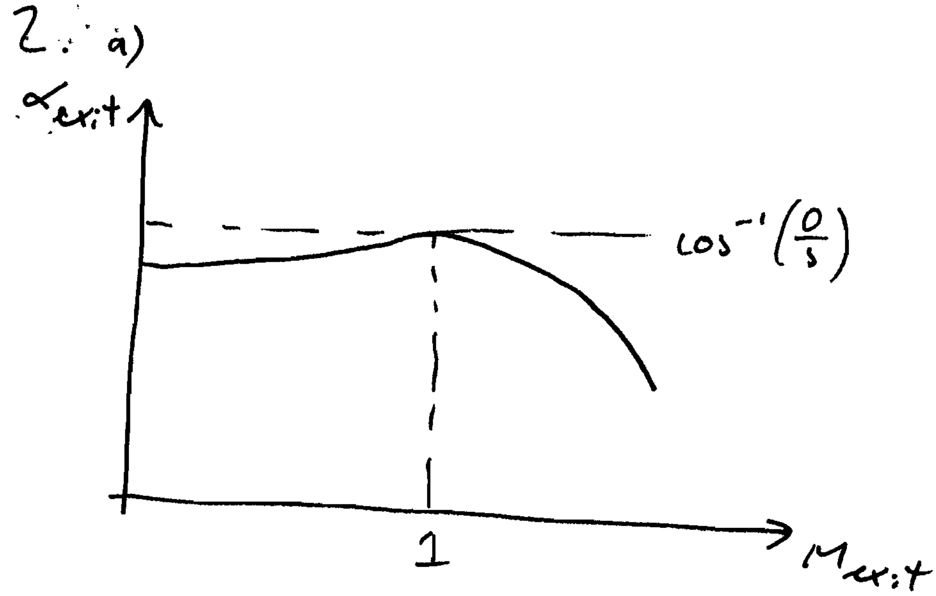
$$T_{03,new} = \frac{\sigma\bar{U}^2}{c_p}\left(\frac{r_I}{\bar{r}}\right)^2 + T_{023}$$

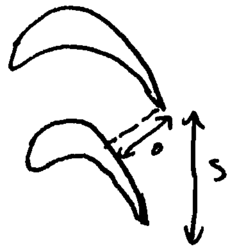
$$T_{03,new} = \frac{0.86.377^2}{1005}\cdot\left(\frac{0.34}{0.24}\right)^2 + 665.5 = \underline{\underline{909.6K}}$$

$$\eta_{is,ov} = \frac{PR^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_{03,new}}{T_{023}} - 1} = \frac{8.5^{\frac{0.4}{1.4}} - 1}{\frac{909.6}{450} - 1} = \underline{\underline{82.5\%}}$$

i.e. η_{is} falls by over 5% with CF compressor

2. a)





For supersonic flow, turbine passage is convergent-divergent nozzle. For the flow to diverge, and the area increase, the exit angle must reduce.

Limit to exit Mach number when $M_{axial} = 1$. At this point pressure waves cannot travel upstream and thus M_{exit} and turbine flow cannot be changed further.

b)

$$\frac{p_{01}}{p_2} = 2.6 \qquad \alpha_1 = -45^\circ$$

$$Y_p = \frac{p_{01} - p_{02}}{p_{02} - p_2} = \frac{\frac{p_{01}}{p_2} - \frac{p_{02}}{p_2}}{\frac{p_{02}}{p_2} - 1}$$

$$Y_p\left(\frac{p_{02}}{p_2} - 1\right) = \frac{p_{01}}{p_2} - \frac{p_{02}}{p_2}$$

$$\therefore \frac{p_{02}}{p_2} = \frac{\left(\frac{p_{01}}{p_2} + Y_p\right)}{Y_p + 1} = \frac{2.6 + 0.098}{1.098} = 2.457$$

$$\therefore \underline{\underline{M_2 = 1.21}} \quad \text{(data book)}$$

2. b) contd.

Continuity.

At inlet, $\dfrac{\dot{m}\sqrt{c_pT_{01}}}{A\,p_{01}} = \dfrac{\dot{m}\sqrt{c_pT_{01}}}{hs\cos\alpha_1\,p_{01}} = Q(M_1)$

At exit, $\dfrac{\dot{m}\sqrt{c_pT_{01}}}{hs\cos\alpha_2\,p_{02}} = Q(M_2)$ $\quad [T_{02} = T_{01}]$

$$\Rightarrow Q(M_1)\,p_{01}\cos\alpha_1 = Q(M_2)\,p_{02}\cos\alpha_2$$

$$\cos\alpha_2 = \frac{Q(M_1)}{Q(M_2)}\cdot\frac{p_{01}}{p_{02}}\cos\alpha_1$$

$$= \frac{Q(0.3)}{Q(1.21)}\cdot\frac{2.6}{2.457}\cdot\cos(-45^\circ)$$

$$= \frac{0.6295}{1.2396}\cdot\frac{2.6}{2.457}\cdot\frac{1}{\sqrt{2}} = 0.380$$

$$\underline{\underline{\alpha_2 = 67.7^\circ}}$$

c) $o/s = 0.354$

At throat, $\dfrac{\dot{m}\sqrt{c_pT_{01}}}{h\,o\,p_{0T}} = Q(1)$

At exit, $\dfrac{\dot{m}\sqrt{c_pT_{01}}}{hs\cos\alpha_2\,p_{02}} = Q(M_2)$

$$\Rightarrow Q(1).o.\,p_{0T} = Q(M_2)\,s\cos\alpha_2\,p_{02}$$

2: b) contd

$$\Rightarrow \quad \frac{p_{02}}{p_{0T}} = \frac{Q(1)}{Q(1.21)} \cdot \frac{o}{s} \cdot \frac{1}{\cos 67.7^\circ}$$

$$= \frac{1.281}{1.2396} \cdot 0.354 \cdot \frac{1}{0.380} = 0.9627$$

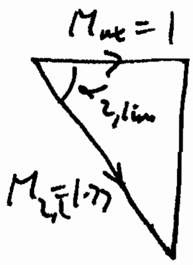
Proportion of loss downstream of throat $= \dfrac{p_{0T} - p_{02}}{p_{01} - p_{02}}$

$$= \frac{\frac{p_{0T}}{p_{02}} - 1}{\frac{p_{01}}{p_{02}} - 1}$$

$$= \left(\frac{1}{0.9627} - 1\right) \Big/ \left(\frac{2.6}{2.457} - 1\right) = \underline{\underline{0.666}}$$

i.e. 66% of the loss occurs downstream of the throat

d) At limit load, $M_{ex} = 1.77$



$$\cos \alpha_{2,lim} = \frac{1}{1.77} \quad \Rightarrow \underline{\alpha_{2,lim} = 55.6^\circ}$$

Use subscript 'l' or 'lim' for limit-load condition.

Continuity between throat and exit:

$$Q(1)\, h.o.\, p_{0T} = Q(M_{2,l})\, h\, s \cos\alpha_{2,l} \cdot p_{02,l}$$

$$\frac{p_{02,lim}}{p_{0T}} = \frac{Q(1)}{Q(M_{2,l})} \cdot \frac{o}{s} \cdot \frac{1}{\cos\alpha_{2,l}} = \frac{1.281}{0.9104} \cdot 0.354 \cdot 1.77$$

$$\frac{p_{02,lim}}{p_{0T}} = 0.8816$$

2: d) contd.

$$Y_{p,l} = \frac{p_{01} - p_{02,l}}{p_{02,l} - p_{2l}} = \frac{\frac{p_{01}}{p_{02,l}} - 1}{1 - \frac{p_{2l}}{p_{02,l}}}$$

$$\frac{p_{01}}{p_{02l}} = \frac{p_{01}}{p_{0T}} \times \frac{p_{0T}}{p_{02,l}} = \left(\frac{p_{01}}{p_{02}} \times \frac{p_{02}}{p_{0T}}\right) \times \frac{p_{0T}}{p_{02,l}}$$

[$\frac{p_{01}}{p_{0T}}$ is unchanged from previous condition]

$$\therefore \quad \frac{p_{01}}{p_{02,l}} = \left(\frac{2.6}{2.457} \times 0.9627\right) \times \frac{1}{0.8816} = 1.1555$$

$$Y_{p,\text{lim}} = \frac{1.1555 - 1}{1 - 0.1822} = \underline{\underline{0.19}} \quad \left(\text{i.e. about } 2\times \text{ greater}\right)$$

e) Loss is higher at 'limit load' due to stronger shocks and greater blockage from the trailing edge flow, which causes a drag force on the blade. Although a turbine with high Mach numbers tends to have ~~to~~ greater losses, the pressure ratios and stage loadings can be higher, leading to fewer stages in a multi-stage turbine

$$\psi = 2(1 - \Lambda - \phi \tan\alpha_1)$$

Λ can be lower, and α_1 more $-$ve (interstage swirl)

$[\Delta h_0 = O(UV_\theta)]$ However, U can also be higher giving greater Δh_0.

3. a) Bookwork

$F_G = \dot{m} V_j$

$F_N = \dot{m} V_j - \underbrace{\dot{m} V_{in}}_{\text{ram drag}}$

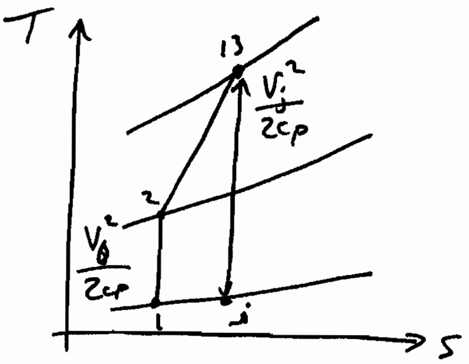
The ram drag is the momentum of the air entering the engine.

$$\eta_P = \frac{\text{Power to Aircraft}}{\Delta KE} = \frac{F_N V_0}{\frac{1}{2}\dot{m}(V_j^2 - V_0^2)} = \frac{\dot{m}(V_j - V_0)V_0}{\frac{1}{2}\dot{m}(V_j + V_0)(V_j - V_0)}$$

$$\eta_P = \frac{2V_0}{V_j + V_0}$$

b) $\frac{T_{02}}{T_2} = \left(1 + \frac{\gamma-1}{2} M^2\right) = 1.1445$, $T_{02} = 255.6K$

$p_{02} = p\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} = 42.5 kPa$



$V_0 = 0.85\sqrt{\gamma R T_a} = 254.7 m/s$

$p_{013} = p_{02} \times FPR = 42.5 \times 1.23$
$= 52.27 kPa$

$$\frac{T_{013}}{T_{02}} = \left(\frac{p_{013}}{p_{02}}\right)^{\frac{\gamma-1}{\eta_{poly}\gamma}} = (1.23)^{\frac{0.4}{0.9 \cdot 1.23}} \Rightarrow T_{013} = 273.0K$$

$$\frac{T_{013}}{T_j} = \left(\frac{p_{013}}{p_j}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{52.27}{26.5}\right)^{\frac{\gamma-1}{\gamma}} = 1.214$$

$$\frac{1}{2}V_j^2 = c_p T_{013}\left(1 - \frac{T_j}{T_{013}}\right) = 1005 \cdot 273\left(1 - \frac{1}{1.214}\right) = 4.76 \times 10^4$$

Propulsive efficiency, $\eta_p = \frac{254.7 \times 2}{254.7 + 308.6} = \underline{0.90}$

c) Work balance:

$$\dot{m}_c C_{pc} \Delta T_{0,LPT} = \dot{m}_c c_p \Delta T_{0,\text{CORE COMPRESSOR}} + BPR \cdot \dot{m}_c c_p \Delta T_{0,FAN} \quad -①$$

Core compressor:

$$\frac{T_{03}}{T_{02}} = \left(\frac{p_{03}}{p_{02}}\right)^{\frac{\gamma-1}{\eta_{poly}\gamma}} = (2.5)^{\frac{0.4}{0.9 \cdot 1.4}} = 1.34$$

$$T_{03} = 341.9K$$

fan: $\Delta T_{0,FAN} = 273 - 255.6 = 17.4K.$

LPT:
$$\Delta T_{0,LPT} = 390 - 0.9 \times \frac{V_j^2}{2c_{pc}} = 390 - \frac{0.9 \times 4.76 \times 10^4}{1250}$$
$$= \underline{355.2K}$$

From ①,

$$1250 \cdot 355.2 = 1005(341.9 - 255.6) + BPR(17.4) \cdot 100$$

$$\Rightarrow \underline{\underline{BPR = 20.4}} \quad \text{(i.e. very high)}$$

d) $F_N = \dot{m}(V_j - V_0) - F_D$, $F_D = 2.5kN$

$$\eta_{P,NEW} = \frac{\dot{m}(V_j - V_0)V_0 - F_D V_0}{\frac{1}{2}\dot{m}(V_j - V_0)(V_j + V_0)} = \frac{2V_0}{V_j + V_0} - \frac{2F_D V_0}{\dot{m}(V_j^2 - V_0^2)}$$

$$= \frac{2 \times 254.6}{311.1 + 254.6} - \frac{2 \times 2500 \times 254.6}{600(311.1^2 - 254.6^2)}$$

$$\underline{\underline{\eta_{P,NEW} = 0.834}}$$

A BPR of 20.4 is very high and the high weight of the engine and nacelle would increase fuel burn further. The engine may also have trouble fitting under the wing.